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Flight path optimization for an airplane

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Problem description

Background and objective

It is recognized that the fuel burn of the aircraft (with powerplants) has to be incorporated in the flight trajectory analysis. In addition, the simplified cruise description should include an increase of the flight Mach number with altitude for a beneficial interaction of kinetic and potential energy.

Hence, the ultimate goal is to reduce the fuel cost and emission particles by an analysis and convert this into a computer code for a parametric study of the appropriate trajectory and aircraft variables.

The purpose of the present simulation study is twofold, i.e.

- Give a simple analytical basis for a parametric variation of selected parameters defining the climb or descent of a representative flight of an airliner.
- Give a focus on a positive flight trajectory (for a greener sky)

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Academic supervisor: Per-Åge Krogstad
Supervisor: Helge Nørstrud

Abstract

The diminution of the fuel consumption during the flight trajectory has an impact on the cost of the travel and answers to the ecologic challenge “Green Sky”. The analysis has for objective to optimize the flight trajectory of the aircraft in order to reduce the fuel consumption. The flight trajectory is defined by a simplified description and depends on some parameters which affect the different phases of the trajectory. The flight description is introduced in a computer code and the different parameters vary in order to define their influence on the fuel consumption. The results which are obtained show the influence of the times of climb and descent and the cruise altitude on the fuel consumption. The variation according to the defined configuration is in the order of few percent. Today, all the few kilograms of fuel which are saved are important. The different phases of the flight trajectory have to be optimized to reduce the fuel consumption.

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Flight path optimization for an airplane

By Dorothée Merle

Introduction

During the last 20 years, the price of the barrel has risen sharply and this augmentation does not seem to stop. The fuel increase accelerates the process of fleet replacement to less polluting models. Today the aircraft self life is 20 years instead of 30 years.
For many years, airlines around the world have seen their fuel bills rise, as the price of crude oil has continued to rise steadily. For the flight companies, the fuel cost represents 30% of the operating costs. The flight companies want to reduce the flight costs, consequently they look for aircrafts more economics and eco-responsible. The demand of the flight companies to reduce the fuel consumption and the difficult in this economically critical period to go into a new aircraft actuate the aircraft constructors to modernize the engine range.
The engine builders propose new engines, like the group CFM international which present the LEAP-X engine, lighter, durability, temperatures extremely high. Pratt&Whitney developed the geared turbofan technique (GTF) which is endowed with speed reducer between the fan and the low pressure compressor, each component running at its optimal speed and improve the reactor performances: reduction of the fuel consumption.

Methods are developed to reduce the fuel consumption, like the control of the extern surfaces of the aircraft to reduce the aerodynamic drag or the cleaning water of the reactor or the reduction of the fuel capacity at takeoff.
The flight trajectory operations are modified. The takeoff phase of flight is the most fuel consuming per unit time and the operation is realised at maximum power for the aircraft engine until the cruise altitude. The maximum power is not necessary; consequently the fuel consumption decreases if the engine power is reduced. For the takeoff phase, the angle of the flaps setting is reduced, the acceleration and the flap retraction are made at lower altitude than the typically 3000 feet. The drag decreases and the aircraft efficiency augments.
During the climb phase, after the flap retraction, the aircraft accelerates to the climb speed the most economic. In general, the aircraft turn around the airport before to land. The flight companies look for reduce the landing phase and to have a straight down landing.

The objective of the optimization of the fuel consumption is double. The reduction of the fuel consumption makes it possible to improve the environmental reputation of aviation. The contribution of the aviation to the production of the main greenhouse gas CO_2 is moderate but the augmentation of airline traffic encourages the airline companies to reduce the fuel consumption.

Symbols

a	Acceleration [m/s²]
α	Parameter defining the dimensional velocity [s/m]
β	The velocity/altitude parameter [-]
c	Speed of sound [m/s]
C_D	Drag coefficient [-]
$C_{D,0}$	Drag coefficient at zero lift [-]
$C_{D,i}$	Induced drag coefficient [-]
CI	Cost Index [kg/min]
C_L	Lift coefficient [-]
D	Aerodynamic drag [N]
d_H	Horizontal distance [m]
ε	Constant in the density function [1/m]
F	Acceleration force [N]
g	Acceleration of gravity [m/s²]
G	Gravity force [N]
γ	Parameter for the climb/descent time determination [-]
L	Lift [N]
l_f	Fuel capacity [l]
M	Mach number [-]
m	Weight of the aircraft [kg]
m_f	Fuel weight [kg]
m_{fb}	Fuel burn mass [kg]
$\dot{m}_{fb}$	Fuel mass flow [kg/h]
n	Number of increment [-]
ρ	Air density [kg/m^3]
ρ_0	Air density at sea level [kg/m^3]
ρ_f	Fuel density [kg/m^3]
S	Wing area [m²]
SFC	Specific fuel consumption [(kg/h)/N]
T	Required thrust [N]
t	Increment time [h]
θ	Climb/descent angle [rad]
τ	Time [h]
v	Vertical speed [m/s]
V	Velocity along the flight path [m/s]
z	Geometric altitude [m]
Z	Dimensionless altitude [-]
z_C	Cruise altitude [m]

Subscripts :

cl	refers to the climb phase
c	refers to the cruise phase
d	refers to the descent phase
mean	refers to the mean value
D	refers to the drag
L	refers to the lift

1. Analytical analysis

1.1. Fuel consumption

The previous years, the Boeing and the Airbus companies competed to find the optimal aircraft. One of the last technologies found is the winglets. The intended effect is to reduce the aircraft's drag by altering the airflow near the wingtips and decreases the vortex (figure 1.1). The winglets increase the effective aspect ratio of a wing without materially increasing the wingspan. This system makes it possible to reduce the fuel consumption.

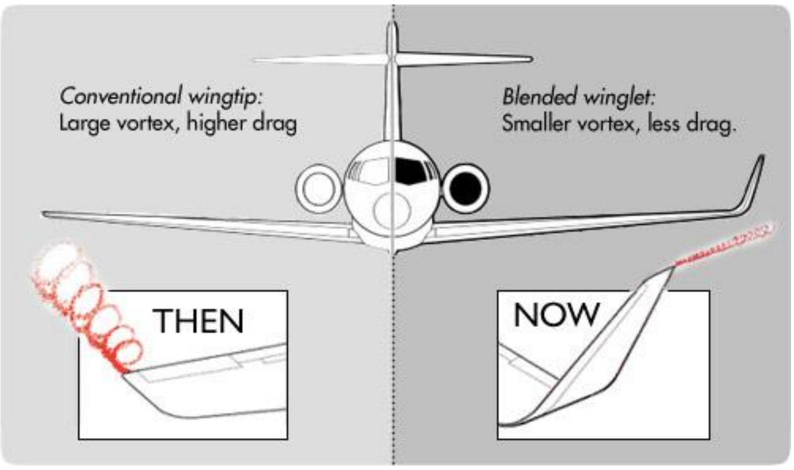

Figure 1.1: Effect of the winglet on the vortex

Today, the companies do not plan to replace the range of the short and medium haul airliners, but predict a modernisation with new range of motors. This choice is guided by a demand in economic and eco-responsible aircrafts and the need of the flight companies to reduce the fuel burn.

In the first part, the strategies of Boeing of fuel conservation are set forth [4], [5], [6]. The company introduced the cost index (CI) to find a compromise between the time cost and the fuel cost of the travel to reduce the flight cost.

In the second part, some benchmarks of the fuel consumption from the Airbus documentation [2] and the PhD of Paul Arentzen [1] are collected to compare with the values from the simulation part [Appendix C].

In the last part, the calculation of the fuel consumption is explained [3]. The equation is used in the computer code [Appendix B] and depends on the time of trajectory and the required thrust which applies on the aircraft.

1.1.1. Strategies of fuel conservation

The fuel conservation strategies by Boeing

The Boeing Company and other flight companies use the cost index. It is a function of fuel and nonfuel costs and has for objective to help airlines to reduce the operating costs.

The cost index (CI) is the ratio of the time-related cost of an airplane operation and the cost of fuel (Equation 1). This value reflects the relative effects of fuel cost on overall trip cost as compared to time-related direct operating costs.

$$CI = \frac{Time\ cost \sim \$/hr}{Fuel\ cost \sim cents/lb} \tag{1}$$

During the travel, the flight crew enters the company calculated CI into the control display unit (CDU) of the flight management computer (FMC). The FMC uses this number and other performance parameters to calculate economy climb, cruise and descent speeds. The airspeed which is used during descent tends to be the most restricted of the three flight phases. The cost index range is 0-500 for the aircraft 737 Boeing Next generation and 0-9999 for the aircraft 777.

For all the aircraft models, if the cost index entering in the FMC is equal to zero, the configuration gives the maximum range airspeed and the fuel consumption is minimal, but this configuration ignores the cost time. If the cost index is maximal, the time flight is minimal, the velocity and the Mach number are maximal, but the speed schedule ignores the cost fuel. In practice, neither of the extreme CI values is used.

In the figures 1.2 and 1.3 are represented the cost index, the influence of the CI on the time and the fuel consumption for different profiles of climb-cruise and descent. During the climb, when the cost index augments, the fuel consumption increases and the time of climb to reach the point B at cruise altitude decreases. For CI=0, the gradient of climb is maximal and the fuel consumption is minimal. During the descent, the cost index is maximal for the maximum gradient. The fuel consumption increases and the time of descent decreases when the cost index augments.

The cost index depends on the fuel cost and the time cost. The latter is based on the flight crew wages, the engines, the auxiliary power units, the airplanes and the maintenance costs. Some costs can be direct and the others are fixe. In the case of high direct time costs, the CI is large to minimize the time. In the case where most costs are fixed, the CI is very low to minimize the fuel cost. The cost index allows finding a compromise between the fuel burn and the time according to the costs of both, to reduce the flight cost.

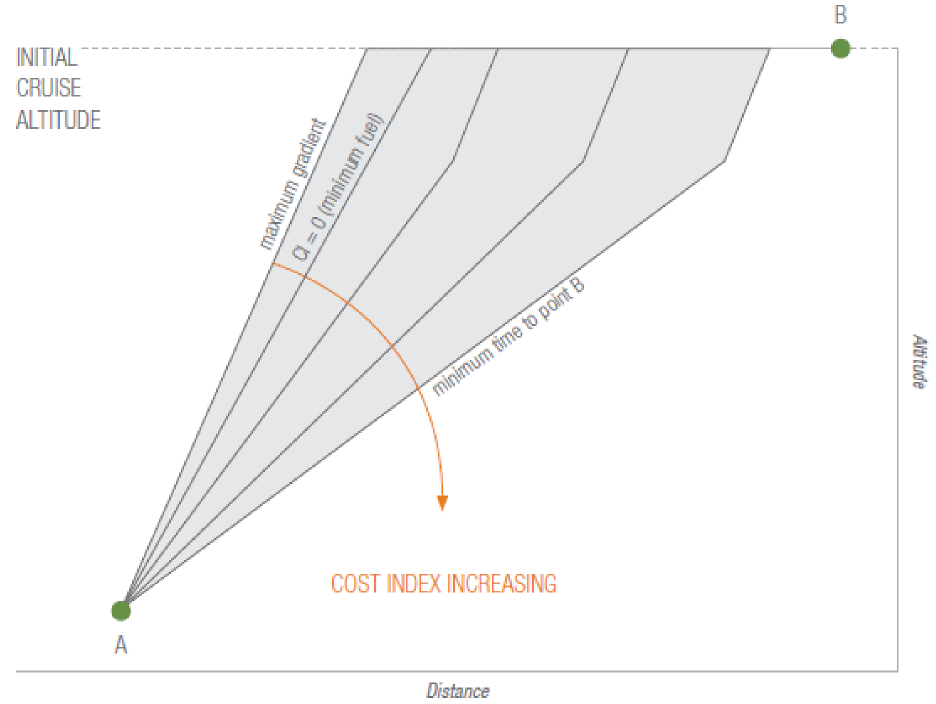

Figure 1.2: The effect of cost index when climbing to cruise altitude [4]

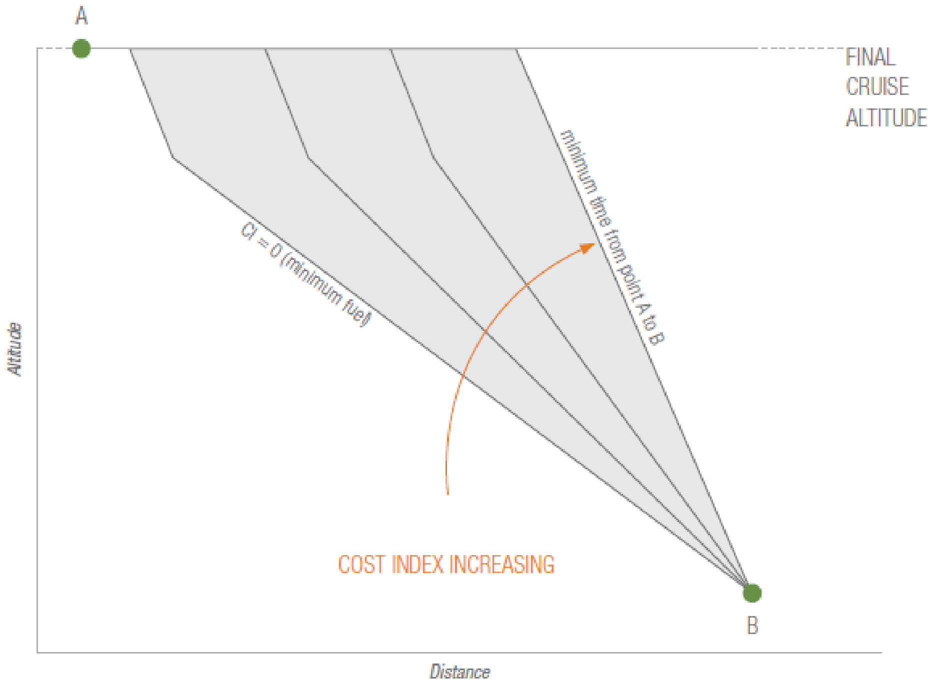

Figure 1.3: The effect of cost index when descending [4]

The strategies of Boeing are to conserve the fuel and minimize the flight cost. They concern all the phases of the flight trajectory. Their study is directed to the takeoff-climb and the cruise.

Takeoff and climb

The fuel consumption per hour is maximal during the takeoff and the climb phases, hence the importance for Boeing to find the best profile of takeoff and climb in order to reduce the fuel burn.

During the takeoff and the climb phases, the takeoff flap setting is important for saving the fuel. According to the flap setting configuration, the fuel consumption varies. For a configuration with an angle smaller, the fuel burn is less important because the drag decreases. The difference is small for the different configurations of flap setting possible, but today it is important to save the fuel and to reduce the cost of the travel.

Table 1.1: Impact of takeoff flaps selection on fuel burn, altitude z=3048m [5]

Airplane model	Takeoff Flap setting [°]	Takeoff Gross weight [kg]	Fuel used [kg]	Fuel differential [kg]
737-800 Winglets	5	72 575	578	-
	10		586	8
	15		588	10
777-200 Extended range	5	249 476	1 635	-
	10		1 668	33
	20		1 692	57

During the climb phase, the aircraft can reduce the fuel burn if the flight crew performs acceleration and flap retraction at lower altitude than the typical 3 000 feet (914 m). The fuel consumption decreases because the drag which is being reduced earlier in the climb-out phase.

The table 1.2 shows two standard climb profiles. The profile 1 is a climb with acceleration and flap retraction beginning at 3 000 feet and the profile 2 is a climb with acceleration to flap retraction beginning at 1000 feet (305 m).

Table 1.2: Fuel saving potential of two climb profiles, cruise altitude [5]

Airplane model	Takeoff Gross weight [kg]	Profile Type	Takeoff Flap setting [°]	Fuel used [kg]	Fuel differential [kg]
737-800 Winglets	72 575	1	10	2 374	-
		2		2 307	- 67
777-200 Extended range	249 476	1	15	6 583	-
		2		6 386	-197

With the profile 2, the aircrafts use 3 to 4 percent less fuel than the profile 1.

The table 1.3 shows the combined effects of using lower takeoff flap setting and flying with the profile 2, compared to using higher takeoff flap setting and flying with the profile 1. The first configuration gives 4 to 5 percent less fuel burn than the second.

Table 1.3: Fuel saving potential of two climb profiles, cruise altitude [5]

Airplane Model	**Takeoff Gross weight [kg]**	**Profile Type**	**Takeoff Flap setting [°]**	**Fuel used [kg]**	**Fuel differential [kg]**
737-800 Winglets	72 575	2	5	2 299	-93
		1	15	2 392	-
777-200 Extended range	249 476	2	5	6 358	-314
		1	20	6 672	-

The reduction of the angle of the takeoff flap setting and the transition between the takeoff and the climb at less altitude can reduce the fuel consumption. The difference of the fuel burn is few kilograms, but each phases of the flight trajectory is important for the fuel conservation.

Cruise

In general, except for the short flight trajectory, the cruise is the largest percentages of the trip time and the trip fuel is consumed typically in this phase of flight, hence the importance to have the best cruise conditions to reduce the fuel consumption. The parameters which affect the travel time and the fuel burn are the cruise speed, the altitude and the centre of gravity of the aircraft. The speed selection depends on the perspective of the pilot, dispatcher, performance engineer, or operations planner.

The objectives can be to minimize the fuel used, the total trip time, the total operating cost for the trip or to maintain the flight schedule. When the flight’s strategic objectives are understood, the cruise speed can be selected.
The problem is that sometimes the pilot has to change the cruise strategy because of some constraints he can meet during the flight.

There are two theoretical speed selections for the cruise phase of flight:

- LCR: Long Range Cruise (the traditional speed)
- MCR: Maximum Range Cruise, has for objective to reduce the fuel burn for a given cruise distance.

The cost index for the two different theoretical speeds is different. The fuel consumption will be more important with the LRC speed, because the cost index is higher than for the MCR speed.

Table 1.4: Entered cost index (CI) [6]

Airplane model	MRC	Typical airline CI values	Approximate LRC equivalent
737-6/7/800	0	10 to 30	35
777	0	90 to 150	180

The objective of Boeing is to help the flight companies to save their money in reducing the operating costs. A compromise has to be finding between the fuel consumption and the time of flight. The company want to reduce the fuel burn and the time of flight.

1.1.2. Benchmarks

With the strategies of each company, the information about the fuel consumption is difficult to obtain. This information is confidential. The Airbus documentation [2] and the PhD of Paul Arentzen [1] give some values about the fuel consumption for different aircrafts and trajectories.

The Airbus estimations for the climb phase

This part relates the different profiles of climb possible according to the cost index. The distance, the time and the fuel consumption vary according to the trajectory profiles.

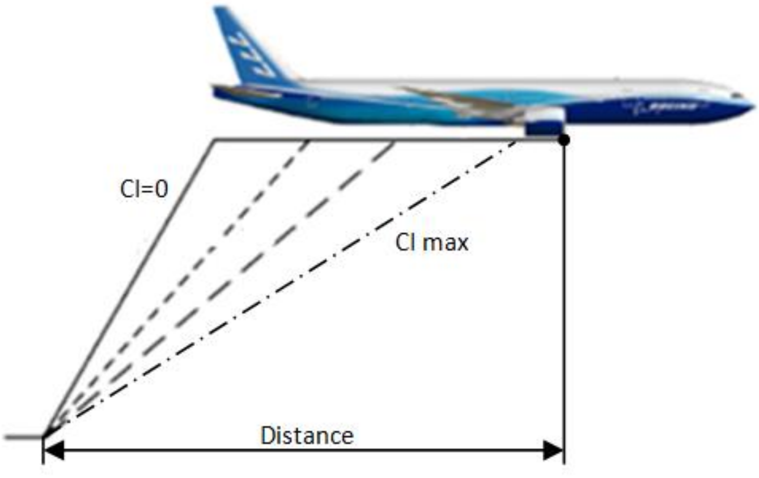

Figure 1.4: Climb profiles

The CI increases when the gradient decreases and the climb distance is longer until the cruise altitude.

The following table 1.5 shows the different climb parameters such as the time, speed, fuel, distance, computed by in-flight performance software for the A320 and the A340. The results of the both aircrafts are interesting to compare with the results from the simulation part for the climb phase. The A320 is used for medium haul flights and the A340 for long haul flights.

Table 1.5: Climb parameters to FL330 (z=10058 m)
ISA conditions, no wind, 250kt (128,6m/s) up to FL 100 (z=30.48 m) [2]

Aircraft type (T/OFF weight)	COST INDEX (kg/min)	Only climb segment			Climb with cruise segment	
		Fuel (kg)	Time (min)	Distance (km)	Fuel (kg)	Time (min)
A 320 (CFM 56) (75000 kg)	0	1757	22,4	227,8	1984	27,5
	20	1838	23,1	294,5	2009	26,9
	40	1897	23,7	305,6	2030	26,6
	60	1980	24,7	324,1	2056	26,3
	80	2044	25,6	338,9	2072	26,2
	100	2080	26,1	346,3	2080	26,1
A 340 (CFM 56) (250000 kg)	0	5363	25,4	311,1	5532	26,8
	50	5450	26	318,5	5551	26,7
	80	5492	26,2	322,2	5560	26,7
	100	5510	26,3	324,1	5563	26,7
	150	5547	26,5	327,8	5570	26,7
	200	5574	26,7	329,6	5574	26,7

In the case of CI max, the climb trajectory corresponds to the distance of climb between the sea level and the cruise altitude (oblique trajectory).
In the other cases, the climb trajectory is divided in two parts: the climb distance between the sea level and the cruise altitude (oblique trajectory) and the cruise segment until the point of cruise for CI max (horizontal trajectory).

For the climb segment, when the cost index diminishes, the time decreases because the distance of climb is shorter to reach the cruise altitude and the fuel consumption decreases, but the aircrafts are not in the same position in each case.
The cruise segment is introduced to define the fuel consumption for the different profiles at the same point. With the cruise segment, the fuel consumption increases, but stays smaller when the cost index is low. In the case of the Airbus A320, the fuel consumption decreases when the cost index diminishes, but the time varies and is a bit longer. For the A340, the time is the same for each profile and the fuel consumption decreases when the cost index diminishes.

The histograms represent the fuel consumption and the curves the time, both depend on the cost index (figure 1.5).

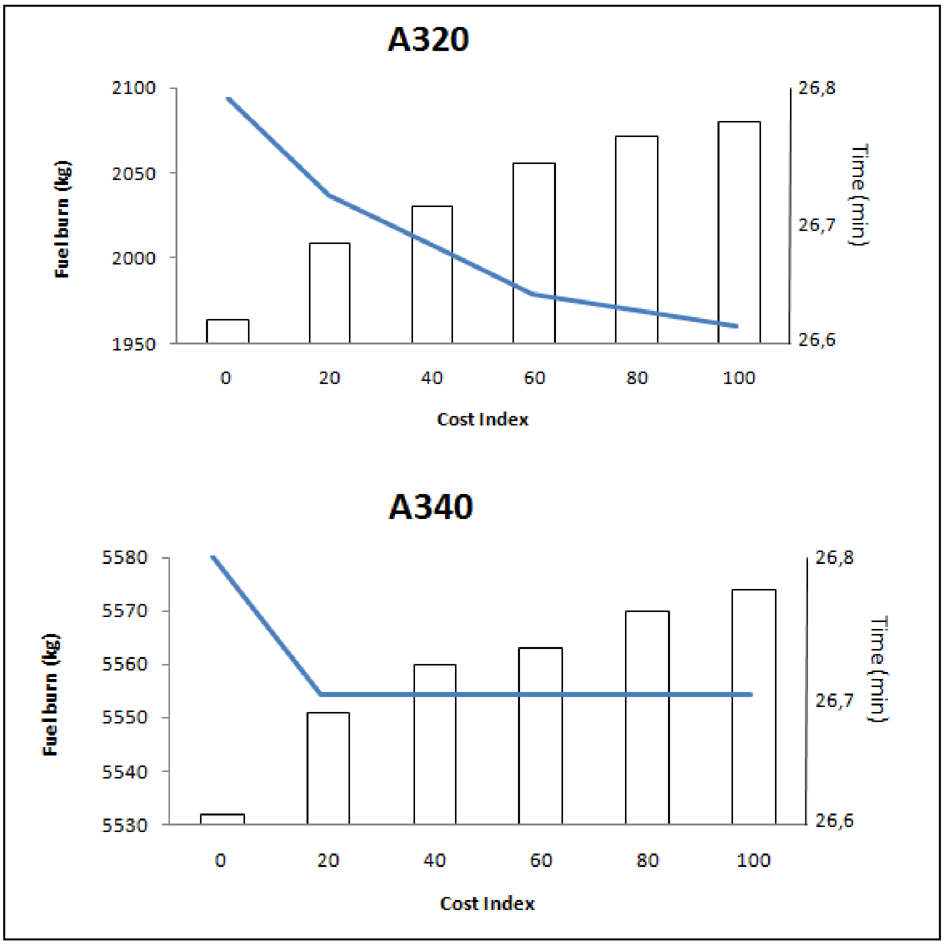

Figure 1.5:Fuel consumption and time, climb with cruise segment

In the computer code, the aircraft A320 can be used for the flight between Trondheim – Nice. The A340 is long-haul airline and can be used for the flight Paris – New York. The information about the fuel consumption during the climb from the Airbus documentation [2] of these two aircrafts is necessary to have some benchmarks for the simulation part.

The fuel consumption for the flight Oslo-Trondheim

The information in this part comes from the PhD of Paul Arentzen [1]. In this study, the range aircraft considered is the Boeing 737 models -3/4/5/800. The type of engine on these aircrafts is the CFM56-3C1. The aircraft area considered is 202,85 m² and the weight at takeoff is 48 000kg. The trajectory profile for Oslo-Trondheim is defined in the table 1.6.

Table 1.6: Flight and weight data for the modelled flight Oslo-Trondheim [1]

Segment	**Altitude [m]**		**Speed [m/s]**		**Distance [km]**	**Climb [°]**	**Time [s]**	**Fuel [kg]**	**A/C Mass [kg] Start**
	Start	**End**	**Start**	**End**					
I	12	12	10,3	79	1,5	0	31	58	48000
II	12	457	79	107	4,3	5,9	45	101	47942
III$_A$	457	2071	107	185	28,3	3,3	170	379	47841
III$_B$	2071	11278	185	208	161,7	3,3	734	1092	47462
Total climb	457	11278	107	208	**190**		**904**	**1471**	47462
IV$_A$	11278	11278	208	220	5,5	0	26	22	46370
IV$_B$	11278	11278	220	220	83,5	0	380	187	46348
Total cruise	11278	11278	208	220	**89**	0	**406**	**209**	46348
V$_A$	11278	5628	220	202	105,5	-3,1	484	102	46161
V$_B$	5628	457	203	138	94,6	-3,1	630	193	46059
Total descent	11278	457	220	138	**200,1**	-3,1	**1114**	**295**	46059
VI	457	6	82	87	9,3	-2,8	110	38	45866
Sum					**494,2**		**2 610**	**2172**	45828

I: taxi-out
II: takeoff
III$_A$ and III$_B$: climb
IV$_A$ and IV$_B$: cruise
V$_A$ and V$_B$: descent
VI: landing

In the table 1.6 is represented the altitude, speed, distance, time and fuel consumption for the different phases of the flight Oslo-Trondheim. The results can be used to compare with the results from the simulation part for the flight Trondheim-Oslo. The principal interesting values are the time, the distance and the fuel burn after the 3 phases: climb, cruise and descent and at the end of the flight. The profile of the flight trajectory which uses in the PhD is different of the profile which is defined in the part 1.2 Flight trajectory and computer code presentation.

The table 1.7 compares the estimates of the aircraft and the engine manufacturer with the Turbomatch/Braathens (model above in the PHD). The principal differences with the three cases are the weight of the aircraft at takeoff, the cruise altitude and the cruise speed. This table gives more benchmark for the simulation part.

Table 1.7: Fuel consumption, different estimates for a 500 km Boeing 737-400 flight two engines [1]

	Boeing estimates	**CFM estimates**	**Turbomach/ Braathens**
Takeoff weight [kg]	62800	56700	48000
Cruise altitude [m]	9450/10670	7620	11278
Cruise Mach no./ speed [m/s]	0,74/223	0,72/223	0,745/220
Climb fuel consumption [kg]	1242	829	1228
Fuel flow in cruise, both engines [kg/s]	0,682	0,761	0,525
Total fuel consumption, 500 km flight [kg]	2212	2021	1948

For the short-haul flight, the highest fuel consumption occurs at take-off. After the climb angle and the drag coefficient decrease and the fuel burn falls accordingly. During the first phase of climb, the fuel consumption increases because the speed is more than doubled and the air density decreases. During the climb, the speed and the climb angle are considered constant and the air density decreases, so the thrust and consequently the fuel consumption decrease. The fuel burn is constant during the cruise phase because the speed is constant. For the descent, the fuel consumption decreases, but in the approach phase and the landing the thrust augments and consequently the fuel consumption too. The variation of fuel during the trajectory is represented in the figure 1.6.

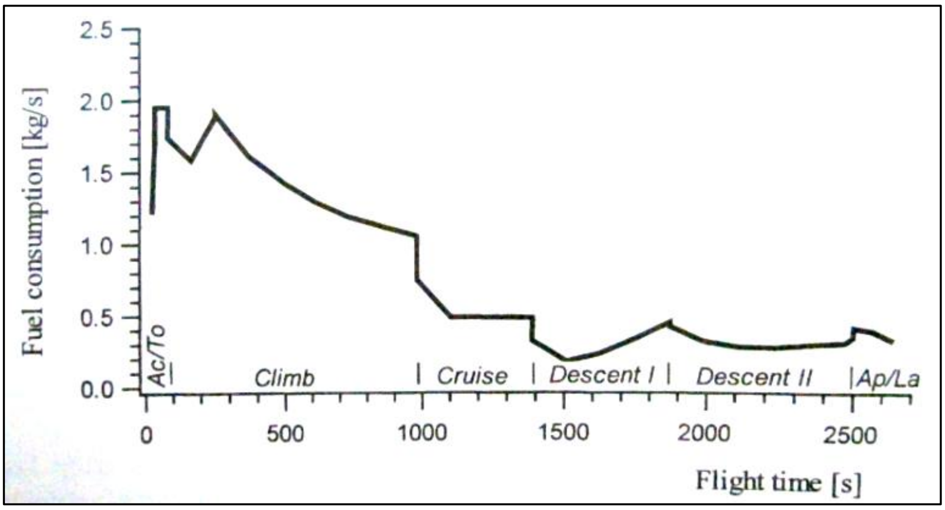

Figure 1.6: Time specific fuel consumption during the complete flight cycle Oslo-Trondheim (Turbomach model) [1]

1.1.3. Calculation of the fuel consumption (SFC)

The objective is to define the fuel consumption during the different phases of the flight trajectory. The engines of the aircraft are defined by the type, the engine characteristics (length, fan diameter, weight), the maximum thrust for the different phases, the overall pressure ratio at maximum power and the specific fuel consumption at maximum power.

The thrust specific fuel consumption (TSFC) or specific fuel consumption (SFC) is an engineering term that is used to describe the fuel efficiency of an engine design with respect to thrust output. The unit is [(lbm/h)/(lbs)] or [(kg/h)/(N)]. TSFC or SFC for thrust engines is the mass of fuel needed to provide the net thrust for a given period. The SFC depends on the engine design and provides important information about the performance of a given engine.

Mathematically, SFC is a ratio of the engine fuel mass flow rate to the amount of thrust produced by burning the fuel [3], i.e.

$$SFC = \frac{\dot{m_{fb}}}{T} \tag{2}$$

where: $\dot{m_{fb}}$ is the fuel mass flow rate [kg/h]
T is the thrust force [N]

The equation to determine the fuel consumption is:

$$m_{fb} = SFC \times \tau \times T \tag{3}$$

where: τ is the time [h]
SFC is the specific fuel consumption [(kg/h)/(N)]
m_{fb} is the fuel burn mass [kg]

The values of the SFC are given in the table 1.8 for the different engines used by the chosen aircraft.

Table 1.8: SFC for the different engines [7]

Aircraft	**Engine**	**SFC [(lbm/h)/(lbs)]**	**SFC [(kg/h)/N]**
737-300	CFM 56-3C1	0,39	0,03977
737-800	CFM 56-7B24	0,38	0,03875
A320	CFM 56-5B4	0,34	0,03467
A340	CFM 56-5C2	0,32	0,03263
777-200	PW 4077	0,33	0,03365

The equation (3) gives the fuel consumption according to the thrust force, the time and the specific fuel consumption (SFC) which depends of the type of engine. This equation is used in the computer code for the simulation part of the project.

In the computer code, the climb and the descent are divided by increment. At each increment the fuel consumption is calculated and the sum of these consumptions gives the fuel burn for the climb and the descent phases. After each increment, the weight of the aircraft decreases. For the cruise, the fuel consumption is constant because the trajectory is straight, the altitude and the cruise speed are constant for the short and medium haul flights. For the long haul flights, the aircraft flies between the cruise altitudes z_{C1} and z_{C2}. The required thrust and the cruise speed are considered constant (1.2.2. Cruise description) consequently the fuel consumption varies linearly according to the air density.

1.2. Flight trajectory and computer code presentation

1.2.1. Climb and descent descriptions

The profiles of the climb and the descent have been defined in the previous analysis [Appendix A]. This analysis is the simplified model of the flight trajectory of an aircraft.

The velocity of the aircraft during the climb and the descent phases is defined by two components: **the velocity along the flight path V [m/s]** and **the vertical speed v [m/s]**. The angle between these two components is **the climb or descent angle θ [rad]**. The equations are given by (4), (5), (6) with the dimensionless altitude $Z = z/z_C$ and z_C is the altitude at the specific cruise height [m]. The curve of the velocity along the flight path [Appendix A, figure 1] indicates two horizontal asymptotes in $z = 0$ and $z = z_C$.

$$V(z) = -\frac{1}{\alpha}\ln\left[\frac{1}{\beta}\left(\frac{z_C}{z} - 1\right)\right] \tag{4}$$

$$v = \gamma Z^2(1 - Z)^2 \tag{5}$$

The vertical speed is equal to zero at $z = 0$ and z_c.

$$sin\theta = \frac{v}{V} \tag{6}$$

where α is the parameter defining the dimensional velocity [s/m], β is the velocity/altitude parameter, γ is the parameter for climb/descent time determination.

In the previous analysis [Appendix A], the value of the parameter α is 0,07 s/m. The velocity along the flight path and the Mach number depend on the parameter α. In the computer code, the aircrafts are different according to the trajectory. Each aircraft is defined by the cruise Mach number. To respect the cruise Mach number theoretical, the value of parameter α has to be defined for each aircraft.

Table 1.9: Values of the parameter α for each aircraft

Aircraft	Cruise Mach number [-]	α [s/m]
737-300	0,74	0,0735
737-800	0,78	0,0697
A320	0,78	0,0697
A340	0,82	0,0663
777-200	0,84	0,0648

The parameter β has an effect on the velocity along the flight path and the Mach number, but it is minimal in comparison with the parameter α. In the previous analysis, the value of the parameter β is 60000. This value is retained for this analysis.

The mean vertical speed $\boldsymbol{v_{mean}}$ [m/s] has to be calculated to evaluate the time τ [s] needed to climb up to the cruise altitude z_c. The vertical speed is integrated between Z=0 and Z=1 and the solution of the mean vertical velocity is given by the equation (7).

$$v_{mean} = 0{,}0333\gamma \tag{7}$$

The time $\boldsymbol{\tau}$ [s] of the climb and the descent is a function of the cruise altitude and the mean vertical speed $\mathrm{v_{mean}}$, i.e.

$$\tau = \frac{z_c}{v_{mean}} \tag{8}$$

The vertical speed can be represented by a parabola. The mean vertical speed depends on the parameter γ and the vertical speed is equal to 0 in $z = 0 \; and \; z = z_c$. These values give the equation of **the parabola** i.e.

$$v = -6v_{mean}Z^2 + 6v_{mean}Z \tag{9}$$

In the simulation analysis, the influence of the vertical speed profile on the fuel consumption is determined.

The air density and the speed of sound have different values during the trajectory because they depend on the geometric altitude z. For an isothermal atmosphere, **the air density [$\mathbf{kg/m^3}$]** is given by the equation (10), i.e.

$$\rho(z) = \rho_0 \exp(-\varepsilon z) = \rho_0 \exp(-\varepsilon z_C Z) \tag{10}$$

where the air density at sea level is $\rho_0 = 1{,}225 \; kg/m^3$, $\varepsilon = 0{,}0001 \; 1/m$ is a factor in order to satisfy an approximation in the range $0 \leq z \leq z_C$.

The speed of sound c [m/s] in the troposphere is a linear decay function of the dimensionless altitude Z.

$$c = 340{,}294 - 45{,}15Z \tag{11}$$

The flight Mach number [-] has an asymptotic behaviour in $z = 0$ and $z = z_c$, to eliminate this behaviour the range will be $0{,}01 \leq Z \leq 0{,}99$, i.e.

$$M = \frac{V}{c} \tag{12}$$

$Climb$

During the climb phase, the forces which apply to the aircraft are the acceleration force F, the gravity G, the lift L, the aerodynamic drag D and the required thrust T. The latter depends on the forces along the flight path; the forces are given by the equations (13), (14), (19), (20) and are represented in the Appendix A, figure 2.

The acceleration force F [N] is a function of the weight of the aircraft m [kg] and the acceleration along the flight path a [m/s²]. The latter depends on the dimensionless altitude. In the previous analysis [Appendix A], the weight of the aircraft is considered constant and equal to the takeoff mass of the aircraft, but in this analysis, the calculation of the forces is realised by increment in the computer code and the weight of the aircraft decreases after each increment because of the fuel consumption. The acceleration force depends on the dimensionless altitude Z, the weight of the aircraft m, the cruise altitude z_C and the parameter for the climb time determination γ.

$$F = ma = \frac{m\gamma}{\alpha z_c} Z(1 - Z) \tag{13}$$

The gravity force is divided in two components: the gravity drag force along the flight path G_D and the gravity lift force perpendicular to the flight path G_L. The value of **the gravity drag force [N]** is necessary to calculate the required thrust force. The gravity force is a function of the climb angle θ [rad] and the weight of the aircraft m [kg], i.e.

$$G_D = mgsin\theta \tag{14}$$

where g [m/s²] is the acceleration of gravity

The aerodynamic drag D [N] is the sum of the drag at zero lift D_0 and the lift induced drag D_i. It is a function of the air density ρ [kg/m³] and the velocity along the flight path V [m/s]. They depend on the altitude z, i.e.

$$D = \left(C_{D,0} + C_{D,i}\right)\frac{1}{2}S\rho V^2 \tag{15}$$

where S [m²] is the reference wing area of the aircraft, $C_{D,0} = 0{,}015$ [-] is the drag coefficient at zero lift and $C_{D,i}$ [-] is the induced drag coefficient related to **the lift force L**. The lift is equal to **the gravity component $\boldsymbol{G_L}$** perpendicular to the flight path. The forces are given by the equations (16) and (17).

$$L = \frac{1}{2} S V^2 C_L \rho \tag{16}$$

$$G_L = mgcos\theta \tag{17}$$

The induced drag coefficient is defined by the equation (18):

$$C_{D,i} = kC_L^2 = k \left[\frac{mgcos\theta}{\frac{1}{2} S\rho V^2} \right]^2 \tag{18}$$

where C_L is the lift coefficient, $k \approx 0{,}045$ [-] is the factor related to the aspect ratio of the wing. With the combination of the equations (15) and (18), the aerodynamic drag is given by the equation (19):

$$D = \frac{1}{2} C_{D,0} S\rho V^2 + \frac{k(mgcos\theta)^2}{\frac{1}{2} S\rho V^2} \tag{19}$$

The aerodynamic drag depends also on the aircraft weight.

The required thrust T [N] is the sum of the forces along the flight path which apply on the aircraft, i.e.

$$T = F + G + D \tag{20}$$

The equations are calculated for the range $0{,}01 \leq Z \leq 0{,}99$ to eliminate the infinite values of the velocity along the flight path, the climb/descent angle, the Mach number, the gravity, the aerodynamic drag, the lift and the required thrust.

The horizontal distance $\mathbf{d_H}$ [m] during the climb between the takeoff position and the cruise altitude depends on the mean horizontal speed v_{Hmean} $[m/s]$ and the time; it is given by the equation (21).

$$d_H = v_{Hmean} \times \tau \tag{21}$$

$$v_{Hmean} = \sqrt{V_{mean}{}^{2} - v_{mean}{}^{2}} \qquad (22)$$

The value of the mean velocity along the flight path V_{mean} is determined by the function “mean” in the software Matlab after the “For” loop.

In the computer code which is realised with the software Matlab, all these equations are necessary and the equation of the fuel burn (Equation 3) to define the time, the horizontal distances and the fuel consumption for the trajectory chosen. A “For” loop is used to calculated by increment (ΔZ=0,01) for the range $0{,}01 \leq Z \leq 0{,}99$, the fuel consumption and to decrease the weight of the aircraft after each increment.

The algorithm for the climb phase is defined by the figure 1.7.

In the computer code, the characteristics of the aircraft are defined at the beginning: the parameter α, the wing area S and the specific fuel consumption SFC. In the first time, all the variables are initialized and the values of the parameters (β, γ, z_C, ρ_0, ε, k and $C_{D,0}$) are entered previously.
The weight of the fuel m_f is defined by the maximum capacity of fuel l_f [l] and the fuel density ρ_f (Equation 23).

$$m_f = l_f \times \rho_f \times 10^{-3} \qquad (23)$$

The mean vertical speed and the time of climb are defined according to the parameter γ and the time increment is the ratio of the time of climb and the number of increments n (Equation 24).

$$t = \frac{\tau}{n} \qquad (24)$$

At each increment, all the equations are executed. At the end of “For” loop, the equations of the weights of the aircraft and the fuel are defined and the values decrease after each increment. The operation is repeated until the dimensionless altitude Z=0,99. After the “For” loop, the values of the fuel burn, the weight of the aircraft and the fuel are determined.
The values of the lift coefficient, the Mach number and the speed of sound at the end of the climb are defined for the next phase, the cruise trajectory.
The value of the horizontal climb distance is calculated after the “For” loop because it depends on the velocity along the flight path which is calculated during the “For” loop for each increment and the function “mean” is used to define the mean value.

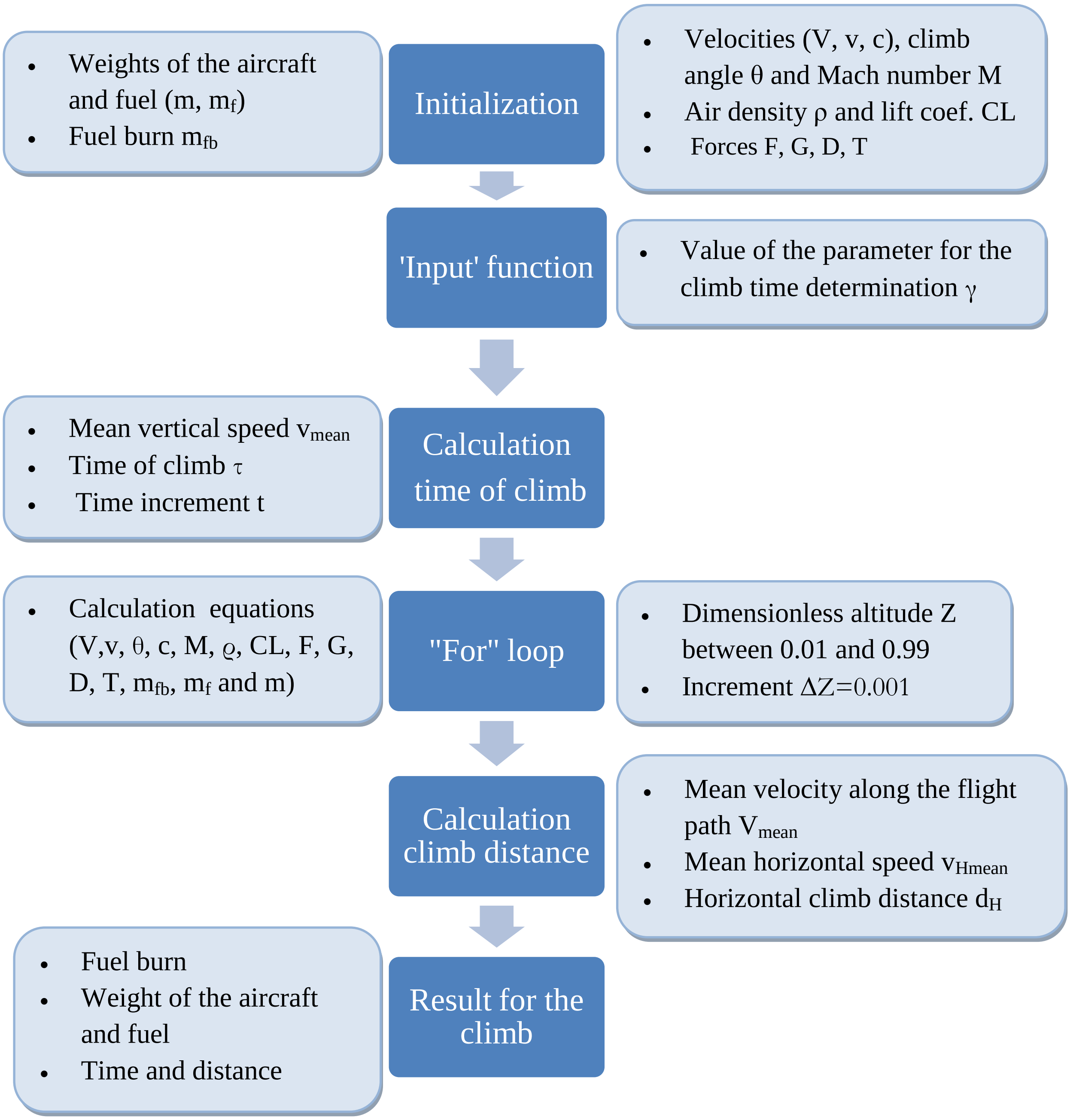

Figure 1.7: Climb algorithm

Descent

During the descent, **the vertical speed v** and **the descent angle θ** are negative. The weight of the aircraft is lighter due to the fuel burn during the climb and the cruise phases.
According to the chosen trajectory, the descent time increases if the parameter γ decreases. The variation of the parameter γ modifies the value of the forces which apply on the aircraft because they depend on the descent angle and the lift coefficient which depend on the parameter γ. The equations for the acceleration force F, the gravity drag force G and the aerodynamic drag D are conserved. The forces which apply on the aircraft are represented in the previous analysis [Appendix A, figure 10]

The force balance for the aircraft during the descent phase can be formulated as

$$T = D - G - F \qquad (25)$$

The thrust is negative for some values of the parameter for the descent time determination γ. In the reality, it is impossible. The thrust is negative because when the parameter γ increases, the gravity force increases and for some values of the dimensionless altitudes Z, the gravity is more important than the aerodynamic drag D and consequently the thrust is negative.
The minimum value of the thrust is required and equal to 10% of the maximum thrust. The latter is determined during the climb phase.

The mean vertical speed, the time and the horizontal distance are determined by the equations (7), (8) and (21).
The fuel burn is determined with the equation (3) as a function of the thrust and the time. Like for the climb, the different equations will be calculated for each increment and the weight of the aircraft will decrease during the descent.

The algorithm for the descent phase is similar to the climb algorithm. The fuel burn is initialized because it is interesting to have the fuel consumption for each phase and after to determine the total fuel consumption for the selected trajectory. The weights of the aircraft and the fuel at the beginning of the descent are equal to the weights at the end of the cruise.
The time of descent and the descent horizontal distance (Equations 8 and 21) are calculated before the cruise phase to define the cruise horizontal distance and time. The time increment for the descent is calculated before the “For” loop.
The loop is realized between the dimensionless altitude Z=0,01 and Z=0,99 with an increment ΔZ=0,001. In the descent algorithm the parameter Z_d is introduced because the aircraft is flying between Z=0,99 and Z=0,01 (Equation 26).

$$Z_d = 1 - Z \qquad (26)$$

During the descent, at some dimensionless altitude Z, the thrust force is negative for some values of the parameter for the descent time determination γ. In the algorithm, this modification is realized by an “If” loop (figure 1.8).

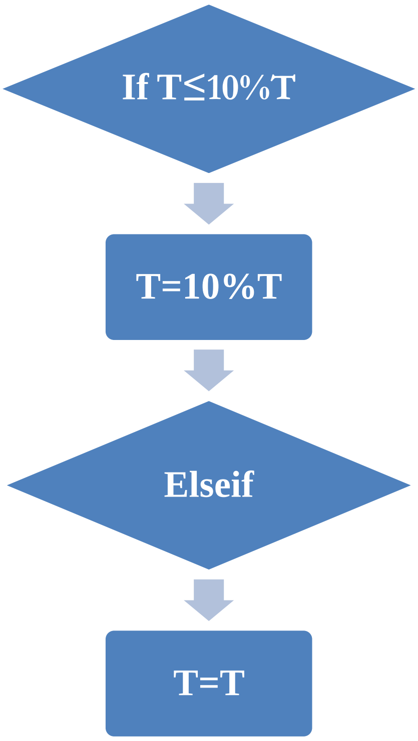

Figure 1.8: Descent algorithm, “If” loop

1.2.2. Cruise description

1.2.2.1. The real cruise trajectory

The cruise trajectory is the phase between the climb and the descent. Except for the short flights, this phase is the period where the aircraft consumes the majority of fuel. During the cruise, the altitude is considered constant, the velocity vector is parallel to the ground and constant, the sum of the opposing forces is equal to zero (Newton’s Third Law). There can no unbalanced forces in steady straight flight.

There is an optimum cruise altitude for the civil aircraft where the performances of the aircraft are maximized. In general, this altitude is around 35000ft (10668m). It is based on the velocity and the weight of the aircraft. The altitude increases when the speed is higher and when the weight of the aircraft is lighter. At high altitudes, the fuel consumption during the cruise is reduced, but the consumption is more important to reach this altitude.

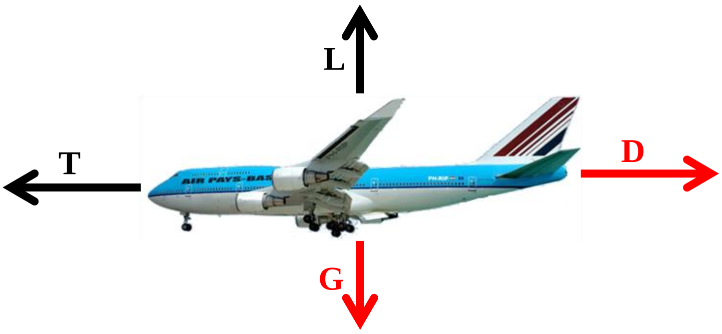

Figure 1.9: Balanced forces during the cruise

In the reality, the aircraft can be confronted to some constraints so the velocity along the flight path and the altitude vary. The pilots have to abandon temporally the cruise strategy one or more time during the cruise. In these cases, the fixed speed has to be compatible with the other traffic, the time of arrival has to be respected and the speed is calculated to minimize the fuel consumption.

The cruise trajectory of the aircraft is not straight because the crew has to follow a trajectory defined by some beacons (figure 1.10).

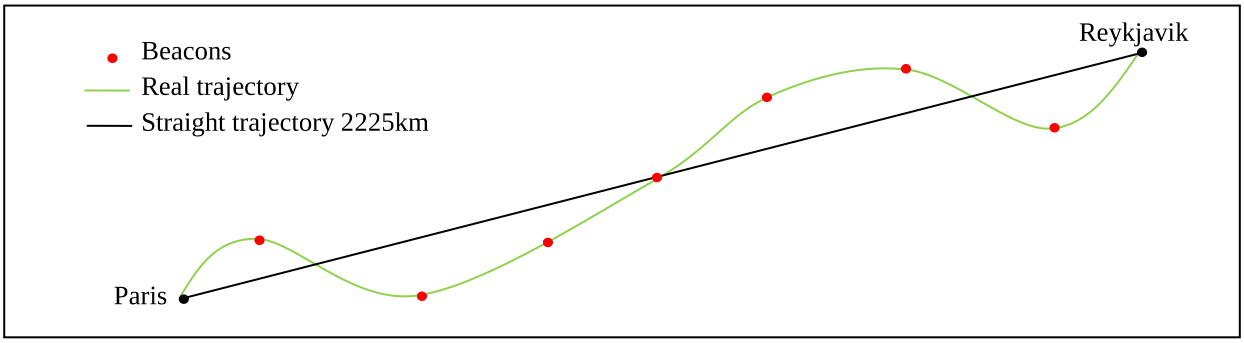

Figure 1.10: Example of the trajectory between Paris and Reykjavik

For a long-haul flight, the initial and final cruise altitudes are quite different since the aircraft weight changes substantially. There is a best altitude for cruise and this optimum altitude augments as the aircraft weight decreases. The cruise trajectory for the flight from Paris to New York is climb, horizontal flight, climb and so until the end of the cruise phase, when the aircraft reaches the maximum cruise altitude for the flight strategy selected.

The cruise trajectory depends on the flight distance. For the short-haul flight, the cruise trajectory is small or inexistent. For the medium-haul flight, the cruise trajectory is assumed horizontal at cruise altitude constant.

1.2.2.2. The previous cruise description

After the climb, the aircraft is at cruise altitude z_c. The velocity along the flight path gives two horizontal asymptotes which one at Z=1. The values obtain at this altitude are indefinites for the velocity along the flight path, the climb and descent angle, the Mach number, the gravity drag force, the aerodynamic drag, the thrust force. The altitude range selected is $0{,}01 \leq Z \leq 0{,}99$ so the cruise altitude is $z_{cruise} = 0{,}99 z_c$. At cruise altitude z_C, the speed of sound and the Mach number are constant so the velocity along the flight path is constant (equation 12).

The values which are obtained in the previous analysis for the speed of sound, the Mach number, the cruise velocity, the air density and the aerodynamic drag are given in the table 1.10. The characteristics of the aircraft are given in the Appendix A.

Table 1.10: Values from the previous cruise description

Cruise altitude [m]	10890
Speed of sound [m/s]	295,6
Mach number	0,754
Cruise velocity [m/s]	222,82
Air density [$\mathrm{kg/m^3}$]	0,41228
Aerodynamic drag [N]	35777
Time of cruise [h]	2,73

The cruise phase is simplified by a straight trajectory at cruise altitude constant. The forces which apply on the aircraft are the gravity force, the lift, the aerodynamic drag and the required thrust. With the Newton’s third law, the gravity is offset by the lift and the required thrust is equal to the aerodynamic drag [Appendix A, figure 8].

In the previous analysis, for the trajectory Trondheim – Nice, if the fuel consumption is given by the equation 3, the estimation of the fuel burn is equal to 3785 kg during the cruise phase. The specific fuel consumption is equal to SFC=0,03875.

1.2.2.3. The cruise description

The cruise description depends on the flight selected. In this analysis, three trajectories are defined:

- Trondheim – Oslo (short-haul flight)
- Trondheim – Nice (medium-haul flight)
- Paris – New York (long-haul flight)

For **the short-haul flight**, the cruise phase is small and sometimes inexistent. The aircraft doesn’t reach the maximum cruise altitude (value defined by the aircraft constructor). In this analysis, the cruise altitude is a parameter and it is modified to define its influence on the fuel consumption. When the cruise altitude augments, the cruise horizontal distance decreases because the descent and climb horizontal distances are longer. The cruise trajectory is straight and the cruise altitude is constant.
In the simulation analysis, for the trajectory Trondheim – Oslo, the fuel burn is calculated for different values of the cruise altitude.

For **the medium-haul flight**, the cruise trajectory is straight and horizontal at cruise altitude constant.

In this cruise description, after the climb, the aircraft is at an altitude z_C. At this altitude, the Mach number and the speed of sound are considered constant during the cruise phase. According to the equation (12), the speed along the flight path is constant. The cruise speed is calculated with the values of the Mach number and speed of sound at the end of the climb phase. The cruise altitude is given by the equation (27) because the dimensionless altitude range is $0{,}01 \leq Z \leq 0{,}99$, i.e

$$z_{C1} = Z(end) \times z_{C1} \tag{27}$$

where Z(end) is the dimensionless altitude at the end of the climb.

The air density is constant and defined by the equation (10).

The forces which apply on the aircraft are defined in the figure 9 and the equations (28) and (29). The lift force L offsets the gravity G and the required thrust T is equal to the aerodynamic drag D.

$$D = T \tag{28}$$

$$G = L \tag{29}$$

The altitude, the cruise speed, the air density and the thrust are constant, consequently the fuel consumption depends on the time of cruise which depends on the parameters for the climb/descent time determination γ.

For **the long-haul flight**, the cruise altitude increases. The cruise trajectory is considered linear and straight. The aircraft reaches the cruise altitude z_{C1} and during the cruise will reach the cruise altitude z_{C2} before the descent phase. The two cruise altitudes have to be defined. The cruise trajectory for the long-haul is described below.

Trajectory

In this cruise analysis, the aircraft trajectory is not parallel to the ground; there is a climb angle θ_c between the horizontal and the flight path, it is constant during the cruise phase. The aircraft is flying between two cruise altitudes z_{C1} *and* z_{C2} (figure 1.11).

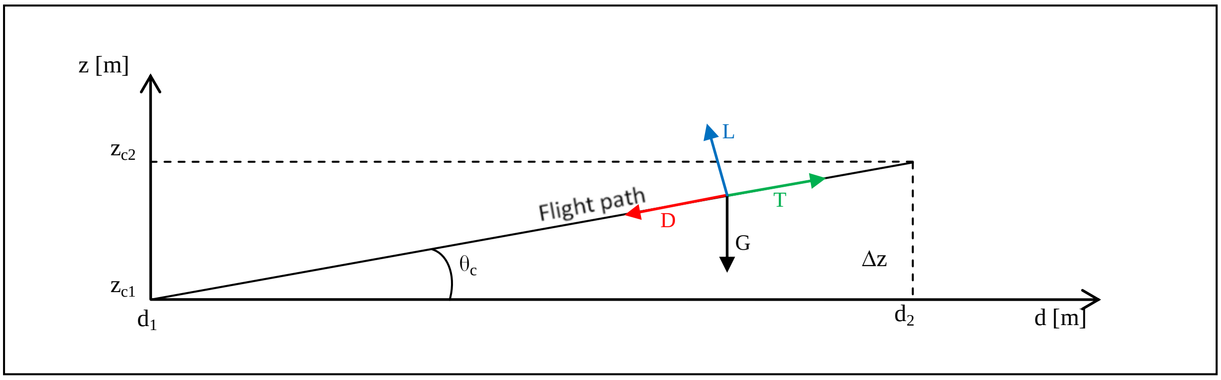

Figure 1.11: Cruise trajectory for the long haul flight

At the cruise altitude z_{C1}, the forces which apply on the aircraft are given by the equations (30) and (31). The gravity force perpendicular to the flight path is given by the equation (17) and the lift is defined by the equation (16).
The climb angle is supposed very small, therefore the cosine of the climb angle is close to the value 1 and the sinus of the climb angle is close to the value zero (equation 32). Consequently the equilibrium equations are given by the equations (28) and (29) for the horizontal cruise trajectory because the climb angle is very low.

Forces along the flight path:

$$T = D + Gsin(\theta_C) \tag{30}$$

Forces perpendicular to the flight path:

$$L = Gcos(\theta_C) \tag{31}$$

$$\theta_C \approx 0 \; so \; \cos(\theta_C) \approx 1 \; and \; \sin(\theta_C) \approx 0 \tag{32}$$

The aircraft weight at cruise altitude z_{C1} is given by the equation (33):

$$m_c = \left(\frac{C_L \times S \times V_c}{2g}\right) \times \rho_{C1} \tag{33}$$

where ρ_{C1} is the air density at the altitude z_{C1}.

The weight of the aircraft at cruise altitude z_{C1} is calculated to verify that is equal to the weight of the aircraft at the end of the climb phase.

At cruise altitude z_{C2}, the weight of the aircraft is lighter. The gravity force perpendicular to the flight path is given by the equation (34). The variation of the weight of the aircraft is defined below, i.e.

$$G = \left(m - m_{fb}\right)g \tag{34}$$

where m_{fb} is the weight of fuel burn [kg] after the cruise phase. The fuel consumption during the cruise is calculated with the equation (3).

For a small climb angle at cruise:

$$k = \frac{SC_L M^2 c^2}{2g} = constant \tag{35}$$

$$m = k\rho \ and \ dm = kd\rho \tag{36}$$

where the lift coefficient C_L is constant during the cruise phase and determined at the end of the climb.

The relation between the delta altitude dz and the ratio $\frac{dm}{m}$ can be defined for a ratio $\frac{d\rho}{z}$ given.

The air density depends on the altitude (Equation 10). For a small variation of the altitude, the air density is approximated by a linear equation (figure 1.12).

The delta altitude is given by the equation (37):

$$dz = \frac{\rho}{4.10^{-5}} \frac{dm}{m} \tag{37}$$

where m is the aircraft weight and ρ the air density at the cruise altitude z_{C1} and dm is the weight variation (fuel consumption).

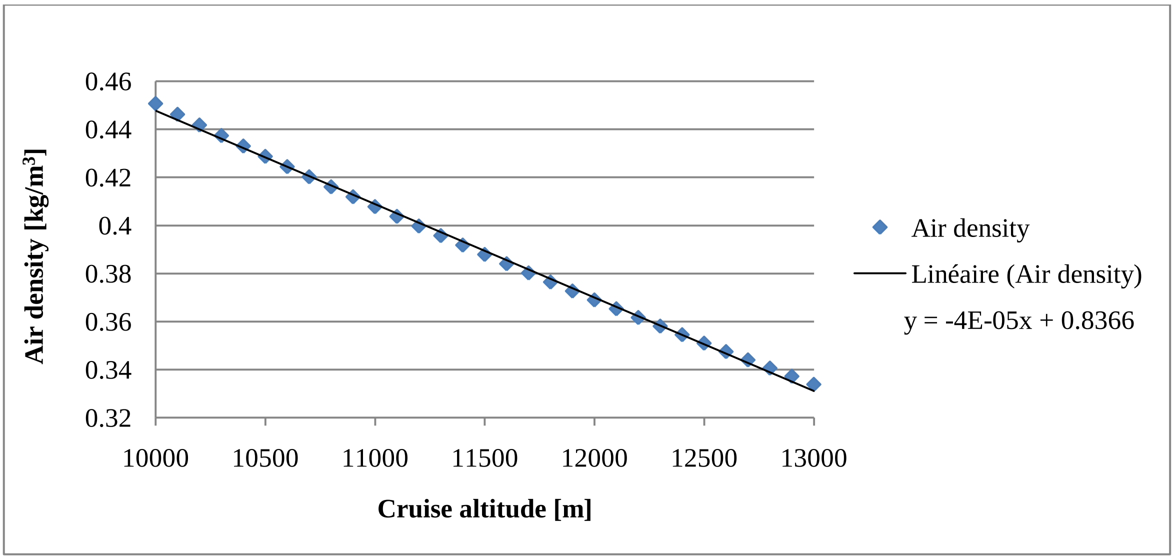

Figure 1.12: Evolution of the air density according to the cruise altitude

The fuel consumption during the cruise is calculated at each increment by the equation (3) between $z_{C1} = 11000\, m$ and $z_{C2} = 12000\, m$, the delta cruise altitude is equal at $\Delta z = 1000m$. The choice of the delta altitude is explained in the part 2.6. Choice of the cruise altitude.
Between the two cruise altitudes $z_{C1}\ and\ z_{C2}$, the air density is considered as a linear function and decreases when the altitude increases. The required thrust is equal to the aerodynamic drag and is a linear function because it depends on the air density (Equation 15). The cruise speed is constant because the speed of sound and the Mach number are considered constant after the cruise altitude z_{C1}, the time of cruise depends only on the cruise horizontal distance. The fuel consumption depends on the time and the thrust, consequently varies as a linear function.

The cruise horizontal distance [m] is determined by the difference between the total distance between the two airports and the climb and descent horizontal distances (equation 38).

$$d_{Hcruise} = d - d_{Hclimb} - d_{Hdescent} \tag{38}$$

The time of cruise [s] is defined by the equation (39):

$$\tau_{cruise} = \frac{d_{Hcruise}}{V_{mean}} \tag{39}$$

The cruise profile depends on the selected trajectory. For the short and medium haul flights, the cruise trajectory is considered horizontal at cruise altitude constant and for the long haul flights; the trajectory is linear between the altitude z_{C1} and z_{C2}.

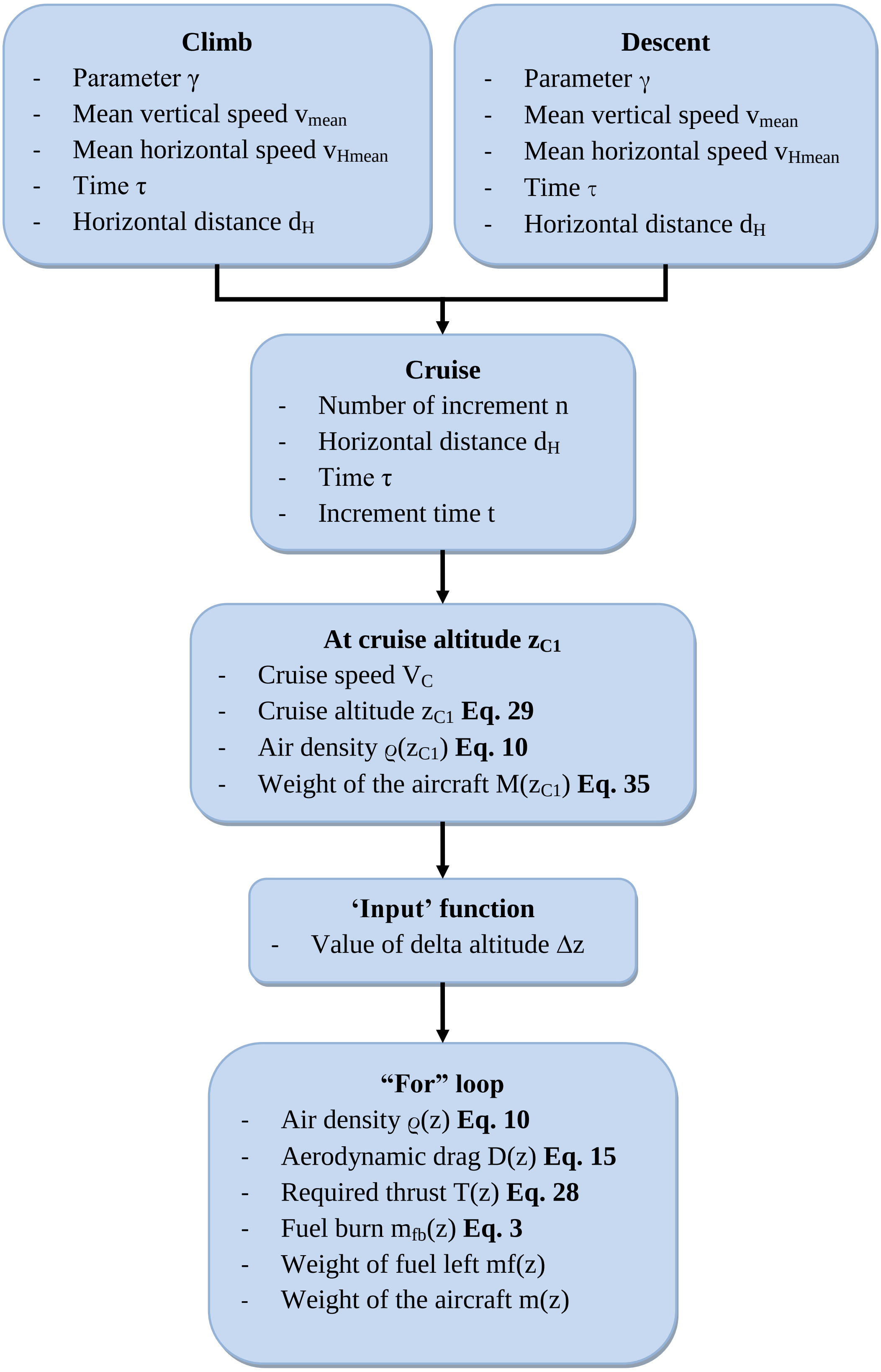

Figure 1.13: Cruise algorithm

In the computer code, the cruise phase for the trajectory Paris – New York is defined by the algorithm figure 13. The increment time is defined by the equation (24) and depends on the number of increment n in the cruise phase and the time of cruise.

2. Sample flight routes

In this analysis, three flight travels are considered:

- Trondheim – Oslo
- Trondheim – Nice
- Paris – New York

The three trajectories correspond at different type of flight: short, medium and long haul flights. The distance between the two airports is different and more or less important according to the trajectory. For each trajectory, in the computer code [Appendix B], two different aircrafts are defined with different characteristics: maximum takeoff weight m, wing area S, cruise Mach number and specific fuel consumption SFC.

For these trajectories, the parameters for the climb and the descent time determination γ modify the trajectory and have an influence on the fuel consumption. This influence is defined in the simulation analysis for different values of the parameters for the climb and descent time determination.

For the flight Trondheim – Oslo, the aircraft does not reach the maximal cruise altitude, consequently the cruise altitude can various and the influence of the cruise altitude on the fuel consumption can be defined.

The distance between the two airports, the aircraft characteristic and the parameters which have an influence on the trajectory are identified and defined below for each trajectory.

2.1. Trondheim – Oslo

The flight Trondheim-Oslo is a domestic flight. The straight distance between the two airports is 390km and the time of flight is 40 – 45 minutes between the takeoff and the landing. The times necessary for the taxi-out and the taxi-in are not considered. The type of aircraft is a short-range like the Airbus A320 Family, the Boeing 737 Classic and Next generation. The characteristics of the Airbus A320 and the Boeing 737 are given in the table 2.1.

Table 2.1: Characteristics Boeing 737-Classic, 737-Next generation and Airbus A320

	737-Classic (-300/-400/-500)	**737-Next generation (-600/-700/-800/-900)**	**A320**
Wing area [m²]	**105,4**	**125,58**	**122,6**
Maximum takeoff weight MTOW [kg]	**62 800** – 68 000	65 500 – 85 100 **79100 (-800)**	**77 000**
Cruise speed	**0,74** (780 km/h)	**0,78** (823 km/h)	
Maximum speed	0,82 (876 km/h)		
Maximum fuel Capacity [l]	**20 100**	**26 020**	**24050**
Engine x2	CFM 56-3	CFM 56-7B	CFM 56-5B
Thrust x2 [kN]	89 – 105	87 – 121	111 – 120
Service selling [m]	11300	12500	11890
SFC [(kg/h)/N]	**0,03977**	**0,03875**	**0,03467**

The red values on the table 2.1 correspond to the aircraft characteristics necessary for the simulation analysis.

For the flight Trondheim – Oslo, the analysis is realised with two aircrafts: the Boeing 737-300 Classic and the Boeing 737 – 800 Next generation. The weight of the aircraft at takeoff which is used in the computer code corresponds to the maximum weight at takeoff.

In the analysis of the flight Oslo-Trondheim by Paul Arentzen, the aircrafts which are considered, are the Boeing 737 – Classic and the Boeing 737 – 800 [1]. The value of the weight at takeoff does not correspond to the maximum takeoff weight which is used for the simulation part. In the computer code, the value of the aircraft weight can be modified and replaced with the value which is used by Paul Arentzen, to compare the results of the fuel consumption obtain with the result of the analysis of the flight Oslo – Trondheim (table 1.6).

For the domestic flight, when the distance between the two airports is short, the aircraft does not reach the maximum cruise altitude in each case. In the simulation analysis, the fuel consumption is analysed for different values of the cruise altitude.
In the part 2.6. Choice of the cruise altitude, the values of the cruise altitude which are considered, are defined and the influence on the choice of the parameters for the climb and the descent time determination.

In the computer code, the flight has to respect two conditions: the flight time and the straight distance between the two airports. These two parameters depend on the choices for the cruise altitude and the parameters γ. The analysis is realised for different values of these parameters.

The figure 2.1 gives some possible trajectories for the flight Trondheim – Oslo according to the cruise altitude and the parameter for the climb/descent time determination.

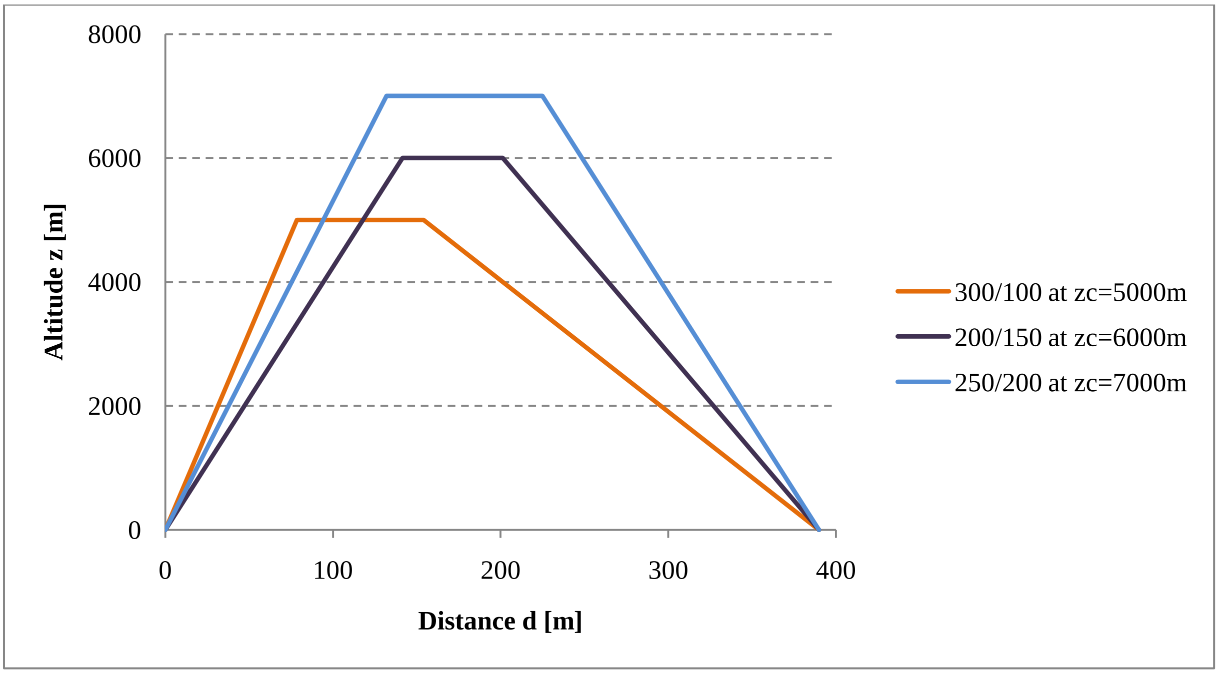

Figure 2.1: Trajectory profiles for the flight Trondheim – Oslo

The values 300 / 100, 200 / 150 and 250 / 200 correspond to the parameters for the climb and descent time determination.

2.2. Trondheim – Nice

The flight Trondheim – Nice is an international flight. The straight distance between the two airports is 2204km and the time of flight is approximately 4 hours.
The types of aircraft are medium – range like the Airbus A320 Family, the Boeing 737 Classic and Next generation. The characteristics of the aircrafts are given in the table 2.1.

For the flight Trondheim-Nice, the aircrafts chosen are the Airbus A320 and the Boeing 737 – Next generation (-800). For the A320, the airbus documentation [2] gives some values of the fuel consumption, the time of climb and the distance for the climb phase (Table 1.5).
In the computer code, the aircraft weight corresponds to the maximum weight at takeoff which is different to the weight used by the Airbus documentation. The value of the aircraft weight can be modified to compare the results of the simulation part with the results of the Airbus documentation concerning the climb phase.

The trajectory profile is defined previously in the part 1.2. Flight trajectory and computer code presentation. The cruise trajectory is horizontal at cruise altitude constant.

2.3. Paris – New York

The flight Paris – New York is a long haul flight. The straight distance between the two airports is 5851km and the time of flight is between 8 hours - 8 hours and 40min. The type of aircraft is long-range like the Airbus A330 or A340 and the Boeing 757 or 767 or 777.

The characteristics of the Airbus A340 and the Boeing 777 – 200 are given in the table 2.2.

Table 2.2: Characteristics Boeing 777 – 200 and Airbus A340

	777-200	**A340 (-200/-300)**
Wing area [m²]	427,8	361,6
Maximum takeoff weight MTOW [kg]	**247 200**	**275 000**
Cruise speed	**0,84** Mach (905 km/h)	**0,82** Mach (871 km/h)
Maximum speed	0,89 Mach (950 km/h)	0,86 Mach (913 km/h)
Maximum fuel Capacity [l]	117 348	155 040
Engine	(x2) PW 4077 (x2) RR 877 (x2) GE90-77B	(x4) CFM 56-5C
Thrust [kN]	(x2) PW: 342 (x2) RR: 338 (x2) GE: 342	(x4) 139 – 151
Service selling [m]	13 140	12527
SFC [(kg/h)/N]	**0,03365**	**0,03263**

The red values in the table 2.2 correspond to the aircraft characteristics necessary for the trajectory Paris – New York in the computer code.

For the flight Paris – New York, the aircraft chosen is the Airbus A340 to compare with the values given by the Airbus documentation for the climb phase [table 1.5].
The aircraft 777-200 is used to compare the results between the two aircraft and to define the influence of the choice of the aircraft.

In the computer code, the weight of the aircrafts corresponds to the maximum weight at takeoff. In the Airbus documentation, the aircraft weight is lighter. The weight of the aircraft can be modified to compare more precisely the values of the fuel burn for the climb phase.

The cruise altitude and the definition of the cruise trajectory are determined in the part 1.2.2.3. Cruise description.

In the simulation part, different trajectories are defined according to the choices of the parameters γ. The latter affects the times and the horizontal distances for the climb, cruise and descent phases and has an influence on the fuel consumption.

In the figure 2.2, are represented some trajectory profiles for the flight Paris – New York.

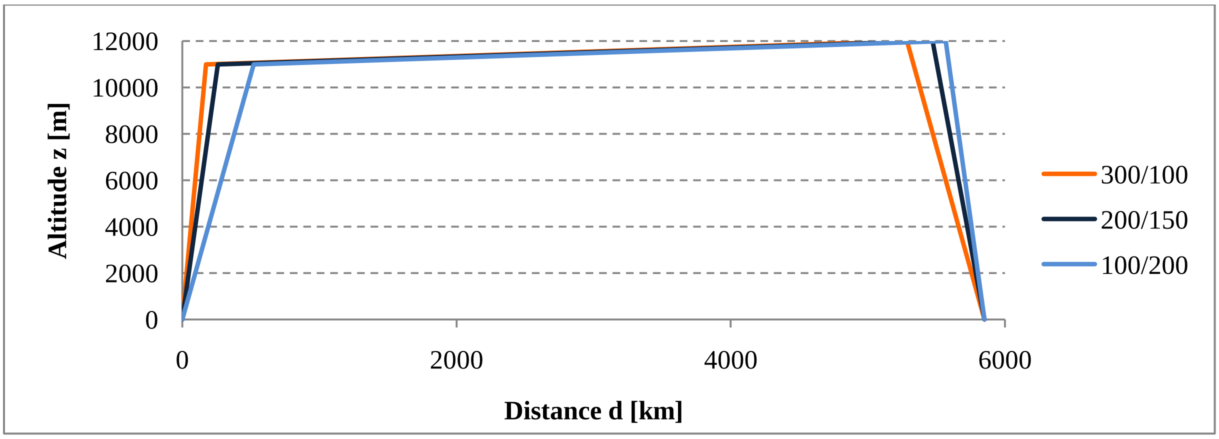

Figure 2.2: Trajectory profiles for the flight Paris – New York

300 / 100: the value 300 corresponds to the parameter for the climb and 100 the parameter for the descent time determination, similarly for 200 / 150 and 100 / 200. The variation of the parameters γ has an influence on the horizontal distances for the climb, cruise and descent phases and on the angles of climb and descent.

2.4. Choice of the flight trajectory

The first question when the computer code is running, is the choice of the destination. Three trajectories which are defined previously are possible: Trondheim – Oslo, Trondheim – Nice and Paris – New York. For each trajectory, the straight distance between the two airports is defined in the computer code.

In the software Matlab, the function “Menu” is used. A window is open with the three different destinations and the user has to select the desired destination (figure 2.3).

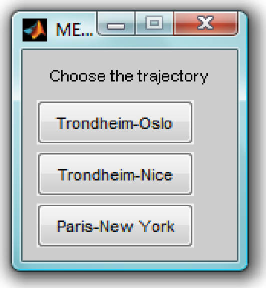

Figure 2.3: Function “Menu” on Matlab, choice of the destination

When the user chosen the destination, another window “Menu” is open with the choice of the aircrafts (figure 2.4). In the computer code, for each trajectory, the user has the choice between two aircrafts. The own characteristics of each aircraft are defined:

- Maximum takeoff weight m [kg]
- Wing area S [m^2]
- Maximum fuel capacity l_f [l]
- Parameter defining the dimensional velocity α [s/m], to define the cruise Mach number.
- Specific fuel consumption SFC [(kg/h)/N]

The different aircrafts and their characteristics which are used in the computer code are defined previously.

Figure 2.4: Choice of the aircraft for the flight Trondheim-Nice

The choice of the destination affects the cruise trajectory.

Two different cruise trajectories are defined previously (1.2.2. Cruise description). For the flights Trondheim – Oslo and Trondheim – Nice, the cruise trajectory is considered straight and horizontal at cruise altitude constant. While for the trajectory Paris – New York, the trajectory is linear and the aircraft flies between two cruise altitudes. At the end of the cruise, the aircraft reaches the maximum cruise altitude which is defined by the user.

In the computer code, two models define the cruise trajectory and they depend on the chosen destination, one model or the other is selected to calculate the trajectory and the fuel consumption. The fuel burn depends on the aircraft selected because the aircraft characteristics are different and have an influence on the different parameters which step in the fuel consumption equation (3).

The destination and the aircraft are selected.

2.5. Time of climb

In this part, the different estimations of the climb and cruise times and horizontal distance are calculated for the trajectory Trondheim – Nice, with the Airbus A320. The aircraft characteristics are given previously (2.1. Trondheim – Oslo).

The time of climb depends on the cruise altitude and the mean vertical speed, the latter depending on the parameter for the climb time determination γ (equations 8 and 7). The cruise altitude is defined in the next part and depends on the selected destination.
After that the user chosen the destination and the aircraft, the parameter for the climb time determination γ has to be entered. The function "input" is using to ask the user to enter the value of the parameter γ. The selected range is $100 \leq \gamma \leq 300$. The parameter γ has an influence on the mean vertical speed and the time of climb.

The mean vertical speed v_{mean} and the climb time τ are calculated for different values of the parameter γ for the trajectory Trondheim – Nice. The cruise altitude selected is $z_c = 10058\ m$ (value of the cruise altitude used in the Airbus documentation).

In the table 2.3, when the value of the parameter γ augments, the mean vertical speed increases and consequently the time of climb decreases. The maximal value of the vertical speed increases when the parameter γ augments.

Table 2.3: Climb times and mean vertical speed according to the parameter γ [Appendix A]

	γ=100	**γ=150**	**γ=200**	**γ=250**	**γ=300**
$\tau\ [s]$	3020	2014	1510	1208	1007
$\tau\ [min]$ $\tau\ [hr]$	50 0,84	33,5 0,56	25 0,42	20 0,33	17 0,28
$v_{mean}\ [m/s]$	3,33	4,99	6,66	8,33	9,99
$v_{max}\ [m/s]$ **at Z=0,5**	6,25	8,375	12,50	15,625	18,75

The maximum value of the parameter γ for the climb time determination is 300 which correspond to a time of climb at 17 min. Over $\gamma = 300$, the time of climb is considered too small. In the reality, the range for the time of climb is 20 – 25 min to reach the cruise altitude.

The choice of the parameter γ affects the climb horizontal distance (equation 21). The distance depends on the mean vertical speed, the mean velocity along the flight path and the time of climb.

The velocity along the flight path does not depend on the parameter γ. The mean velocity along the flight path is constant for all the γ values.

The mean vertical speed is smaller than the mean velocity along the flight path. The mean horizontal velocity is supposed constant because the influence of the mean vertical speed is low.

Table 2.4: Climb horizontal distances according to the parameter γ

$\boldsymbol{\alpha}$	[s/m]	0,0697
$\boldsymbol{\beta}$	[-]	60000
$\boldsymbol{V_{mean}}$	[m/s]	157,85
$\boldsymbol{v_{Hmean}}$	[m/s]	157,5

$\boldsymbol{\gamma}$	[-]	100	150	200	250	300
$\boldsymbol{\tau}$	[h]	0,84	0,56	0,42	0,33	0,28
$\boldsymbol{d_{Hclimb}}$	[km]	477	318	238	190	159

The climb horizontal distance decreases when the parameter γ augments. For a value of the parameter γ low, the climb horizontal distance is important.

The distances between the two airports for the flights Trondheim – Nice and Paris – New York are important, superior at 2000 km. The maximum climb horizontal distance corresponds to less a quarter of the total distance. The trajectories can be calculated for the different values of the parameter for the climb time determination γ which are defined above.

On the other hand, the distance between the airports of Trondheim and Oslo is equal to 390km which is inferior to the climb horizontal distance for some values of the parameter γ.
The parameter γ range for the trajectory Trondheim – Oslo is $200 \leq \gamma \leq 300$, to respect the distance between the two airports.

The climb distance for the flight Trondheim – Oslo varies according to the selected cruise altitude (2.6.Choice of the cruise altitude). The choices of the parameter for the climb time determination γ and the cruise altitude z_C affect the choice of the parameter for descent time determination γ.

The variation of the parameter γ for the climb time determination modifies the climb horizontal distance, consequently the cruise horizontal distance. When the parameter γ augments, the climb horizontal distance decreases and the cruise horizontal distance increases. The cruise horizontal distance is the difference between the total distance and the climb and descent horizontal distances (equation 38). It depends on the parameters for the climb and the descent time determination γ. The cruise time depends only on the cruise horizontal distance because the cruise speed is considered constant during the cruise phase (equation 39). When the cruise horizontal distance increases, the time of cruise increases.

The parameter γ affects the time of climb and consequently the fuel consumption because the fuel consumption depends on the time and the required thrust (equation 3).

The required thrust depends on the parameter γ. Indeed, the thrust depends on the drag due to the gravity (equation 14), the aerodynamic drag (equation 19) and the acceleration force (equation 13). The first depends on the climb angle θ which is a function of the vertical speed, the second depends on the lift coefficient which depends on the climb angle θ and the latter depends on the parameter γ. The influence of the parameter γ on the different forces is not identical. On the aerodynamic drag, the influence of the parameter γ is negligible. The acceleration force has a low influence on the required thrust. Consequently, the variation of the parameter γ has principally an influence on the gravity force. The latter varies linearly according to the parameter γ.

The fuel consumption depends on the parameter γ. For the Airbus A320, the specific fuel consumption value is equal to 0,03467(kg/h)/N.

The estimations of the fuel consumption are identified in the table 2.5 and realized with the file Excel (the aircraft weight is considered constant). The fuel burn is calculated with the equation (3) for the mean required thrust.

Table 2.5: Fuel consumption according to the parameter γ

γ	[-]	100	150	200	250	300
τ	[h]	0,84	0,56	0,42	0,33	0,28
G_{mean}	[N]	16220	24330	32440	40550	48660
T_{mean}	[N]	58045	67060	76069	85070	94064
m_{fb}	[kg]	1690	1302	1108	973	913
m_{fb}/h	[kg/h]	2012	2325	2638	2948	3261

When the parameter γ is lower, the time of climb increases and the mean required thrust decreases. The fuel consumption per hour augments when the parameter γ decreases. For the fuel consumption equation (3), the parameter γ has an influence more important on the time of climb than on the required thrust, consequently when the parameter γ augments the fuel consumption decreases.

The objective is to find the value of the parameter γ for the climb time determination enable to give the configuration with the less fuel consumption for the climb. The time of climb has to respect the time available for the trajectory which is chosen.

The optimal configuration to reduce the fuel burn depends on the parameters γ for the climb and the descent time determination and the cruise altitude. The values possible for the cruise altitude and the parameter γ for the descent time determination have to be defined.

2.6. Choice of the cruise altitude

The cruise altitude depends on the trajectory. Each aircraft is defined by the maximum cruise altitude which is given by the constructor characteristics. In this analysis, the selected cruise altitude does not correspond to the maximum cruise altitude and depends on the trajectory that the user chosen.

For the flight trajectory **Trondheim – Nice**, the aircraft can reach the maximum cruise trajectory given by the characteristics of the aircraft. In this analysis, the value of the cruise altitude is equal to $z_C = 33000\ ft$ (10058m), to compare with the values of the climb phase from the Airbus documentation. The cruise altitude is constant during the cruise phase, so has no influence on the fuel consumption. The latter depends on the values of the parameter for the climb and the descent time determination γ.
In the computer code, the function "input" is used, the user can choose the desired cruise altitude.

For the flight **Paris – New York**, the distance between the two airports is more important and consequently the cruise phase is longer. The trajectory of the cruise phase is defined previously (1.2.2.3. The cruise description), it is linear and the aircraft flies between the cruise altitude $z_{C1} = 11000\ m$ and the cruise altitude $z_{C2} = 12000\ m.$ The delta altitude between the two cruise altitudes is $\Delta z_C = 1000\ m.$ The aircraft does not reach the maximum cruise altitude given by the characteristics of the aircraft, but it is close to this altitude at the end of the cruise.

During the cruise, the weight of the aircraft decreases because the quantity of fuel decreases and consequently the gravity force perpendicular to the flight path decreases. The forces perpendicular to the flight path which are applied on the aircraft during the cruise are the gravity and the lift. At the cruise altitude z_{C1}, the gravity is equal to the lift. When the weight decreases, the gravity decreases and the lift is more important that the gravity force so the aircraft is going up.

The gravity perpendicular to the flight path at the cruise altitude is given by the equation (17), the climb angle θ is considered very small consequently the cosine of the climb angle θ is considered equal to 1. In the table 2.6, the values of the aircraft weight and the gravity force are given for the different values of the parameter γ climb at cruise altitude z_{C1}. The aircraft is the Airbus A340 and the aircraft characteristics are given previously (2.3. Paris – New York). The value of the specific fuel consumption is $SFC = 0{,}03263.$

Table 2.6: Aircraft weights and gravity forces

τ	[h]	0,92	0,61	0,46	0,37	0,30
γ	[-]	100	150	200	250	300
T_{mean}	[N]	208905	239562	270191	300793	331367
m_{fb}	[kg]	6255	4782	4045	3602	3307
$m(z_{C1})$	[kg]	268745	270218	270955	271398	271693
G	[N]	2636390	2650841	2658070	2662410	2665307

where m is the aircraft weight at the cruise altitude z_{C1}, G the gravity force perpendicular to the flight path and m_{fb} the fuel burn during the climb phase.
The values are obtained with the Excel file, consequently the weight does not vary during the climb phase.

The aircraft is at the cruise altitude z_{C1} and begins the cruise phase between the two cruise altitudes. To determine the variation of the cruise altitude, the fuel consumption during the cruise has to be calculated.

The fuel burn is defined by the equation (3) and depends on the required thrust and the time of climb. The latter depends on the horizontal distance which varies according to the parameters for the climb and the descent time determination. The distance between the two airports is equal to 5851km and the mean horizontal velocity is equal to 166 m/s. The horizontal distance and the time of cruise are given in the table 2.7.
The parameter for the descent time determination is considered constant and equal to 100. Therefore, the time of descent is equal to $\tau_d = 1h$ and the horizontal distance is $d_H = 597867m$. The required thrust is equal to 142kN.The air density at cruise altitude $z_{C1} = 11000m$ is equal to $\rho_1 = 0{,}408\ kg/m^3$ and the cruise velocity is $V = 235{,}25\ m/s$.

Table 2.7: The horizontal distance, the time and the fuel consumption during the climb

		Climb		**Cruise**			
γ_{cl}	**m_1**	**τ**	**d_H**	**τ**	**d_H**	**m_{fb}**	**Δz**
100	268745	0.918	548054	6.261	5302946	29010	1101
150	270218	0.612	365277	6.477	5485723	30011	1133
200	270955	0.459	273861	6.585	5577139	30511	1149
250	271398	0.367	218990	6.650	5632010	30812	1158
300	271693	0.306	182390	6.693	5668610	31012	1164

The variation of the cruise altitude is defined with the equation (37) and the values which are obtained for the different configurations are identified in the table 2.7.
The value of the delta cruise altitude is close to $\Delta z_C = 1000m$, this value is used in the computer code.

For the flight **Trondheim – Oslo**, the distance between the two airports is short, $d = 390\ km$. The aircraft does not reach the maximum cruise altitude. The trajectory of the flight is defined for different values of the cruise altitude. The latter affects the choice of the parameters for the climb and the descent time determination γ. The values range for the cruise altitude is $5000 \leq z_C \leq 7000$. The variation of the cruise altitude modifies the time and the horizontal distance for the climb and the descent phase, consequently the horizontal distance and the time for the cruise phase.

The time depends on the cruise altitude (Equation 8). When the altitude augments, the times for the climb and the descent increase. The horizontal distance depends on the time and increases when the cruise altitude augments (Equation 21). The cruise altitude has an influence on the fuel consumption, the latter increases when the time augments so when the cruise altitude is higher.

The cruise altitude has an influence on the time, the horizontal distance and the fuel consumption for the climb and descent phases. This influence is represented in the table 2.8 for the aircraft 737-300 which the characteristics are defined previously (2.1. Trondheim – Oslo). The specific fuel consumption is equal to 0,03977.

Table2.8: Variation of the time, the horizontal distance and the fuel consumption for the climb

z_C	[m]	5000	5500	6000	6500	7000
τ	[h]	0,139	0,153	0,167	0,181	0,194
d_H	[m]	78845	86730	94614	102499	110383
T_{mean}	[N]	81583	80580	79743	79040	78446
m_{fb}	[kg]	451	490	529	568	607
m_{fb}/h	[kg/h]	3245	3203	3168	3138	3129

where the parameter for the climb time determination equal to $\gamma = 300$.

The choice of the cruise altitude affects the fuel consumption during the climb phase. The latter increases when the cruise altitude is higher.

The fuel consumption per hour is more important for the shorter cruise altitude, but the time of climb decreases. The time of climb has an influence more important on the fuel consumption than the mean required thrust. The climb horizontal distance is longer when the cruise altitude is higher.

In the computer code, the function “Input” is used to ask the value for the cruise altitude for each trajectory. For the flights Trondheim – Oslo and Trondheim – Nice, the aircraft is flying at cruise altitude constant and the user has to enter in the computer the desired value for the cruise altitude. For the flight Paris – New York, the cruise altitude and the delta altitude have to be defined by the user, in the computer code.

2.7. Time of descent

The choice of the parameter for the descent time determination has an influence on the time of descent and the horizontal distance. If the descent horizontal distance varies, the cruise horizontal distance and consequently the time of cruise are modified. The parameter for the descent time determination γ affects the fuel consumption for the cruise and the descent phases. The choice of the parameter γ depends on the selected trajectory.

For **the flights Trondheim – Nice and Paris – New York**, the distances between the airports are important, superior at 2000km. The maximum horizontal distance for the climb and the descent phase is inferior at 600km for the minimum value of the parameter γ considered. The parameter for the descent time determination γ can vary between 100 and 200.

On the other hand, for **the flight Trondheim – Oslo**, the distance between the two airports is short, d=390km. The values of the parameter for the descent time determination γ depend on the cruise altitude and the value of the parameter for the climb time determination γ. In the table 2.9, the different possible values of the parameter γ are defined according to the parameter for the climb time determination and the cruise altitude.

Table 2.9: Values of the parameter for the descent time determination γ

z_C $[m]$	**γ climb [-]**	**γ descent [-]**
5000	200 – 300	100 – 200
5500	200 – 300	100 – 200
6000	200 – 250 300	150 – 200 100 – 200
6500	200 – 300	150 – 200
7000	200 – 300	150 – 200

In the software Matlab, the function “input” is used to ask the value of the parameter for the descent time determination γ. In the computer code, the value is asking at the beginning of the cruise phase to define the cruise horizontal distance (equation 38) and the time of cruise (equation 39). The cruise horizontal distance depends on the climb and descent horizontal distances. It is necessary to know the descent horizontal distance consequently the parameter for the descent time determination γ. The parameter for the climb time determination γ is defined during the climb phase.

The parameter γ affects the value of the gravity force, consequently the required thrust. The required thrust during the descent phase is given by the equation (25). When the parameter γ increases, the sinus of the descent angle increases and the gravity force increases, consequently the required thrust decreases. With the Excel file, for the values of the parameter γ superior to 100, the required thrust is negative at some dimensionless altitude. In the reality, it is impossible to have a required thrust negative. In the part 1.2.1. Climb and descent descriptions, a limit thrust is defined and equal to 10% of the maximum required thrust.

In the table 2.10, the estimations of the time and horizontal distance for the descent phase are identified for the flight Trondheim – Oslo according to the values of the cruise altitude.
The mean velocity along the flight path is constant, V_{mean}=157,85m/s, but depends on the parameter α according to the aircraft chosen.

Table 2.10: Times and horizontal distances for the descent, flight Trondheim – Oslo

Altitude	[m]	**5000**			**6000**			**7000**	
γ	[-]	100	150	200	100	150	200	150	200
τ	[h]	0,417	0,278	0,208	0,500	0,334	0,250	0,389	0,292
v_{mean}	[m/s]	3,33	4,99	6,66	3,33	4,99	6,66	4,99	6,66
d_H	[km]	237	158	118	284	190	142	221	166

2.8. Profile of the vertical speed

The vertical speed depends on the parameter for the climb or the descent time determination γ and the dimensionless altitude Z (Equation 5). In $Z = 0$ and $Z = 1$, the vertical speed is equal to zero. The mean vertical speed depends only on the parameter γ (Equation 7). The profile of the vertical speed is given in the figure 2.5. It is represented for the parameter for the climb time determination γ equal to 300.

As defined previously, the vertical speed can be represented by a parabola. The values which define the parabola are the mean vertical speed and the speeds at Z=0 and Z=1. The parabola is given by the equation 9.

In the computer code, the user can choose the profile of the vertical speed. In the results and discussion part, the influence of the vertical speed profile on the fuel consumption is determined.

For the both profiles, the mean and the minimum vertical speeds are identical, but the maximal values of the vertical speed are different.

The vertical speed has an influence on the climb and descent angles (Equation 6). With the parabola profile of the vertical speed, the climb angle has a parabolic behaviour (figure 18).

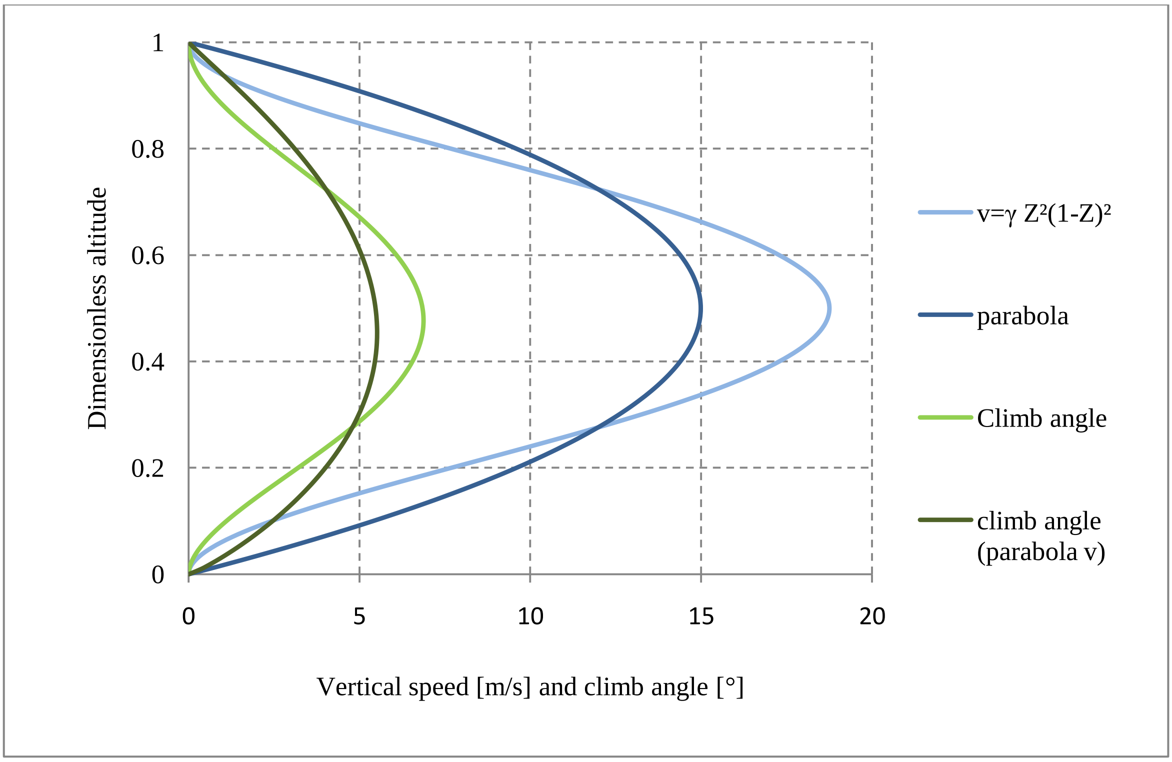

Figure 2.5: The two configurations of the vertical speed and the influence on the climb angle

The profile of the vertical speed has an influence on the gravity and the aerodynamic drag. This latter depends on the lift coefficient which is a function of the climb or descent angle.

The aerodynamic drag is considered constant for the two profiles of the vertical speed because their influence on the lift coefficient is insignificant.
The variation of the gravity force according to the profile of the vertical speed generates the modification of the required thrust profile. The gravity force and the required thrust have a parabolic behaviour when they are calculated with the parabolic profile of the vertical speed.

The forces which apply on the aircraft are represented in the figure 2.6 for the two profiles of the vertical speed. The forces are calculated for the flight Trondheim – Nice with the aircraft Airbus A320.

The mean values of the gravity and the required thrust are conserved.

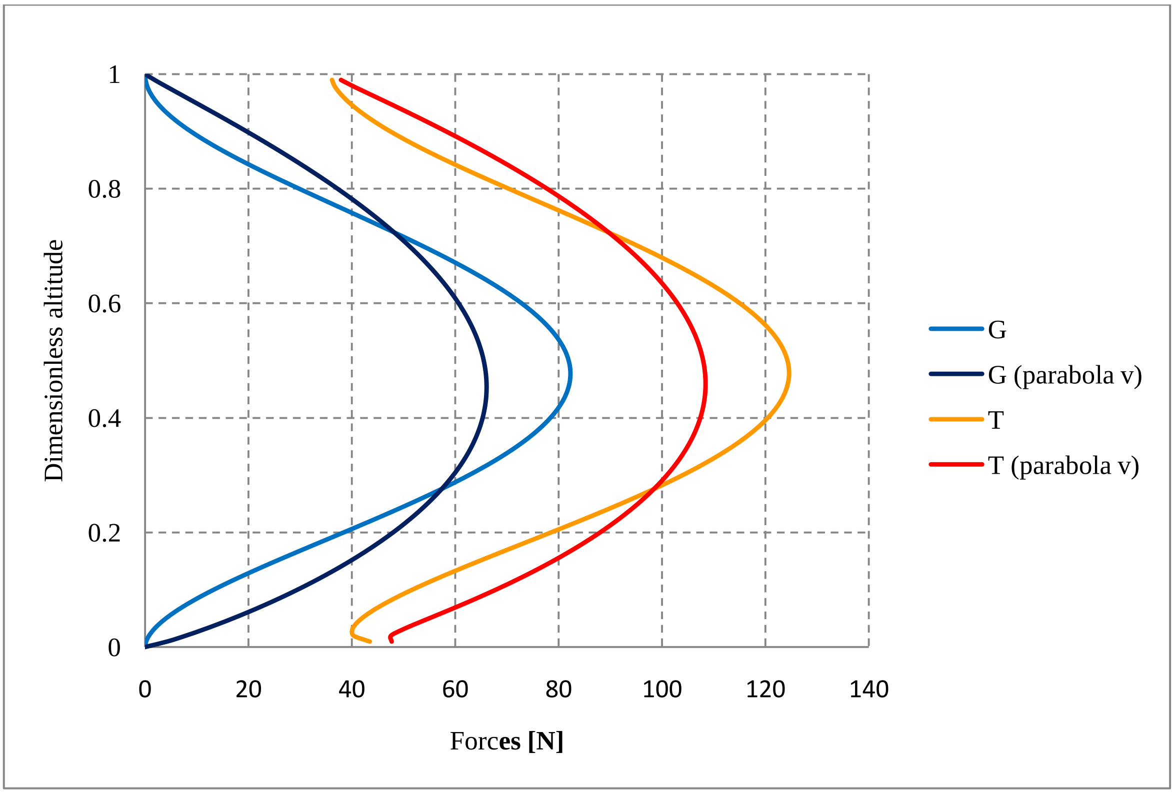

Figure 2.6: Forces which apply on the aircraft for the two vertical speed profiles

In the computer code, the different equations are calculated step by step. The influence of the vertical profile on the fuel consumption can be defined.

The destination, the parameters for the climb and descent time determination γ, the cruise altitude and the profile of the vertical speed have an influence on the fuel consumption more or less important. These influences are defined in the part (3. Results and discussion).

3. Results and discussion

3.1. Influence of the aircraft choice

3.1.1. Trondheim – Oslo

The flight trajectory Trondheim – Oslo is realized and calculated for two different aircrafts: the Boeing 737 – 300 Classic and the 737 – 800 Next Generation.
The characteristics of the two aircrafts, the parameters and constants are defined in the tables 3.1 and 3.2. The vertical speed is defined by the equation (5).

The quantity of fuel at takeoff depends on the distance of the flight trajectory. In this analysis, the quantity of fuel is considered equal to the maximum fuel capacity so the weight of the aircraft corresponds to the maximum takeoff weight. In the reality, for this flight trajectory, the weight of the aircraft is lighter.

Table 3.1: Characteristics aircrafts

	Units	**737 – 300**	**737 – 800**
Wing area	m²	105,4	125,58
MTOW	kg	62800	79100
Max. fuel capacity	l	20100	26020
A	s/m	0,0735	0,0697
Cruise speed	m/s	212,207	223,776
Cruise Mach number	-	0,74	0,78
Speed of sound at zc	m/s	286,6	286,6
SFC	(kg/h)/N	0,03977	0,03875

Table 3.2: Parameters and constants

	Units	**Values**
$\boldsymbol{\beta}$	-	60000
$\boldsymbol{\rho_0}$	kg/m^3	1,225
$\boldsymbol{\varepsilon}$	1/m	0,0001
$\boldsymbol{k}$	-	0,045
$\boldsymbol{C_{D,0}}$	-	0,015
$\boldsymbol{\rho_f}$	kg/m^3	817,15
Distance	m	390000

The cruise altitude varies and the results are obtained for three different cruise altitudes: z_c=5000m, 6000m and 7000m.

The parameters for the climb and the descent time determination vary according to the cruise altitude (table 19).
The results are given in the Appendix C.

In the tables 3.3 and 3.4, the configurations with the minimum fuel consumption are identified for the three different cruise altitudes with the Boeing 737 – 300 and the 737 – 800.

Table 3.3: Fuel consumption for the Boeing 737 – 300, Trondheim – Oslo

	Tmax	**d_H**			**Time**				**Fuel burn**			
	N	**cl.**	**c.**	**d.**	**cl.**	**c.**	**d.**	**total**	**cl.**	**c.**	**d.**	**Total**
		km			h				kg			
1	1,07E+5	89	81	220	0,165	0,106	0,409	0,680	490	152	289	931
2	1,19E+5	89	37	264	0,165	0,048	0,491	0,704	535	66	359	960
3	1,18E+5	104	81	205	0,193	0,106	0,382	0,680	615	141	253	1009

Table 3.4: Fuel consumption for the Boeing 737 – 800, Trondheim – Oslo

	Tmax	**d_H**			**Time**				**Fuel burn**			
	N	**cl.**	**c.**	**d.**	**cl.**	**c.**	**d.**	**total**	**cl.**	**c.**	**d.**	**Total**
		km			h				kg			
1	1,31E+5	94	64	232	0,165	0,079	0,409	0,653	591	144	357	1092
2	1,46E+5	94	18	278	0,165	0,022	0,491	0,678	642	38	441	1121
3	1,44E+5	109	64	217	0,193	0,079	0,382	0,654	738	131	309	1178

The three cases correspond to:

- Configuration 1:
    - Cruise altitude 5000m
    - Parameter for the climb time determination 250
    - Parameter for the descent time determination 100
- Configuration 2:
    - Cruise altitude 6000m
    - Parameter for the climb time determination 300
    - Parameter for the descent time determination 100
- Configuration 3
    - Cruise altitude 7000m
    - Parameter for the climb time determination 300
    - Parameter for the descent time determination 150

The configurations with the less fuel conception are identical for the two aircrafts according to the cruise altitude.
The fuel consumption is more important for the Boeing 737 – 800 Next Generation. The variation of the fuel consumption between the two aircraft is 15% at the end of the trajectory.

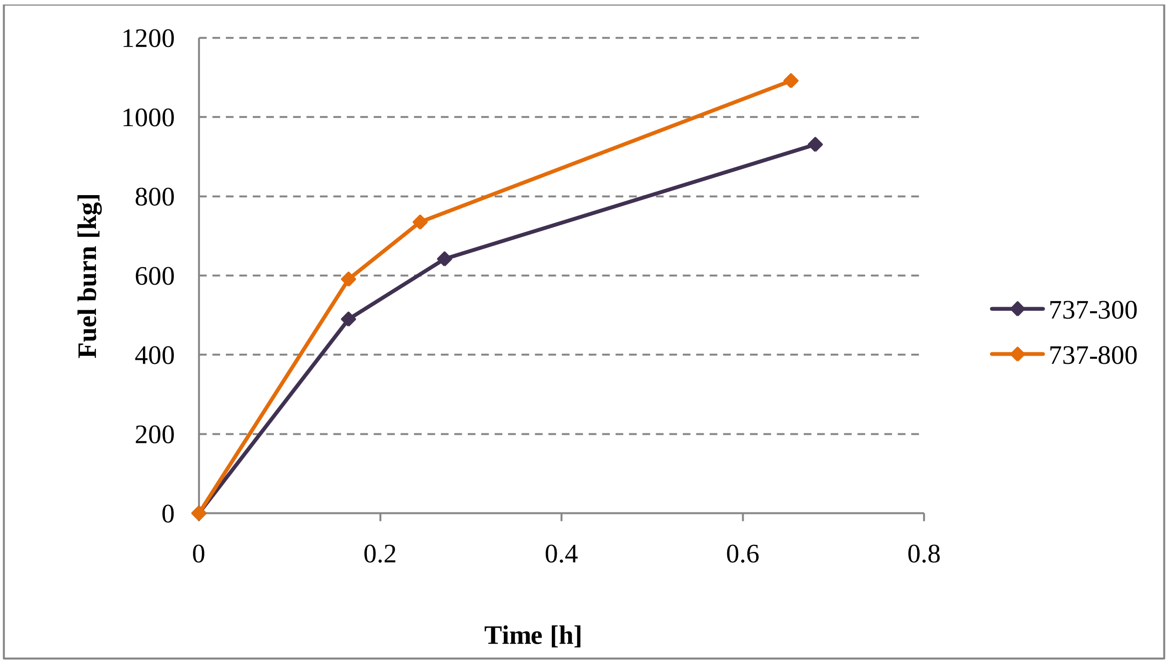

Figure 3.1: The fuel consumption according to the time for z_c=5000m, Trondheim – Oslo

The fuel consumption depends on the required thrust, the time and the specific fuel consumption (Equation 3).

For each configuration, the times of climb and descent do not depend on the characteristics of the aircraft so they are identical for the two aircrafts. They depend on the parameter for the climb and descent time determination and the cruise altitude. The time of cruise depends on the cruise speed. For the aircraft 737 – 800, the cruise Mach number is more important, so for the same cruise distance, the time of cruise is shorter. The variation of the time of cruise and consequently the time of the flight trajectory is $\Delta\tau = 0{,}026h$.

The thrust depends on the aerodynamic drag, the acceleration force and the gravity (Equation 20 for the climb phase and equation 25 for the descent). All these forces depend on the weight of the aircraft and the parameter α. The aerodynamic drag depends also on the wing area. The forces are given by the equations (13), (14) and (19).
The characteristics of the aircrafts are different. The wing area and the maximum takeoff weight are more important for the Boeing 737 – 800. The parameter α which affects the cruise speed is smaller for the Boeing 737 – 800.

The acceleration force is more important for the Boeing 737 – 800. Indeed, the acceleration force increases if the weight of the aircraft increases or/and if the parameter α decreases.

The drag gravity component depends on the weight of the aircraft and the climb or descent angle. The latter depends on the velocity along the flight path, consequently to the parameter α. When the aircraft weight increases, the gravity force increases, but when the velocity along the flight path increases, the sinus of the angle decreases and the gravity decreases. In this case, the influence of the weight of the aircraft is more important than the parameter α. The value of the gravity is higher for the Boeing 737 – 800.

The aerodynamic drag depends on the wing area, the weight of the aircraft and the velocity along the flight path. All of these characteristics are more important for the Boeing 737 – 800, so the value of the aerodynamic drag is higher.

The required thrust is more important for each phases of the flight trajectory, with the Boeing 737 – 800. The specific fuel consumption is different for the two aircrafts and the value is higher for the flight trajectory with the Boeing 737 – 300.
The variation of the thrust has more influence on the fuel burn than the time of the cruise and the specific fuel consumption, consequently the fuel burn is more important with the Boeing 737 – 800.

3.1.2. Trondheim – Nice

The flight trajectory Trondheim – Nice is realized and calculated for two different aircrafts: the Airbus A320 and the Boeing 737 – 800 Next Generation.
The characteristics of the two aircrafts are defined in the table 3.5. The parameters and constants are conserved and given in the table 3.2, but **the distance** between the two airports is different and is equal to **2204km**. The vertical speed is defined by the equation (5).

Table 3.5: Characteristics aircrafts

	Units	A320	737 – 800
Wing area	m^2	122,6	125,58
MTOW	kg	77000	79100
Max. fuel capacity	l	24050	26020
A	s/m	0,0697	0,0697
Cruise speed	m/s	223,776	223,776
Cruise Mach number	-	0,78	0,78
Speed of sound at zc	m/s	286,6	286,6
SFC	(kg/h)/N	0,03467	0,03875

In the tables 26 and 27, the three configurations with the minimum fuel consumption are identified for the Airbus A320 and the Boeing 737–800. The cruise altitude is $z_c = 10058m$.

Table 3.6: Horizontal distances and times, Trondheim – Nice

	Horizontal distance			Time			
	Climb	**Cruise**	**Descent**	**Climb**	**Cruise**	**Descent**	**Total**
	km			h			
1	189	1548	467	0,332	1,922	0,822	3,077
2	157	1580	467	0,277	1,961	0,822	3,1
3	157	1736	311	0,277	2,154	0,548	2,980

The horizontal distances and the times for the different phases of the flight trajectory are identical for the two aircrafts because the values of the speed, the cruise altitude and the parameters for the climb and descent time determination are equal.

Table 3.7: Fuel consumption for the Airbus A320 and the Boeing 737 – 800, Trondheim - Nice

	Airbus A320					Boeing 737 – 800				
	Tmax	**Fuel burn**				**Tmax**	**Fuel burn**			
	N	**Climb**	**Cruise**	**Descent**	**Total**	N	**Climb**	**Cruise**	**Descent**	**Total**
		kg					kg			
1	1,21E+5	977	2589	614	4180	1,24E+5	1122	2968	701	4791
2	1,37E+5	902	2644	630	4176	1,40E+5	1035	3031	720	4786
3	1,37E+5	902	2905	378	4185	1,40E+5	1035	3330	432	4797

The three configurations which are identified correspond to the less fuel consumption configurations. The maximal difference of the fuel burn between them is small, 8kg for the Airbus A320 and 11kg for the Boeing 737 – 800 compare to the total fuel consumption.

The three cases correspond to:

- Configuration 1:
  - Parameter for the climb time determination 250
  - Parameter for the descent time determination 100
- Configuration 2:
  - Parameter for the climb time determination 300
  - Parameter for the descent time determination 100
- Configuration 3
  - Parameter for the climb time determination 300
  - Parameter for the descent time determination 150

In the figure 3.2, the accumulated fuel consumption for the two aircrafts is represented with the configuration 2 which corresponds to the less fuel burn during the flight trajectory.

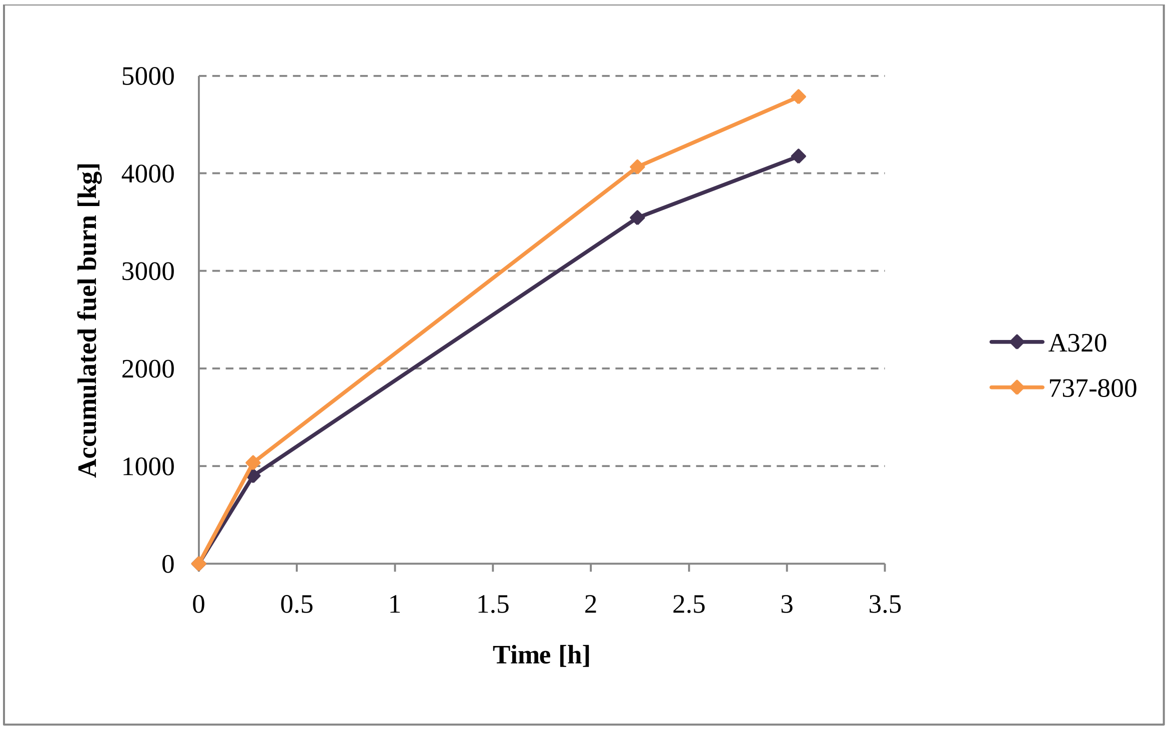

Figure 3.2: Accumulated fuel consumption according to the time of flight, Trondheim – Nice

For the trajectory Trondheim – Nice, with the aircraft Boeing 737 – 800, the fuel consumption is superior. The variation of the fuel burn between the two aircraft at the end of the trajectory is equal to 13% of the value of fuel burn for the Boeing 737 – 800.

The fuel consumption is given by the equation (3). The time of the trajectory is identical for the two aircrafts; consequently the fuel burn depends on the required thrust and the specific fuel consumption.

The specific fuel consumption is more important for the Boeing 737 – 800, so the fuel consumption increases.

The required thrust depends on the weight of the aircraft, the wing area and the parameter α which defines the cruise Mach number. The latter is equal for the two aircrafts. On the other hand, the values of the weight of the aircraft and the wing area are higher for the Boeing 737 – 800. Accordingly, the required thrust augments for the Boeing 737 – 800, hence the augmentation of the fuel consumption.

3.1.3. Paris – New York

The flight trajectory Paris – New York is realized and calculated for two different aircrafts: the Airbus A340 and the Boeing 777 – 200.

The characteristics of the two aircrafts are defined in the table 3.8. The parameters and constants are conserved and given in the table 22, but **the distance** between the two airports is different and is equal to **5851km**. The vertical speed is defined by the equation (5).

Like for the other flight trajectories, the weight of the aircraft corresponds to the maximum takeoff weight.

Table 3.8: Characteristics aircrafts

	Units	737 – 300	737 – 800
Wing area	m²	361,6	427,8
MTOW	kg	275000	247200
Max. fuel capacity	l	155040	117348
A	s/m	0,0663	0,0648
Cruise speed	m/s	235,252	240,698
Cruise Mach number	-	0,82	0,84
Speed of sound at zc	m/s	286,6	286,6
SFC	(kg/h)/N	0,03263	0,03365

In the tables 3.9 and 3.10, the four configurations with the minimum fuel consumption are identified for the Airbus A340 and the Boeing 777–200. The cruise altitude varies between $z_{c1} = 11000m \text{ and } z_{c2} = 12000m$.

The four cases correspond to:

- Configuration 1:
    - Parameter for the climb time determination 250
    - Parameter for the descent time determination 100
- Configuration 2:
    - Parameter for the climb time determination 250
    - Parameter for the descent time determination 150
- Configuration 3
    - Parameter for the climb time determination 300
    - Parameter for the descent time determination 100
- Configuration 4
    - Parameter for the climb time determination 300
    - Parameter for the descent time determination 150

Table 3.9: Fuel consumption for the Airbus A340, Paris – New York

	Tmax	**d_H**			**Time**				**Fuel burn**			
	N	**cl.**	**c.**	**d.**	**cl.**	**c.**	**d.**	**total**	**cl.**	**c.**	**d.**	**Total**
		km			h				kg			
1	4,20E+5	217	5048	586	0,363	5,960	0,982	7,306	3554	25768	2443	31765
2	4,20E+5	217	5243	391	0,363	6,191	0,655	7,209	3554	26766	1430	31750
3	4,76E+5	181	5084	586	0,303	6,003	0,982	7,288	3266	25984	2497	31747
4	4,76E+5	181	5279	391	0,303	6,234	0,655	7,191	3266	26983	1492	31741

Table 3.10: Fuel consumption for the Boeing 737 – 800, Paris – New York

	Tmax	**d_H**			**Time**				**Fuel burn**			
	N	**cl.**	**c.**	**d.**	**cl.**	**c.**	**d.**	**total**	**cl.**	**c.**	**d.**	**Total**
		km			h				kg			
1	3,69E+5	222	5029	600	0,363	5,804	0,982	7,149	3196	23622	2194	29012
2	3,69E+5	222	5229	400	0,363	6,035	0,655	7,053	3196	24563	1280	29039
3	4,18E+5	185	5066	600	0,303	5,847	0,982	7,132	2941	23816	2242	28999
4	4,18E+5	185	5266	400	0,303	6,078	0,655	7,036	2941	24757	1337	29035

The four configurations which are identified correspond to the less fuel consumption configurations. The maximal difference of the fuel burn between them is not important compare to the total fuel consumption, 24kg for the Airbus A340 and 40kg for the Boeing 777 – 200.

For the two aircraft, the minimum fuel consumption is not obtained for the same configuration. For the Airbus A340, the values of the parameter for the climb and the time determination for the less fuel burn are equal to 300 and 150. Whereas the parameter for the descent time determination is equal to 100 for the less fuel consumption with the Boeing 777 – 200.

In the figure 3.2, the accumulated fuel consumption for the two aircrafts is represented with the configuration 3 for the Boeing 777 – 200 and the configuration 4 for the Airbus A340 which correspond to the less fuel burn during the flight trajectory.

The time of climb and descent are equal for the two aircrafts because they depend on the parameters for the climb and the descent time determination and the cruise altitude. These parameters are equal in the two cases. The time of cruise with the Boeing 777 – 200 for a same cruise distance is shorter because the cruise Mach number is more important.

For the climb and the descent, the horizontal distances are longer with the aircraft 777 – 200 because they depend on the velocity along the flight path. The cruise Mach number is more important for the Boeing 777 – 200, consequently the velocity along the flight path increases. For the cruise phase, the horizontal distance is longer with the Airbus A340 (Equation 38).

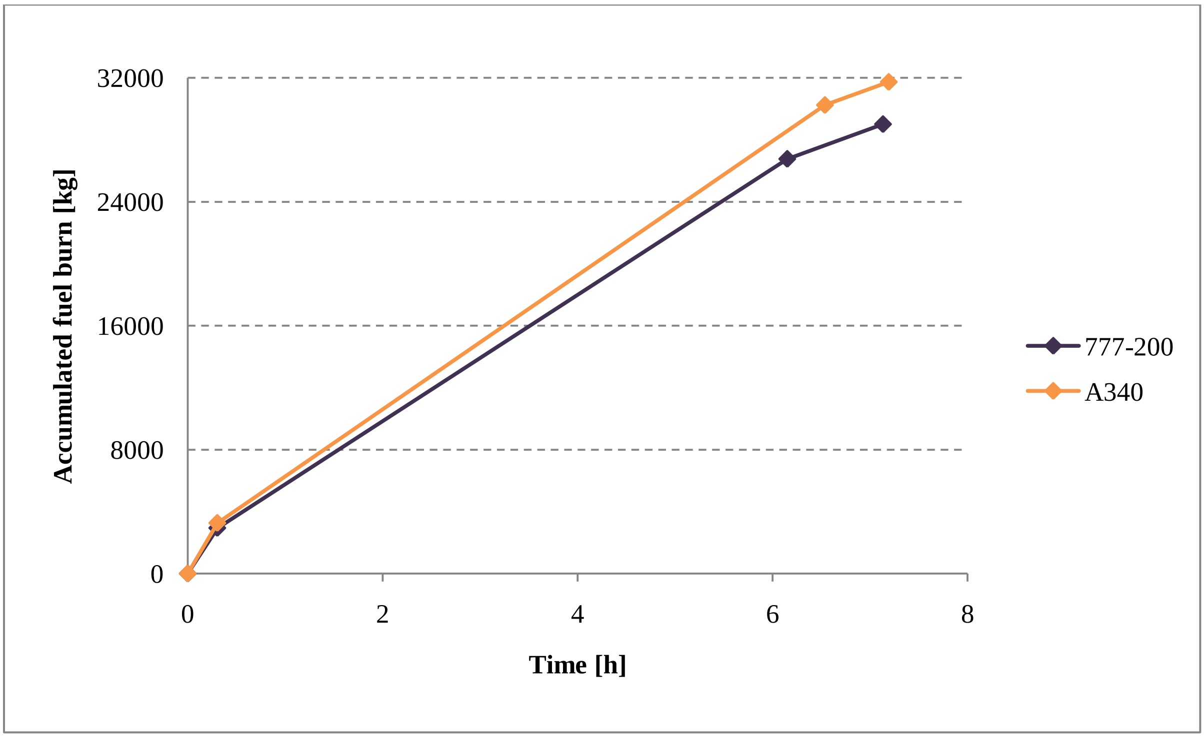

Figure 3.3: The accumulated fuel consumption according on the time of the flight, Paris – New York

With the aircraft Airbus A340, the fuel consumption during the climb and the descent phases are more important than with the Boeing 777 – 200. The time of climb and descent are equal for the two aircrafts, but the required thrust is higher for the A340. The specific fuel consumption is smaller, but the influence of the required thrust is more important, consequently the fuel consumption increases. For the cruise, the time and the thrust are higher with the Airbus A340, so the fuel consumption augments.

The total fuel consumption is more important for the flight trajectory with the Airbus A340. The variation of the fuel burn between the two aircraft at the end of the trajectory is equal to 9%. The difference corresponds to the variation of the less fuel burn configurations (figure 3.3).

3.2. Influence of the parameters

3.2.1. Trondheim – Oslo

Influence of the cruise altitude

The cruise altitude has an influence on each phases of the trajectory.

- Climb

When the cruise altitude augments, the horizontal distance and the time of climb increase, but the maximum thrust decreases.
The fuel consumption increases (Equation 3), consequently the time has an influence more important than the required thrust on the fuel burn in this case.
The variation of the fuel consumption is around 15% more for an augmentation of the cruise altitude equal to 1000m.

- Cruise

When the cruise altitude augments, the cruise horizontal distance, the time and thrust decrease, consequently the fuel consumption is less important.
The variation of the fuel consumption is not constant and varies according to the parameter for the climb time determination. The range is $-38\% \leq \Delta m_{fb} \leq -20\%$.

- Descent

When the cruise altitude augments, the horizontal distance and the time increase, but the required thrust decreases. The fuel consumption increases because the influence of the time is more important than the thrust. The variation of the fuel consumption is low, $m_{fb} = 2 - 3\%$.

- Total trajectory

When the cruise altitude increases, the maximum thrust decreases and the time of the trajectory augments. The influence of time is more important than the required thrust.
The variation of the fuel consumption in percent is higher for the cruise trajectory, but the fuel consumption is maximal during the climb phase and when the altitude increases the fuel consumption during the climb phase increases. The influence of the fuel consumption during the climb phase is more important than during the cruise phase for the short haul flight. Accordingly, the fuel consumption increases when the cruise altitude is higher.

The influence of the cruise altitude is illustrated in the figures 3.4 and 3.5 for the Boeing 737 – 300 and the values of the parameters for the climb and the descent time determination respectively equal to 300 and 150.

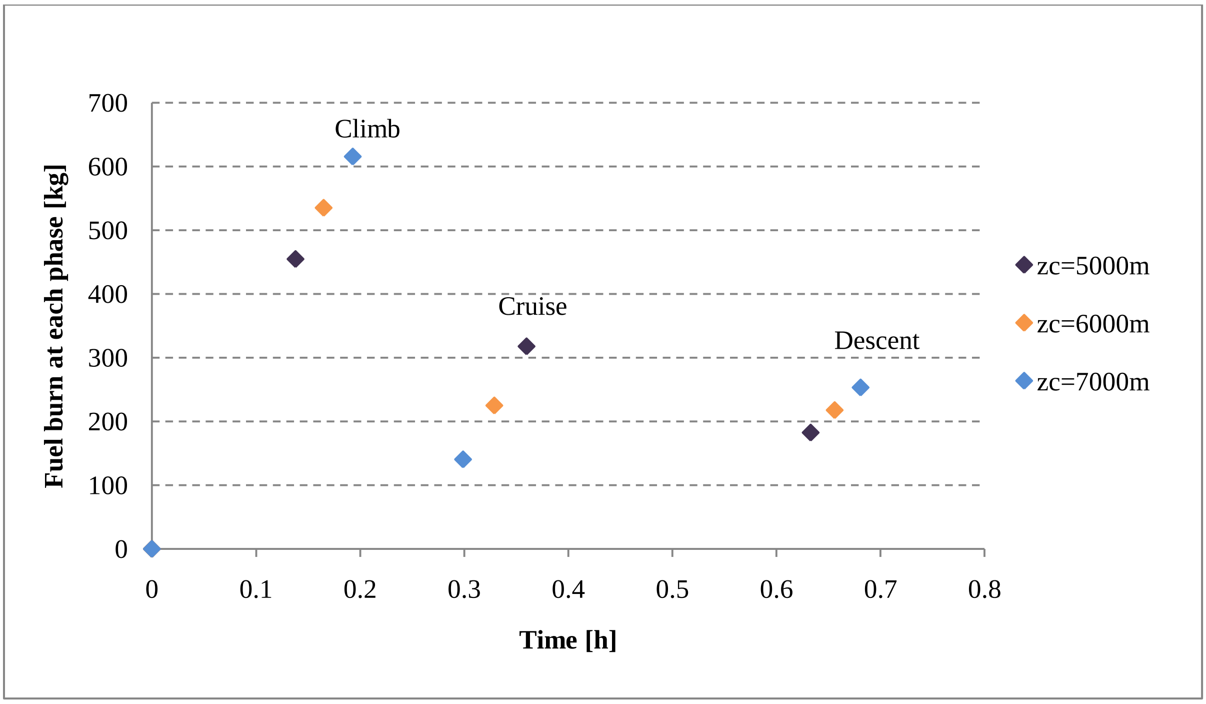

Figure 3.4 : The variation of the fuel consumption according to the cruise altitude, Trondheim – Oslo

The figure 3.4 represents the fuel burn at each phase of the flight trajectory according to the time of flight and for the three cruise altitude. The variation of the fuel burn is superior during the cruise phase. The value of fuel burn is the highest during the climb and the variation is important according to the cruise altitude.

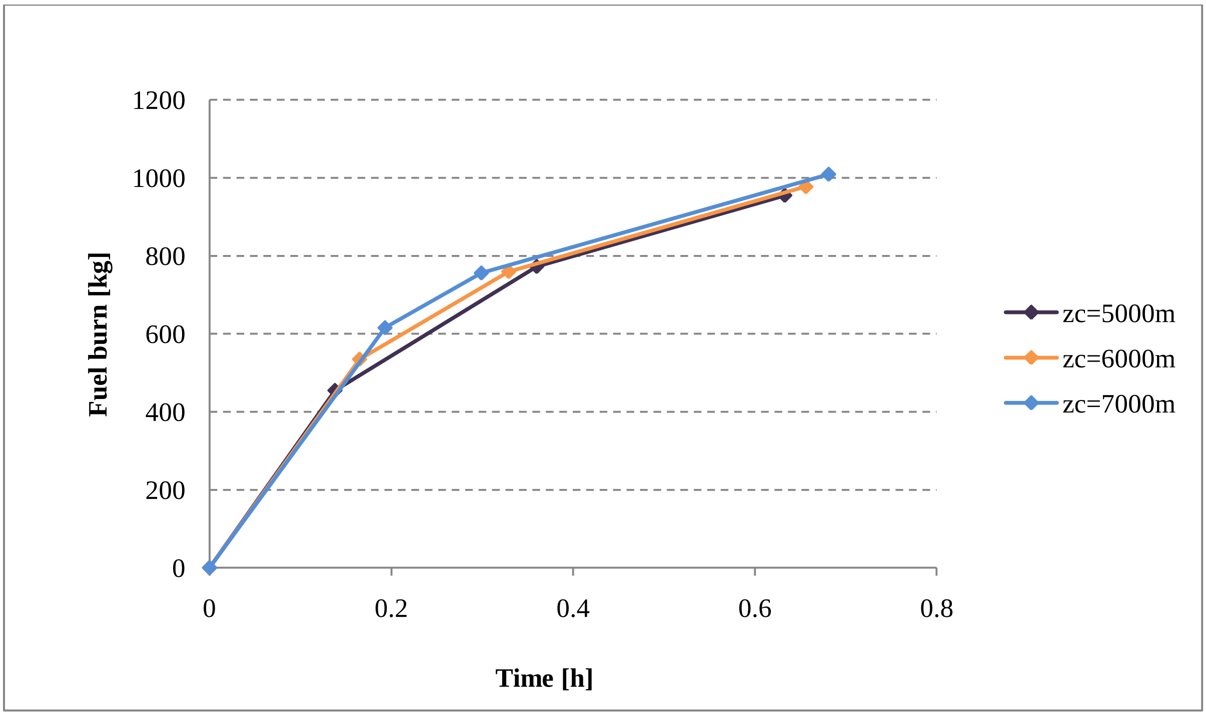

Figure 3.5: The accumulated fuel consumption according to the cruise altitude, Trondheim – Oslo

The figure 3.5 represents the accumulation of the fuel burn after each phase according to the time of flight and for the three cruise altitudes. The variation of the fuel burn at the end of the trajectory is not important.

Influence of the parameter for the climb time determination

The range of the parameter for the climb time determination is $200 \leq \gamma \leq 300$.

- Climb:

When the parameter for the climb time determination γ augments, the required thrust increases, the horizontal distance and the time of climb decrease. The influence of the time of climb is more important than the thrust, therefore the fuel consumption decreases.

- Cruise:

When the parameter γ increases, the fuel consumption is more important because the cruise horizontal distance increases and consequently the time of cruise augments.

- Descent:

When the parameter γ increases, the horizontal distance, the time of descent are constant because they depend on the parameter for the descent time determination and the cruise altitude. The required thrust is more important, accordingly the fuel consumption increases.

The parameter for the climb time determination has a small influence on the total fuel consumption. For example, at cruise altitude $z_C = 5000m$, the variation between the maximum and the minimum fuel burn is inferior to 0,5%.

Influence of the parameter for the descent time determination

For some cases, $\gamma_d = 100$ is not possible because the distance between the two airports is too small. The different possibilities are defined previously in the table 2.9.

- Cruise:

When the parameter for the descent time determination γ augments, the horizontal distance and the time of cruise increase, therefore the fuel consumption too.

- Descent:

When the parameter γ increases, the horizontal distance and the time of descent decrease and reduce the fuel consumption.

The parameter for the descent time determination has an influence more significant on the total fuel consumption than the parameter for the climb time determination. The total fuel consumption augments when the parameter γ increases because the variation and the value of the fuel burn during the cruise are more important than during the descent (figure 25).

At cruise altitude $z_C = 5000m$, the variation of the fuel consumption is 3-4%.

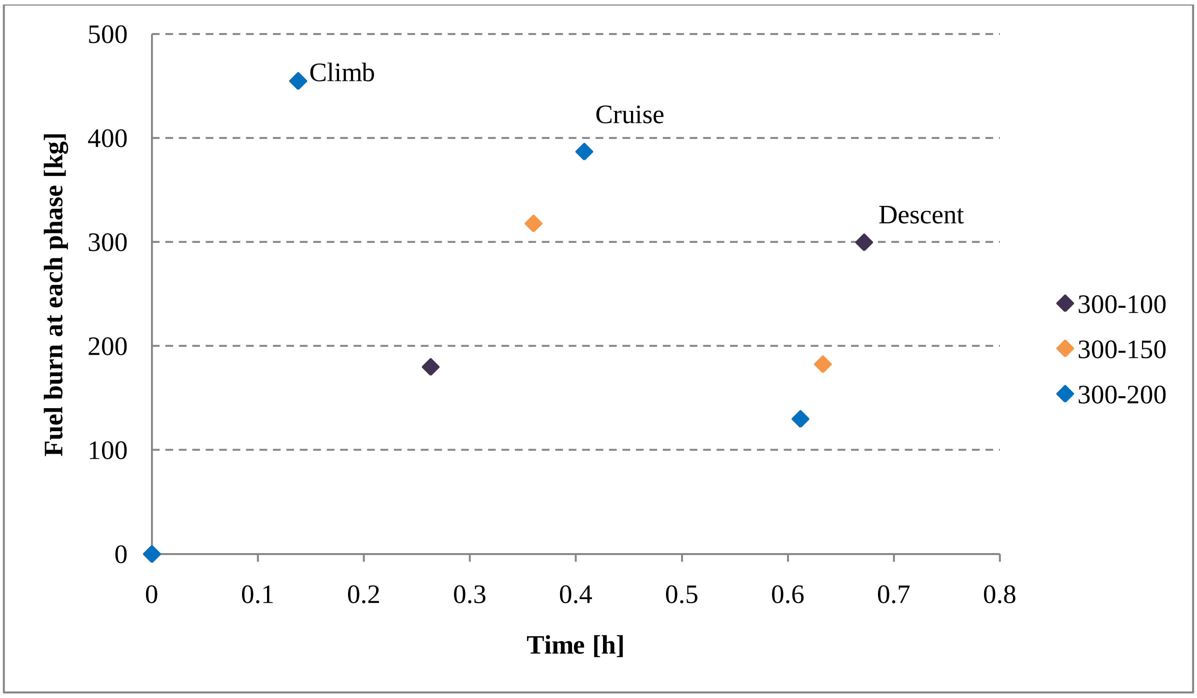

Figure 3.6: The fuel consumption at each phase at z_c=5000m, Trondheim – Oslo

The variations of the fuel burn during the cruise and the descent phase are important according to the fuel burn during these phases.

General

The configurations where the fuel consumption is minimal for the different cruise altitude are defined previously in the tables 3.3 and 3.4.
The configurations are defined for a parameter for the climb time determination high and a parameter for the descent time determination low. Except at cruise altitude $z_C = 5000m$, the values of the parameters γ are 300 for the climb. For the cruise altitude $z_C = 5000m$, the parameter for the climb time determination is equal to 250 because the cruise altitude is very low and the influence of the fuel consumption during the climb is less important.

For the parameter for the descent time determination, the value is equal to 100, except for the cruise altitude $z_C = 7000m$, where the parameter γ is equal to 150 to respect the distance between the two airports.

The fuel consumption increases when the cruise altitude is higher, because the phase where the aircraft consumed the maximal fuel is during the climb.

The fuel burn per hour is constant during the cruise phase, but for the climb and the descent varies according to the values of the parameters for the climb and descent time determination. The minimum value of the fuel burn per hour for the climb and the descent is obtained for $\gamma_{cl} = 200\ and\ \gamma_d = 200$.

At cruise altitude $z_{C1} = 5000m\ and\ z_{C1} = 7000m$, the minimum fuel burn per hour correspond to the configuration $\gamma_{cl} = 200\ and\ \gamma_d = 100/150$. At cruise altitude $z_{C1} = 6000m$, the configuration with the less fuel consumption per hour corresponds to the optimal configuration with the less fuel consumption which is defined previously.

3.2.2. Trondheim – Nice

Influence of the parameter for the climb time determination

The range of the parameter for the climb time determination is $150 \leq \gamma \leq 300$.

The analysis of the fuel consumption for the different phases of the flight trajectory corresponds to the previous analysis which is realized for the trajectory Trondheim – Oslo. According to the variation of the parameter for the climb time determination, the horizontal distance, the time, the required thrust and the fuel consumption vary.

When the value of the parameter γ augments, the fuel consumption during the climb decreases, increases during the cruise because the horizontal distance is more important and augments during the descent phase.

The maximal influence of the parameter for the climb time determination on the total fuel consumption is inferior to 1%.

Influence of the parameter for the descent time determination

The range of the parameter for the descent time determination is $100 \leq \gamma \leq 200$.

The influence of the parameter γ is identical to the influence on the fuel consumption for the trajectory Trondheim – Oslo. When the parameter for the descent time determination augments, the fuel consumption increases during the cruise phase and decreases during the descent phase.

The variation of the fuel consumption for the different values of the parameter γ is very low, inferior at 1% of the total fuel consumption.

General

The influence of the parameters for the climb and the descent time determination is not important and the maximal value of the fuel burn variation is inferior to 2% of the total fuel consumption. The variation of these parameters affects the distance for the climb and the descent phases.

For the trajectory Trondheim – Nice, the distance of the climb and the descent are small compare to the cruise horizontal distance. For the flight trajectory Trondheim – Oslo, the variation of the fuel consumption for the climb and the climb distance has an influence significant on the total fuel consumption. In this case, the variation of the fuel consumption during the climb and the descent is important, but the values are small compare to the fuel consumption during the cruise. The latter has a linear variation.

The maximal difference of the fuel consumption between two configurations is represented in the figure 3.7. The aircraft is the Airbus A320.

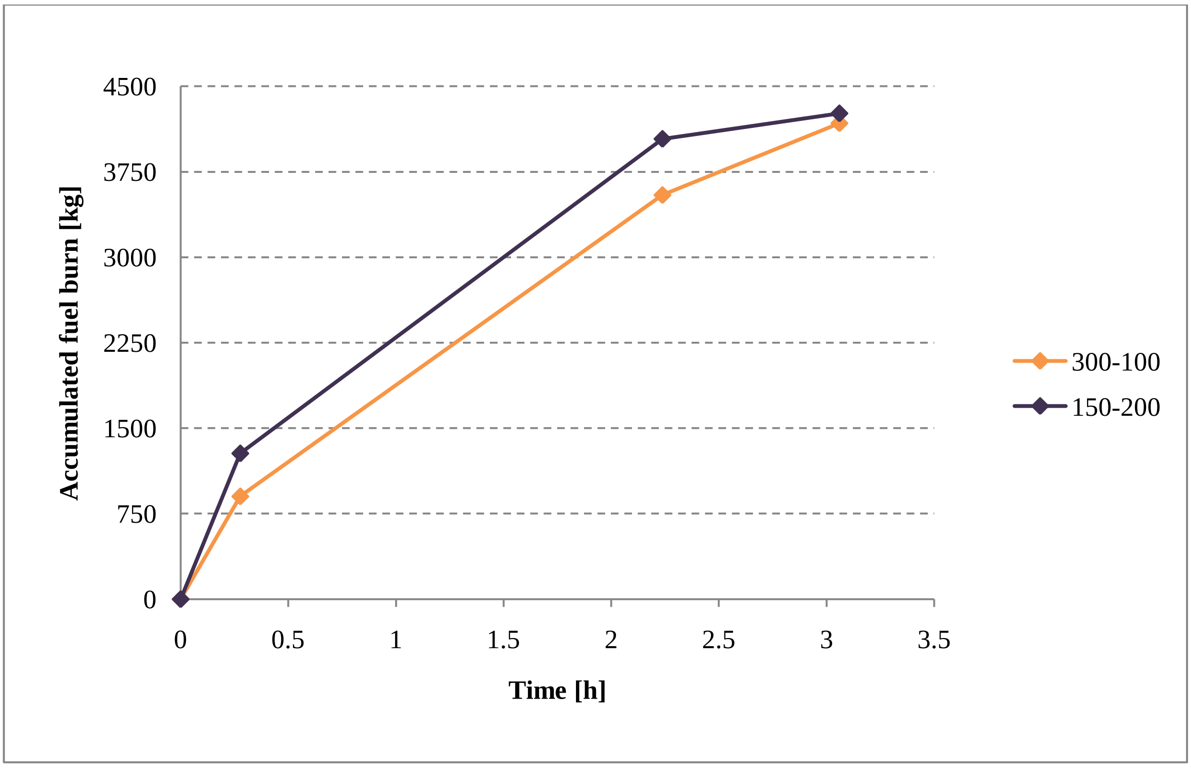

Figure 3.7: The accumulated fuel consumption according to the time of flight, Trondheim – Nice

The fuel consumption is more important when the parameter for the climb time determination is smaller, because the time of climb augments and consequently the fuel during the climb increases. During the climb phase, the fuel consumption per hour is the most important.

When the parameter for the descent time determination is high, the cruise distance increases and the fuel consumption too. The fuel burn per hour is more important during the cruise than the descent, so the variation of the fuel burn is higher for the cruise phase.

For the fuel burn per hour, the consumption during the climb phase is the most important. The influence of the time of climb is more important than the thrust force, so the climb distance and time have to be reduced and for this the value of the parameter for the climb time determination has to be high.

The optimal fuel consumption is obtained for a short time of climb and the longest time of descent. The objective is to reduce the cruise distance because it is the longest distance for this trajectory and the fuel consumption is important. To reduce the cruise distance, the distance of the descent has to be more important because it is the phase where the fuel consumption is the less important.

3.2.3. Paris – New York

Influence of the parameter for the climb time determination

The range of the parameter for the climb time determination is $150 \leq \gamma \leq 300$.

For the two aircrafts, the analysis of the fuel consumption for the different phases of the flight trajectory corresponds to the previous analyses which are realized for the trajectory Trondheim – Oslo and Trondheim – Nice. According to the variation of the parameter for the climb time determination, the horizontal distance, the time, the required thrust and the fuel consumption vary.
The fuel consumption decreases during the climb and increases during the cruise and the descent phases when the value of the parameter for the climb time determination augments.
The maximal influence of the parameter for the climb time determination on the total fuel consumption is inferior to 1%.

Influence of the parameter for the descent time determination

The range of the parameter for the descent time determination is $100 \leq \gamma \leq 200$.

The variation of the fuel consumption according to the parameter γ corresponds to the description which is done for the trajectories Trondheim – Oslo and Trondheim – Nice. The parameter for the descent time determination affects the cruise and the descent fuel consumption. When the parameter γ augments, the fuel consumption increases during the cruise phase and decreases during the descent phase.
The variation of the fuel consumption for the different values of the parameter for the descent time determination is not significant, inferior to 1% of the total fuel consumption.

General

As the flight trajectory Trondheim – Nice, the parameters for the climb and the descent time determination do not affect significantly the total fuel consumption because the majority of the fuel is burning during the cruise phase. The fuel burn during the climb represents 10 – 15% of the total fuel consumption according to the choice of the parameters γ and the fuel burn during the descent represents 3 – 7% of the total fuel consumption.

For the long haul flight, the parameters for the climb and the descent time determination have not a significant influence on the total fuel consumption. They affect the distances of the climb and the descent, but the horizontal distances are smaller than the cruise distance. The time during these phases are short compare to the total time of flight, consequently the fuel consumption during the climb and the descent is minim in comparison with the fuel burn during the cruise phase.

With the Boeing 777 – 200, the configuration with the less fuel consumption is obtained for the maximal parameter for the climb time determination $\gamma_{cl} = 300$ and the minimal parameter for the descent time determination $\gamma_d = 100$.

On the other hand, with the Airbus A340, the parameter for the descent time determination is equal to 150 for the configuration with the less fuel burn. Between the two configurations with the parameter $\gamma_d = 150$ and $\gamma_d = 100$, the difference of the fuel burn is small and equal to 6kg. The fuel burn increases during the cruise and decreases during the descent when the parameter γ augment. The variation of the fuel consumption is more important during the descent phase, consequently the parameter for the time determination is equal to 150 for the less fuel configuration.

The maximal difference of the fuel consumption between the two configurations is represented in the figure 3.8. The aircraft is the Airbus A340.

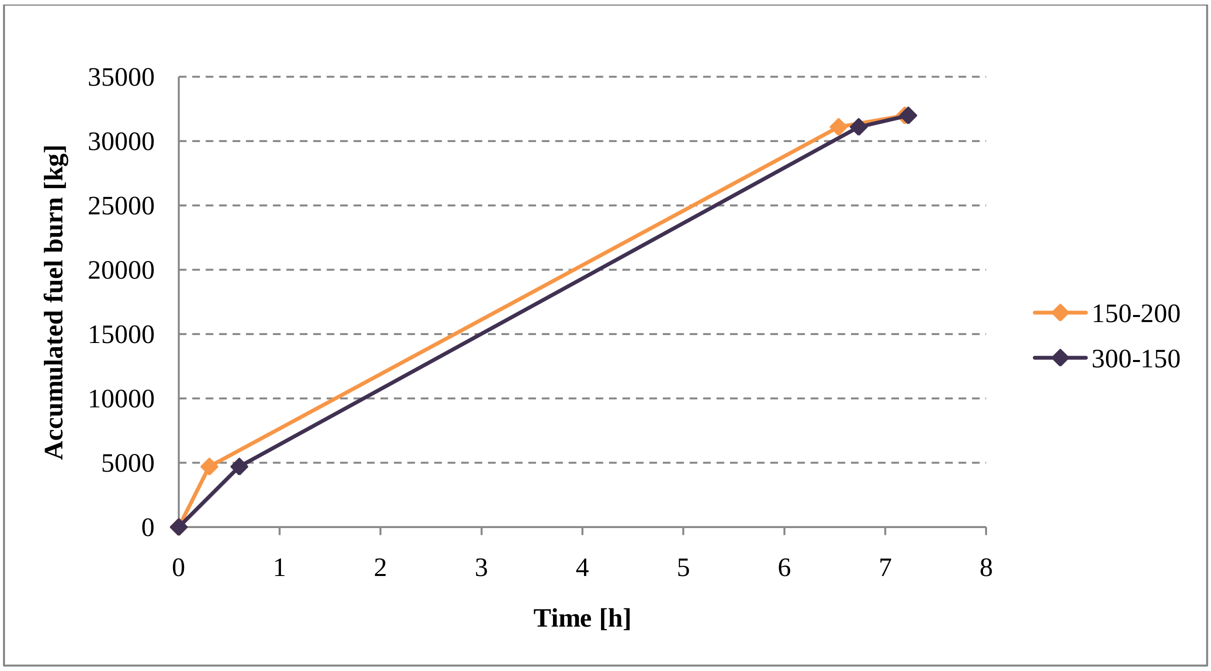

Figure 3.8: The accumulated fuel consumption according to the time of flight, Paris – New York

On the graph, the principal difference visible between the two configurations is during the climb and the cruise phase. However the influence of the parameters for the climb and the descent time determination is minim for the long haul flight, the majority of the fuel is burning during the cruise phase. The variation of the fuel burn at the end of the trajectory is not visible on the figure 3.8.

3.3. Comparison with the theoretical values

3.3.1. Trondheim – Oslo

In this part, the results which are obtained are compared with the results from the PhD of Paul Arentzen for the flight trajectory Oslo – Trondheim.

The aircraft and the configuration which are chosen to compare with the benchmarks are the Boeing 737 – 300, at cruise altitude $z_C = 7000m$ and the parameters for climb and descent time determination are equal to $\gamma_{cl} = 300 \; and \; \gamma_d = 150$.

The profiles of the two flight trajectories, the weight at takeoff, the distance between the two airports and the cruise altitudes are different in the two configurations of the flight trajectory Trondheim – Oslo (table 3.11).

Table 3.11: Difference of the trajectory definitions

	Distance	**Weight at takeoff**	**Cruise altitude**
	[km]	[kg]	[m]
Analysis (1)	390	62800	7000
PhD (2)	494,2	48000	11278

In the PhD, the trajectory of the aircraft is defined by different segments and the taxi-out, the takeoff and the landing phases are considered. In the model which is described in this report, the trajectory is defined by the different equations and the flight corresponds to the three principal phases: climb, cruise and descent.

The difference of the distances between the two airports is significant: $\Delta d = 104m$, that corresponds to 25% more than the distance which is defined in this analysis.

The weight at takeoff is different because in this analysis the weight of the aircraft corresponds to the maximal takeoff weight and in the reality for the short distance, the weight of aircraft is lighter. The aircraft does not need the maximum fuel capacity to reach the destination. The difference of the aircraft weight at takeoff between the two configurations is equal to $\Delta m = 14800kg$.

The cruise altitude is equal to 11278m in the PhD, whereas in this analysis the aircraft cannot reach the maximum cruise altitude because the distance between the two airports is not respected in this case; therefore the cruise altitude is equal to 7000m. The difference between the definitions of the two cruise altitudes is superior at 4000m and significant.

The comparison is realized between the climb and the descent phases; the taxi – out, the takeoff and the landing from the PhD are not considered.
The results for the speed, the distance, the time and the fuel consumption for the two configurations are given in the table 3.12.

Table 3.12: Altitudes, speeds, distances, times and fuel burn

	Altitude		**Speed**			**Distance**			**Time**			**Fuel burn**		
	Start	End	Cl.	C.	D.	Cl.	C.	D.	Cl.	C.	D.	Cl.	C.	D.
Unit	m		m/s			km			s			kg		
1	70	6930	87-212	212	212-87	104	81	205	695	382	1375	615	140	253
2	457	11278	107-208	208-220	220-138	190	89	200	904	406	1114	1471	209	295
Diff.	323	4348	20-(-4)	(-4)-8	8-41	86	8	-5	209	24	-261	856	69	42

The values of the difference correspond to the values from the PhD minus the values from this analysis.

The values of the speeds during the three phases have the same order of magnitude between the two analyses. The values of the distances and the times are close for the cruise and the descent phases. However the distance and the time of climb are different, because the cruise altitude difference is significant, consequently the distance to reach the cruise altitude in the PhD is more important. The value of the climb speed is similar so the time of climb is longer.

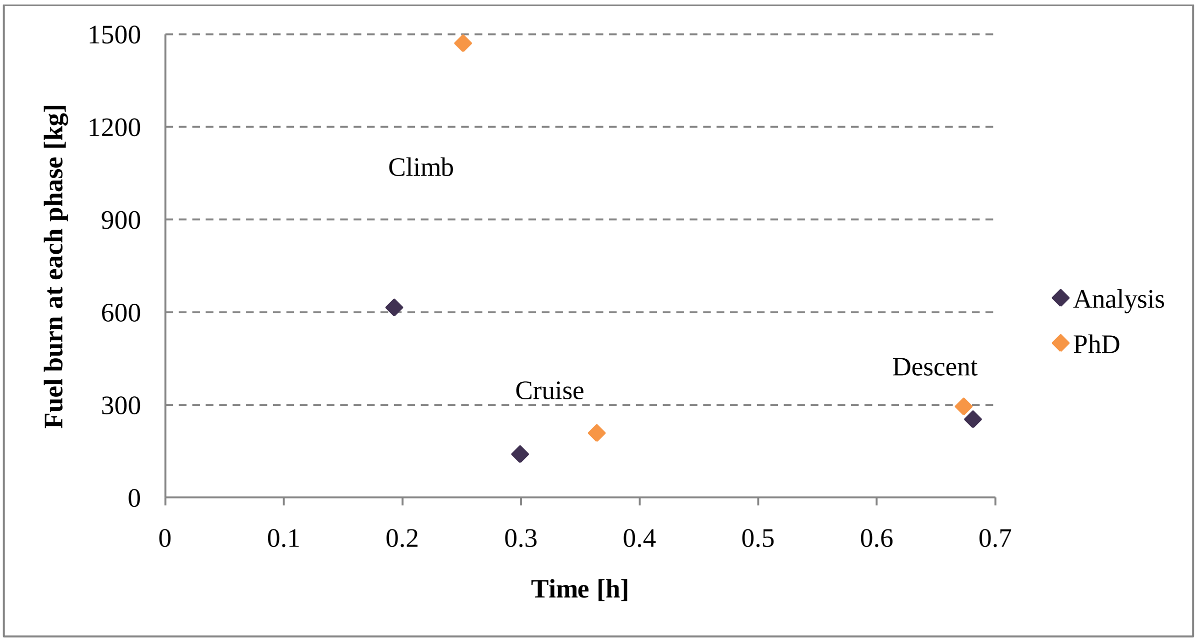

Figure 3.9: The variation of the fuel consumption at each phase, Trondheim – Oslo

The values of the fuel consumption are different in the two analyses. The difference varies according to the flight phases.

For the climb phase, the fuel consumption which is calculated in the PhD corresponds at more than twice of the value of the fuel consumption which is determined in this analysis. This significant difference can be explained by the higher cruise altitude that the aircraft reaches in the analysis from the PhD.
In the part 3.2. Influence of the parameters, the augmentation of the cruise altitude generates the increase of the horizontal distance, accordingly the time of climb augments and the fuel consumption too.

The horizontal distance for the climb which is defined in the PhD is close to twice of the value of the horizontal distance from the analysis configuration. The difference of the time of climb is equal to one third of the time of climb from the PhD. The variation of the time is less important than the climb horizontal distance because the mean climb speed is more important and the profile of the climb trajectory is different.
The difference of the fuel consumption between the two analyses remains too significant.

During the cruise and the descent phases, the variation of the fuel consumption is significant, but less important compare to the variation of the fuel burn during the climb phase.

The cruise horizontal distance from the PhD is longer consequently the time of cruise is more important for a same order of magnitude of the cruise speed. Accordingly, the fuel consumption augments for the results from the PhD.
The descent horizontal distance is longer from the analysis and the speed is slower, consequently the time of descent increases. But the fuel consumption is inferior to the value from the PhD. Nevertheless, the variation of the fuel consumption is small compare to the cruise phase.

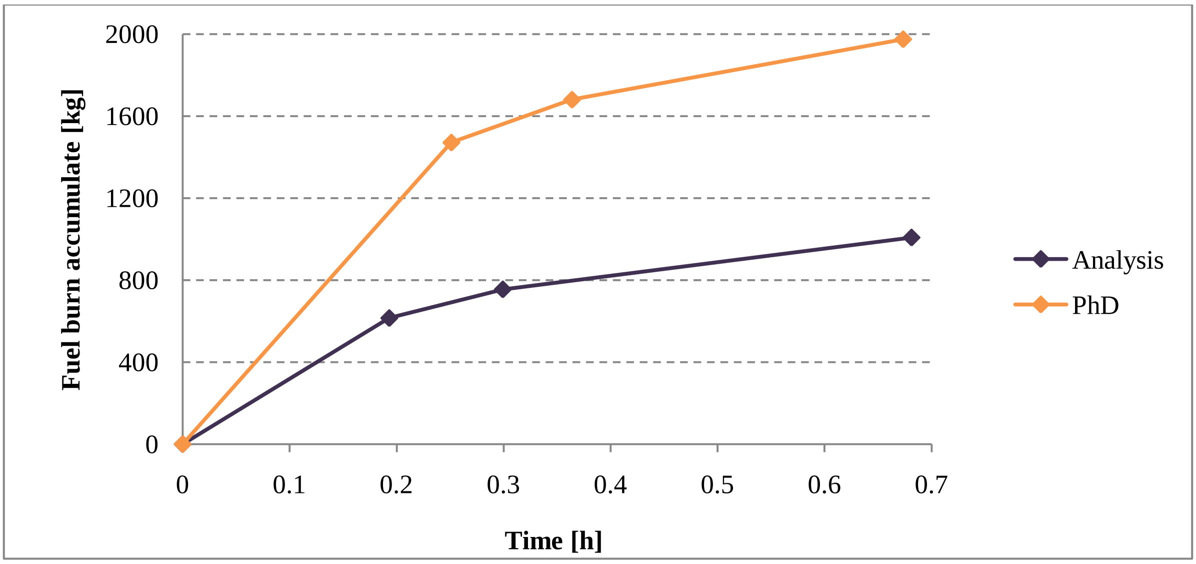

Figure 3.10: The variation of the accumulated fuel consumption, Trondheim - Oslo

In the figure 3.10, the variation of the accumulated fuel consumption between the two configurations is significant. This variation is the result of the significant difference of the fuel consumption during the climb phase.
The comparison of the two models is difficult because of the important difference which exists between them (cruise altitude, horizontal distance, and trajectory profile).

3.3.2. Trondheim – Nice

In this part, the results which are obtained are compared with the results from the documentation Airbus [2] for the climb phase.
In this document, the cruise altitude is equal to $z_C = 10058\ m$ and the aircraft is the Airbus A320.

The weight of the two aircrafts differs little; the variation of the takeoff weight is equal to 2000kg.

The comparison between the two models for the climb phase is difficult because the distances and the time of the climb are different due to the trajectory profiles.
In the documentation Airbus [2], the fuel consumption depends on the choice of the cost index.
The configuration with the less fuel consumption for the two analyses is given in the table 3.13.

Table 3.13: Fuel consumption during the climb phase

	Fuel burn (kg)	Time (min)	Distance (km)	Fuel burn / hour
Airbus documentation	1757	22,4	227,8	4706
Analysis	902	16,6	157	3260

The fuel consumption from the documentation Airbus [2] corresponds to twice the fuel consumption from this analysis. This variation depends on the time of climb and the climb distance.

The difference between the two distances is equal to 30% of the distance from the documentation Airbus. This variation is significant.
The variation of the time between the two models corresponds to more than 25% of the time from the documentation Airbus.

The fuel burn per hour is defined in the table 3.13. The difference is equal to 10% of the fuel consumption per hour from the documentation Airbus.

The results are different and depend on the trajectory profile which is defined in a different way in the two analyses. The variation of the fuel consumption per hour show a coherence in the results which are obtained in this analysis.

3.3.3. Paris – New York

The Airbus documentation [2] gives some values of the fuel burn, the distance and the time for the climb phase. The values depend on the cost index. They concern the aircraft Airbus A340 which is used in this analysis to calculate the fuel consumption during the flight trajectory Paris – New York.

In the analysis, the weight of the aircraft corresponds to the maximum takeoff weight. The difference between the weights of the two aircrafts is equal to 25000kg, so 1% of the weight at takeoff.

The times of climb, the climb distances are different for the two models, so the comparison is difficult to make.

In the table 3.14, is identified the configurations with the less fuel consumption for the two models.

Table 3.14: Values for the climb phase

	Fuel burn (kg)	Time (min)	Distance (km)	Fuel burn / hour
Airbus documentation	5363	25,4	311,1	12669
Analysis	3266	18,2	181	10767

The climb distance and the time of climb vary according to the analysis. The variation of the climb distance is superior to 35% and the variation of the time is close to 30% of the time from the documentation Airbus.

These differences are significant and affect the values of the fuel consumption. The variation of the latter is equal to 40% of the fuel consumption from the documentation Airbus. This variation is very important and depends on the time of climb, the definition of the calculation of the fuel consumption and the profile of the climb trajectory. But the two last information are not given on the documentation Airbus.

The variation of the fuel consumption per hour is superior to 15% of the fuel consumption per hour which is obtained with the values from the documentation Airbus.

Like for the flight trajectory Trondheim – Nice, the comparison between the two analyses is difficult.

3.4. Influence of the model of the vertical speed

The vertical speed has no influence on the cruise trajectory, so the modification of its profile affects only the climb and the descent phases.

The values of the times, distances and fuel burn according to the profile of the vertical speed are given in the appendix C, for the configurations with the less fuel consumption.

The profile of the vertical speed has an influence insignificant on the climb phase. The variation of the fuel burn according to the profile of the vertical speed is equal to 1 kg for the flight trajectory Trondheim – Oslo, inferior to 2kg for Trondheim – Nice and close to 5kg for

Paris – New York. Consequently the choice of the vertical speed profile has no influence on the fuel consumption. The vertical speed can be represented by a parabola.

During the descent phase, the profile vertical speed has an influence on the fuel consumption. For the trajectory Trondheim – Oslo, it is included between 9 and 17% of the fuel burn during the descent with the vertical speed which is defined by the equation 5. The variation depends on the cruise altitude and is maximal for the highest cruise altitude (figure 3.11).

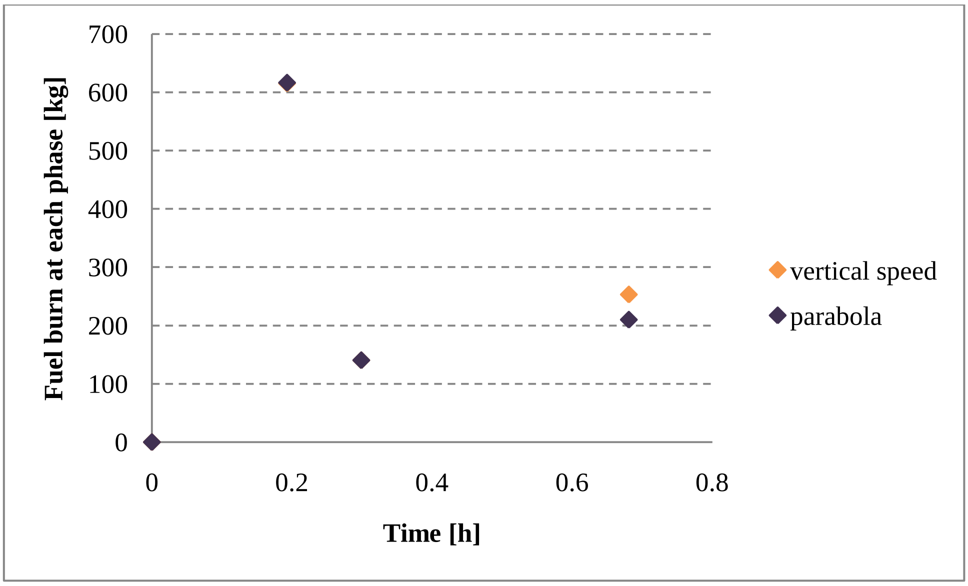

Figure 3.11: Fuel consumption according to the vertical speed profile, Trondheim – Oslo

For the trajectories Trondheim – Nice and Paris – New York, the influence of the vertical speed profile on the fuel consumption has the same order of magnitude than the trajectory Trondheim – Oslo.

The vertical speed profile affects only the descent phase, consequently its influence on the total fuel consumption is less important.

For the flight trajectories Trondheim – Nice and Paris – New York, the total fuel consumption according to the vertical speed profile has a variation to 1%, because the influence of the fuel burn during the descent is negligible compare to the fuel consumption during the cruise.
The influence of the vertical speed on the total fuel consumption for the trajectory Trondheim – Oslo is gently superior and equal to 4%.

The influence of the vertical profile has not an important influence on the fuel consumption for the flight trajectory. Consequently, the vertical speed can be represented by a parabola.

3.5. Conclusion

The optimal trajectory for the three flights is obtained for a parameter for the climb determination high and a parameter for the descent time determination low. The three flight trajectories are defined in two categories: the short haul flight and the medium – long haul flight.

For the short haul flight, the cruise distance is small or inexistent. The maximum fuel is consumed during the climb phase. To reduce this consumption, the parameter for the climb time determination is high because when the climb distance decreases, the time decreases too and the fuel consumption decreases because the influence of the time is more than important than the variation of the thrust force which increases. The fuel consumption per hour is more important during the cruise than the descent so the cruise distance has to decrease. Consequently the descent phase is longer and is obtained for the small value of the parameter for the descent time determination. For the cruise altitude $z_C = 6000m \; and \; 7000m,$ the values of the parameters γ are $\gamma_{cl} = 300 \; and \; \gamma_d = 100/150$. At cruise altitude $z_C = 5000m$, the altitude is too low so the influence of the fuel consumption during the climb phase is smaller and consequently the optimal configuration is obtained for $\gamma_{cl} = 250$.

For the medium – long haul flight, the fuel is consumed mainly during the cruise phase and represents 56 to 72% of the total fuel burn for the flight trajectory Trondheim – Nice and 78 to 87% of the total fuel burn for Paris – New York. The objective is to reduce the cruise phase. Consequently, the time and the distance of the descent are longer because it is the phase where the fuel burn per hour is the less important. The influence of the parameters for the climb and the descent time determination is low because they modify the cruise trajectory but the variation of the cruise trajectory is minim in front of the cruise distance.
The optimal configuration to reduce the fuel burn for the trajectory Trondheim – Nice is $\gamma_{cl} = 300 \; and \; \gamma_d = 100$. For the flight trajectory Paris – New York, the parameters γ are equal to 300 for the climb and 100 or 150 for the descent according to the aircraft.

The choice of the aircraft has an influence important on the fuel consumption and varies according to the characteristics of the aircraft. The weight of the aircraft has to be optimized to reduce the fuel consumption. Indeed, the fuel capacity at takeoff does not need to be maximal and depends on the distance between the two airports. Consequently the aircraft weight at takeoff is lighter.

The profile of the vertical speed can be defined by a parabola because the results which are obtained previously show that the influence of the vertical speed profile is negligible on the fuel consumption.

4. Conclusion

For the last 20 years, the cost of the fuel barrel does not stop to increase. In this economically critical period, the flight companies want to reduce the cost of the travel and the fuel consumption. The latter has a double objective: lower the operating costs of the companies and reduce the emission of the greenhouse gas for a greener sky. This analysis has for objective to define the fuel consumption according to the defined flight trajectory which is represented by a simplified analysis.

The definition of the flight trajectory is divided in three phases: the climb, the cruise and the descent. They are defined by different equations and parameters. The optimization of the flight trajectory is realised with the assistance of a computer code. The parameters for the climb and the descent time determination and the cruise altitude vary according to the desired trajectory and the influence of each parameter on the fuel consumption is defined. The analysis includes three flight trajectories which correspond to three different hauls flight: short, medium and long.

The results show that the variation of the fuel consumption according to the variation of the parameters for the climb and descent time determination is low. But each percent of the reduction of the fuel burn is important and represents a diminution of the cost of the travel.
For the short haul flight, when the cruise altitude is shorter, the fuel consumption decreases and the variation is important according to the selected cruise altitude. In this case, the fuel consumption is maximal during the climb phase and the cruise is small or inexistent.
In the other cases, during the cruise phase, the aircraft consumes the majority of the fuel but the fuel consumption per unit time is still higher during the climb phase. The results show that the optimal flight trajectory for the minimum fuel burn is obtained for a short climb phase and a long descent.

The calculation of the fuel consumption for the different flight trajectories can be optimized, if the weight of the aircraft at takeoff is reduced. Indeed, the aircraft weight which is considered in this analysis corresponds to the maximum takeoff weight because the fuel capacity is considerer equal to the maximum fuel capacity. In the reality, the fuel capacity in the aircraft varies according to the distance of the travel. Consequently, the aircraft weight has to be optimized and the fuel consumption will decrease.

Today, the perspectives of the flight companies are to reduce the fuel consumption, different solutions are found. The flight trajectory has an influence important on the fuel consumption and the different phases of the trajectory have to be optimized.

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Appendix A – On a simplified analysis of the flight trajectory of an aircraft

Draft (03.8.2010)

ON A SIMPLIFIED ANALYSIS OF THE FLIGHT TRAJECTORY OF AN AIRCRAFT

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BACKGROUND

The Clean Sky Joint Undertaking (JTI-CS-2010-1-SGO-03-007) of EU has asked for a proposal under the title ”Parametric optimisation software package for trajectory shaping under constraints”. The present Note is written in this context with reference to a representative jet powered airliner (read a Boeing B737-800W simulation flight case from Trondheim (TRD) to Nice (NCE) with both airport elevation at sea level).

1. ANALYSIS

The re-entry of a space plane has been simulated by using an analytical expression for the trajectory [1,2 and 3], i.e.

$$V(z) = V_{entry}\left[1 + b\exp(-cz)\right]^{-\frac{1}{2}} \qquad (1)$$

where V[m/s] represents the velocity and z [m] is the geometric altitude of the spaceplane. The input parameter V_{entry} [m/s] is the velocity at entry into the atmosphere, whereas b [-] and c [1/m] are ajustable parameters for the curve fit of the trajectory to given re-entry data.

A simplified analysis will now be presented for the climb and descent phase of an aircraft, i.e. Eq. (1) will be similar formulated as a logistic curve [4] as

$$z(V) = z_C\left[1 + \beta\exp(-\alpha V)\right]^{-1} \qquad (2)$$

which is a solution of the Riccati differential equation

$$\frac{dz}{dV} = \alpha z(1 - \frac{z}{z_C}) \tag{3}$$

The parameters α [s/m] and β [-] > 0 in Eq. (2) are to be determined from given aircraft peformance data as discussed at the end of this section. Furthermore, z_C [m] is the altitude at the specific cruise height. If we form the second derivative of z by differating Eq. (3) we will obtain

$$\frac{d^2z}{dV^2} = \alpha(1 - \frac{z}{z_C}) - \alpha\frac{z}{z_C} = \alpha(1 - 2\frac{z}{z_C}) \tag{4}$$

and setting Eq. (4) equal zero will yield the inflection point of Eq. (2) as

$$z_{\inf} = \frac{z_C}{2} \tag{5}$$

Inserting Eq. (5) in Eq. (3) we have

$$(\frac{dz}{dV})_{\inf} = \alpha z_C / 4 \tag{6}$$

which shows an independence to the parameter β, see also Figure 1. Furthermore, a reformulation of Eq. (2) will give

$$V(z) = -\frac{1}{\alpha}\ln[\frac{1}{\beta}(\frac{z_C}{z} - 1)] \tag{7}$$

and inserting Eq. (5) yields the result

$$V_{\inf} = -\frac{1}{\alpha}\ln\frac{1}{\beta} = \frac{1}{\alpha}\ln\beta \tag{8}$$

since ln(1/β) = ln1-lnβ = -lnβ.

Eq. (6) can also be formulated as

$$\alpha = \frac{4}{z_C}\frac{1}{(\frac{dV}{dz})_{\inf}} \tag{9}$$

where $(dV/dz)_{\inf}$ is regarded as an input parameter and with Eq. (8) reformulated as

$$\beta = \exp(\alpha V)_{\inf} \tag{10}$$

will define the parameters α and β at the reference altitude $z_{\inf} = z_C/2$. Eq. (7) posseses the two asymptotes at z = 0 and $z = z_C$ and this is indicated graphically in Figure 1. It should be noted that the lower the value of the factor β is, the higher the dimensionless velocity of the aircraft is where it reaches the cruise height.

1.1 Climb

Rewriting Eq. (3) with the dimensionless altitude Z = z/z_C as

$$\frac{dz}{dV}=\frac{dz}{dt}\frac{dt}{dV}=\alpha z(1-Z) \tag{11}$$

will yield

$$\frac{dV}{dt}=\frac{dz}{dt}\frac{1}{\alpha z(1-Z)}$$

or

$$a=v\frac{1}{\alpha z_C Z(1-Z)}$$

(12)

Here a [m/s^2] = dV/dt is the acceleration along the flight path (see Figure 2) and v [m/s] = dz/dt is the vertical speed of the aircraft. Since the vertical speed v must be zero at z = 0 and z_C we will stipulate the following relation

$$v=\gamma Z^2\left(1-Z\right)^2 \tag{13}$$

where the quadratic formulation of the altitude function Z(1-Z) is to ensure its altitude dependance in the result from combining Eqs. (12) and (13), i.e.,

$$a=\frac{\gamma}{\alpha z_C}Z\left(1-Z\right) \tag{14}$$

The ***acceleration force F [N]*** is formulated with Eq. (14) as

$$F=ma=\frac{m\gamma}{\alpha z_C}Z(1-Z) \tag{15}$$

where m [kg] = 70 000 designates the selected take-off mass of the aircraft which we for simplicity assumes constant during the climb.

From geometric consideration we write

$$\sin\theta=\frac{v}{V} \tag{16}$$

where θ [deg] is the climb angle. Inserting Eqs. (7) and (13) in Eq. (16) and solving for θ will give the result

$$\theta = \arcsin\left\{ \frac{\gamma Z^2 (1-Z)^2}{-\frac{1}{\alpha}\ln\left[\frac{1}{\beta}\left(\frac{1}{Z}-1\right)\right]} \right\} \tag{17}$$

which is shown in Figure 3 together with a plot of the vertical speed, Eq. (13). The ***gravity drag force G [N]*** is defined as

$$G = mg\sin\theta \tag{18}$$

were g [m/s^2] is the acceleration of gravity.

Introducing the atmospheric approximation for an isothermal atmosphere as

$$\rho(z) = \rho_0 \exp(-\varepsilon z) = \rho_0 \exp(-\varepsilon z_C Z) \tag{19}$$

where the air density ρ [kg/m^3] as function of the geometric altitude z is defined through the density ρ_0 = 1.225 kg/m^3 at sea level. We introduce the factor ε [1/m] = 0.0001 in order to satisfy an approximation in the range $0 \le z \le 11\,000$ m.

The speed of sound c [m/s] in the troposphere is a linear decay function with altitude [5] and for z_C = 11 000 m we can write

$$c = 340.294 - 45.14Z \tag{20}$$

Equation (17) combined with Eq. (7) will then yield the flight Mach number M [-] = V/c during climb and is plotted in Figure 4. Note that values in Figure 4 (and also in later figures) are given for the range $0.01 \le Z \le 0.99$ in order to eliminate the asymptotic behaviour of Eq. (7) at Z = 0 and 1.

The total ***aerodynamic drag D [N]*** acting on an aircraft in climb is formulated as

$$D = \left(C_{D,0} + C_{D,i}\right)\frac{1}{2}S\rho V^2 \tag{21}$$

where S [m^2] is the reference wing area, $c_{D,0}$ [-] is the drag coefficient at zero lift and $c_{D,i}$ [-] is the induced drag coefficient related to the lift force L [N] = mg cosθ. Furthermore, we can write

$$C_{D,i} = kC_L{}^2 = k\left[\frac{mg\cos\theta}{\frac{1}{2}S\rho V^2}\right]^2 \tag{22}$$

Eq. (22) states that $C_{D,i}$ is a quaratic function of the lift coefficient C_L [-] and the lift force L. The factor k [-] ≈ 0.045 is related to the aspect ratio of the wing. Combining Eqs. (21) and (22) gives

$$D = \frac{1}{2} C_{D,0} S \rho V^2 + \frac{k(mg\cos\theta)^2}{\frac{1}{2} S \rho V^2} \tag{23}$$

Figure 5 shows the total aerodynamic drag D, see Eq. 20, together with the components D_0 and D_i. The almost constant value of D = D(Z) is due to the opposite S-shapes of the components, but D will show similar behaveiour for any function of V = V(Z). The lift-to-drag ratio L/D [-] at climb is depicted in Figure 6 and represent a quality value for the aerodynamics of the reference aircraft.

We can now formulate the ***required thrust T [N]*** as

$$T = D + G + F \tag{24}$$

and Figure 7 presents the results and Table 1 gives an overview of the selected input data.

SYMBOL	VALUE	REMARKS
α =	0,07 s/m	Parameter defining the dimensional velocity
β =	60 000	Velocity/altitude parameter
γ =	300	Parameter for climb/descent time determination
ε =	0.0001 1/m	Constant in the density function
z_c =	11 000 m	Input for the cruise altitude
m =	70 000 kg	Input for the take-off mass of reference aircraft
g =	9,81 m/s^2	Gravitational constant
S =	125 m^2	Input for the wing reference area
$c_{D,0}$ =	0.015	Constant selected for the zero-lift drag coefficient
k =	0.045	Constant selected for the induced drag coefficient

Table 1. Selected input data for the climb simulation

In order to evaluate the time τ [s] needed to climb up to the cruise height z_C, Eq. (14) will be integrated to yield the mean vertical speed v_{mean} as

$$v_{mean} = \gamma \int_{Z=0}^{1} Z^2 (1-Z)^2 dZ = \gamma \int_{Z=0}^{1} (Z^2 - 2Z^3 + Z^4) dZ \tag{25}$$

and with the solution given in [6], i.e.

$$\int_{Z=0}^{1} Z^n dZ = \frac{1}{n+1}$$

we will obtain

$$v_{mean} = \gamma(1/3 - 2/4 + 1/5) = 0.0333\gamma \tag{26}$$

Hence, we can write $v_{mean} = z_C / \tau$ or $\tau = z_C / (0.0333\ \gamma)$. Table 2 shows the results for various values of γ for the given cruise altitude z_C = 11 000 m and for $\gamma = 300$ the climb time to the cruise altitude will be $\tau = 0.31$ h.

	γ = 100	**= 200**	**= 300**
τ [s] =	3 303	1 652	1 101
τ [min] = τ [h] =	55 0.92	28 0.46	18 0.31
v_{mean} [m/s] =	3.33	6.66	9.99
v_{max} [m/s] = at Z = 0.5	6.25	12.50	18.75

Table 2. Climb time to cruise altitude as function of the parameter γ.

1.2 Steady Cruise

Since the postulated flight trajectory V = V(z) has an asymptote at the cruise altitude z_c, we will define a practical cruise altitude $z_{cruise} \approx z_c$ at Z = 0.99, i.e. $z_{cruise} = 0.99\ z_c$ = 10 890 m. This leads to the cruise velocity V_{cruise} = 222.82 m/s (= 802.15 km/h) and the air density ρ_{cruise} = 0.41228 kg/m^3 (≈ 0.37 kg/m^3 from Reference 6). The sound speed c [m/s] at z_{cruise} is c_{cruise} = 295.6 m/s [3] which gives a Mach number M_{cruise} [-] = V_{cruise} / c_{cruise} = 0.754. Furthermore, the aerodynamic drag is calculated to D_{cruise} = 35 777 N and corresponds to the required thrust T_{cruise}, see Figure 8. This means that the overall drag coefficient

$$C_D[-] = \frac{2D_{cruise}}{S\rho_{cruise}V_{cruise}^2} \tag{27}$$

is evaluated to C_D = 0.0279 when appropriate values are inserted in Eq. (24).

Assuming a fuel burn of Δm = 1 500 kg during the climb phase [7], we can express the lift coefficient as

$$C_L[-] = \frac{2(m-\Delta m)g}{S\rho_{cruise}V_{cruise}^2} \tag{28}$$

and obtain the values C_L = 0.525 and C_L/C_D = 18.83 with Eq. (28). These aerodynamic coefficients are marked as a filled circle in Figure 9. The drag polars shown is taken from reference [8] and the arrow is added to illustrate a tangent going through the circle. This is to demonstrate that the circle is close to an optimal value for the ratio C_L/C_D. Hence, this value would give the best condition for the aircraft in cruise. Assuming for simplicity a horizontal cruise with a fuel-burn of 8500 kg we will reach the descent phase (after 2.73 h) at an aircraft weight of 60 000 kg.

1.3 Descent

The descent of an aircraft is depicted in Figure 10. It should be noted that the decent angle θ and the vertical speed v are now negative as compared to the climb phase, see Figure 11. Since we have a lighter aircraft mass (m = 60 000 kg) due to fuel burn and that we have assumed an increase of time of descent (gamma = 100), the aerodynamic drag has changed and is shown in Figure 12 with its components, see also Figure 13.

The ***force balance*** for the reference aircraft at descent can then be formulated as

$$T = D - G - F \tag{29}$$

and the result is graphically given in Figure 14. The flight idle is also indicated which is a flight situation for which the gravity force gradually takes over as a thrust force.

2. DISCUSSION OF RESULTS

The present Note starts with an analytical forcing function between the flight velocity of an airliner and the flight geometric altitude, i.e. V = V(z). This function is defined through the two parameters α and β and the flight state at $z_{inf} = z_c / 2$ is proposed as a suitable reference point. For a given speed of sound distribution with altitude, the Mach number distribution is also obtained, see Figure 4. Table 3 shows some selected input parameter (identified in bold numbers) for the climb or descent phase, where the computed values are found with reference to Eqs. (9), (10) and (7).

$(dV/dz)_{inf}$ [1/s] =	**0.004**	**0.005**	0.005195	**0.006**	**0.007**
V_{inf} [m/s] = M_{inf} [-] =	**160.00** 0.504	**160.00** 0.504	157.17 0.495	**160.00** 0.504	**160.00** 0.504
α [s/m] =	0.0909	0.0727	**0.07**	0.0606	0.0519
$(\alpha V)_{inf}$ [-] =	14.544	11.632	11.002	9.696	8.304
β [-] =	2 071 948	112 645	**60 000**	16 252	4 040
$V_{Z=0.01}$ [m/s] = $M_{Z=0.01}$ [-] =	109.45 0.322	96.79 0.285	91.53 0.270	84.17 0.248	71.40 0.210
$V_{Z=0.99}$ [m/s] = $M_{Z=0.99}$ [-] =	210.55 0.712	223.21 0.755	222.8 0.754	235.83 0.798	248.54 0.841

Table 3. Selected and computed parameter values for climb or descent

By introducing the Mach number M=V/c the selected input in Eq. (9) at the inflection point $z_{inf} = Z/2$ (see Figure 15) can also be expressed through the Mach number $M_{inf} = (V/c)_{inf}$ and the gradient $(dM/dz)_{inf}$, see Eq. 30.

$$(\frac{dV}{dz})_{\text{inf}} = (c\frac{dM}{dz})_{\text{inf}} + (M\frac{dc}{dz})_{\text{inf}} = (c\frac{dM}{dz})_{\text{inf}} - 0.0041 M_{\text{inf}}$$
(30)

where the gradient for the sound speed is dc/dz = -0.0041 and c_{inf} = 317.72 m/s.

Another important parameter is γ which is a function of the selected time of climb or descent, see Table 2. It also defines the vertical speed of the aircraft and the climb angle as can be seen from Figures 3 and 11. This leads to another parameter in the simulation study.

The aerodynamic qualification value of the reference airliner is basically given by the aircraft manufacturer and Figs. 5 and 12 is based on appropriate values for the lift- and drag coefficients. This indicates that the lift-to-drag ratio values shown in Figs. 6 and 13 have already, in a sense, been optimized by the aircraft manufacturer.

3. CONCLUDING REMARKS

The purpose of the present simulation study is twofold, i.e.

- Give a simple analytical basis for a parametric variation of selected parameters (α, β and γ) defining the climb or descent of a representative flight of an airliner.
- Give a focus on a positive flight trajectory (for a greener sky)

It is recognized that the fuel burn of the aircraft (with powerplants) has to be incorporated in the flight trajectory analysis as presented. In addition, the simplified cruise description should include an increase of the flight Mach number with altitude for a beneficial interaction of kinetic and potential energy.

Hence, the ultimate goal is to reduce the fuel cost and emission particles by expanding the present analysis into a **computer code** for a parametric study of the appropriate trajectory and aircraft variables.

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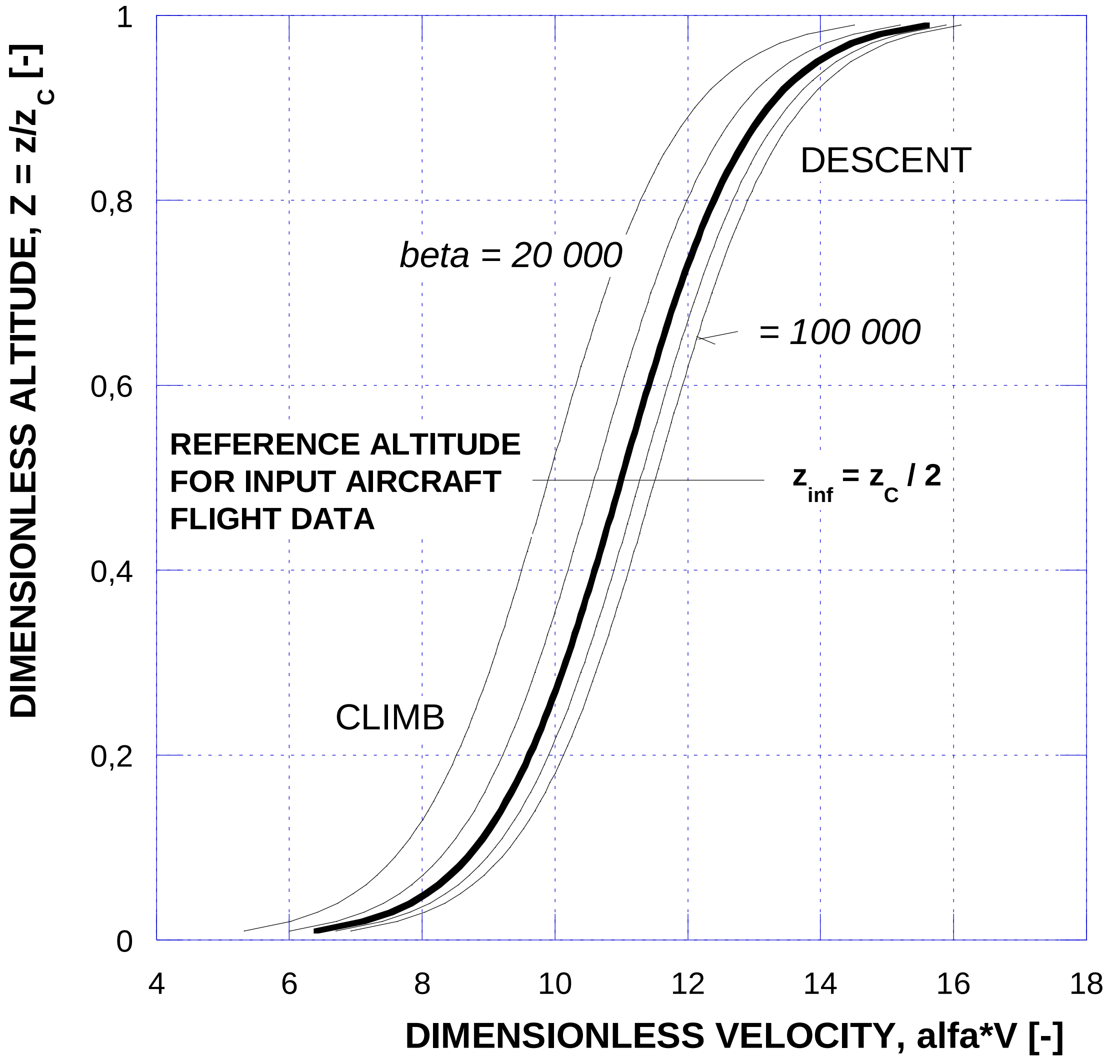


Figure 1. Simulated flight trajectories for climb and descent.

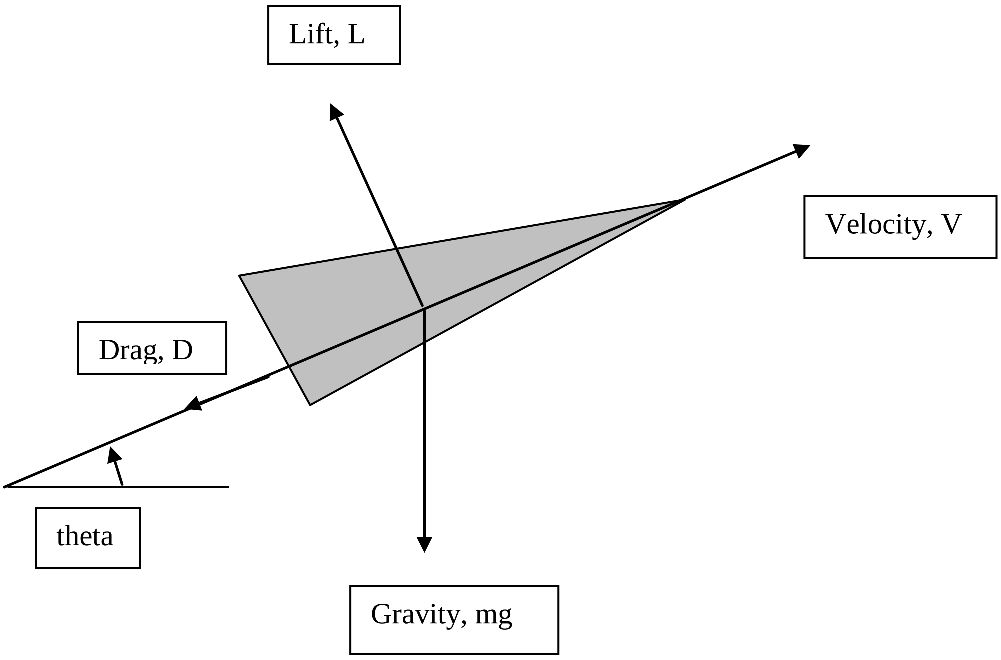


Figure 2. Simplified sketch of an aircraft at climb

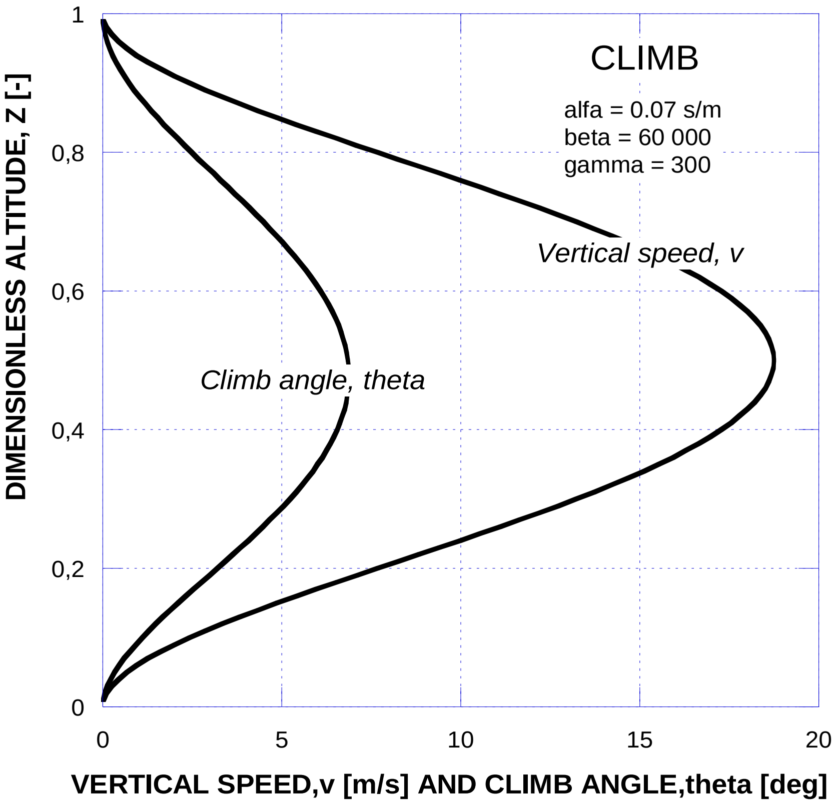


Figure 3. Vertical speed v and climb angle as function of Z

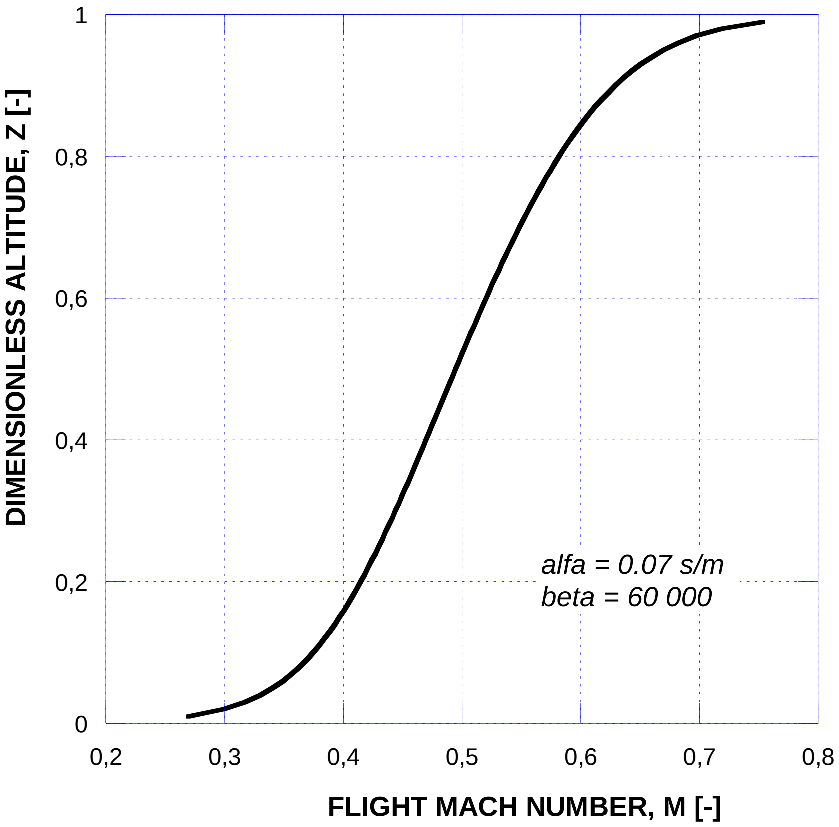


Figure 4. Flight Mach number at climb (and descent)

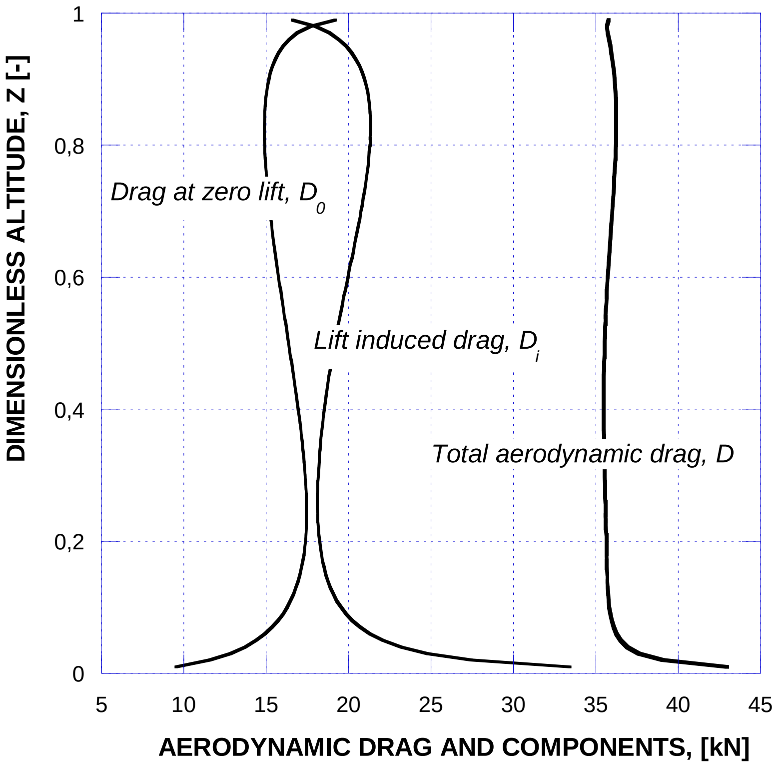


Figure 5. Aerodynamic drag and its components at climb

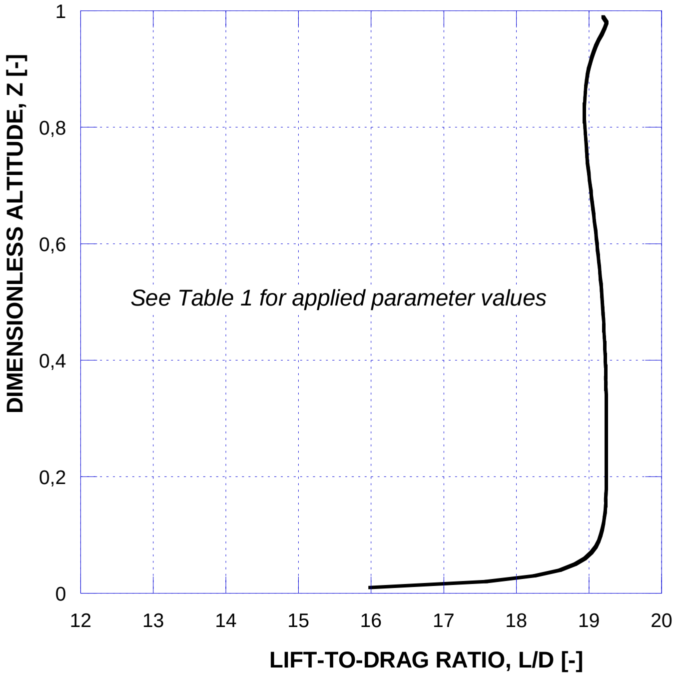


Figure 6. Lift-to-drag ratio at climb for constant aircraft mass

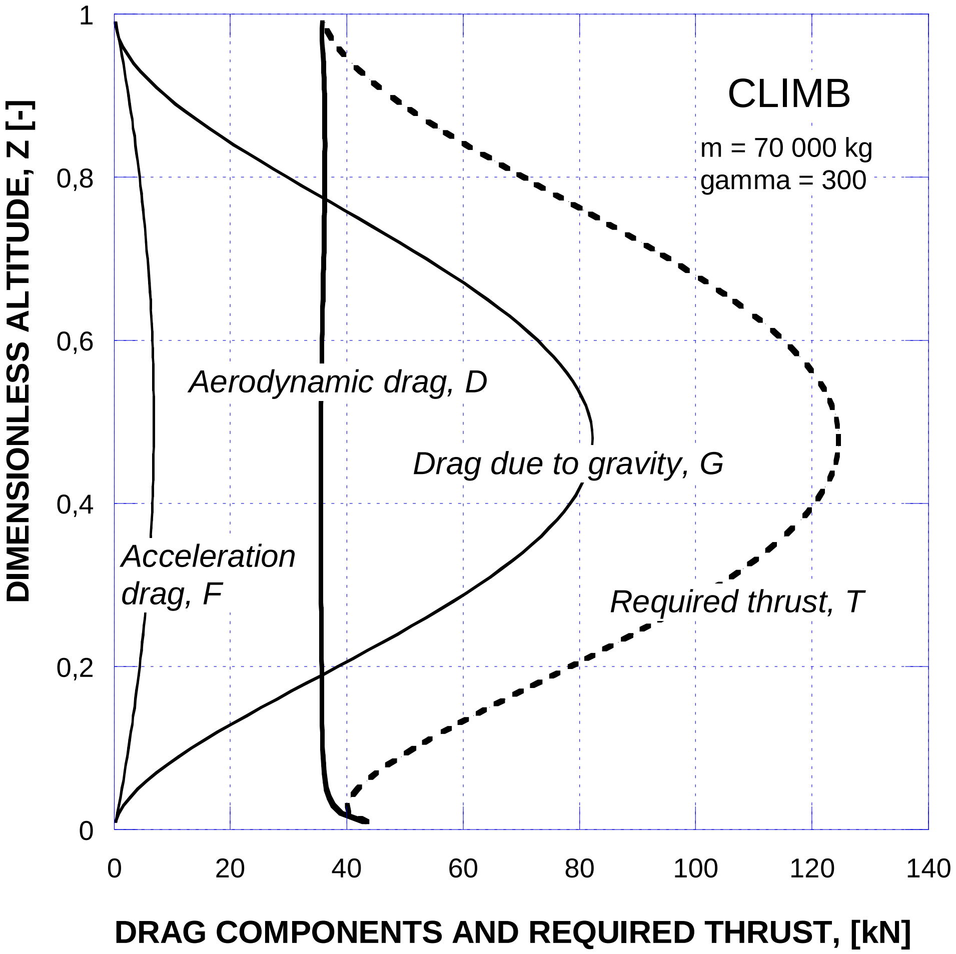


Figure 7. Force balance on an aircraft at climb

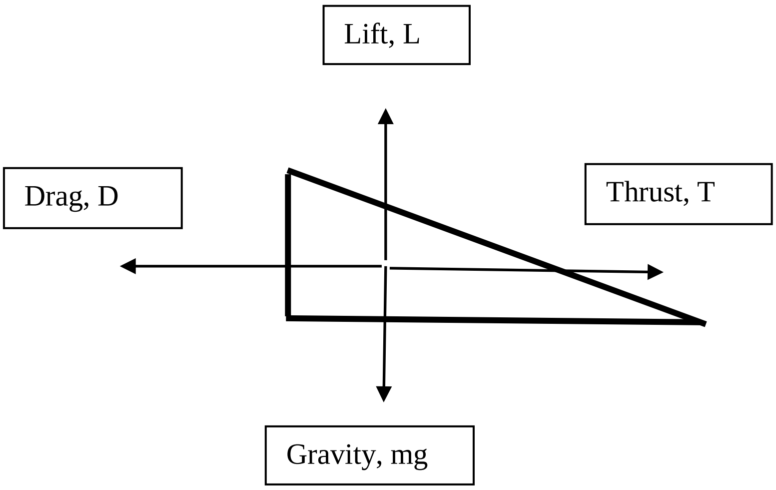


Figure 8. Force balance at steady cruise

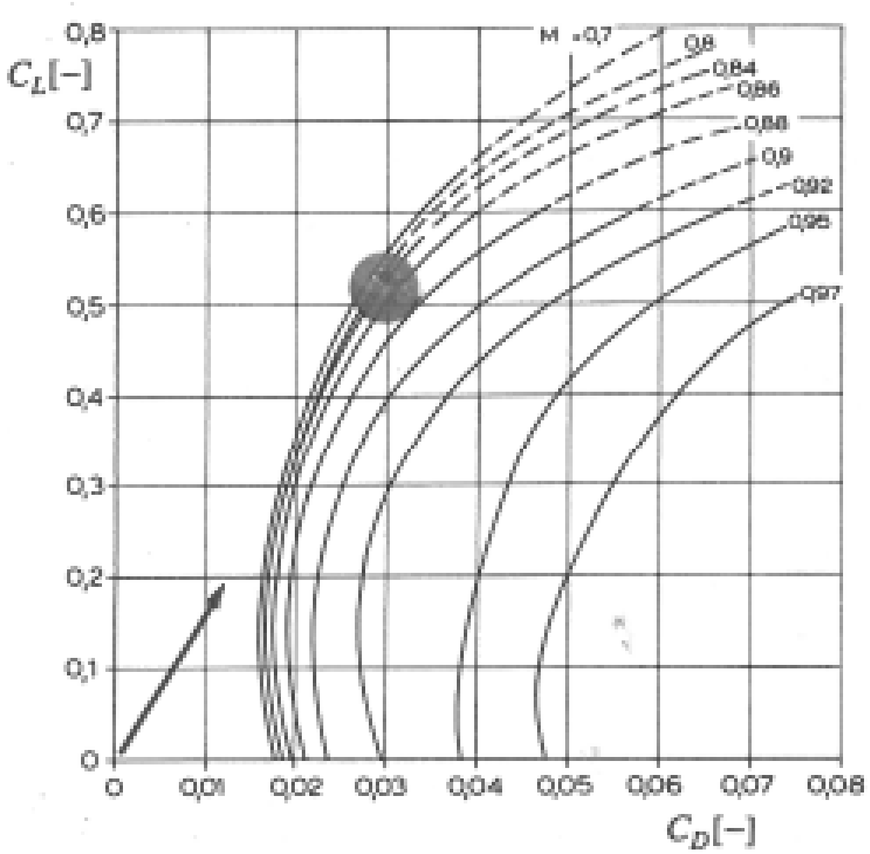


Figure 9. Various drag polars for a jet powered airliner [8]

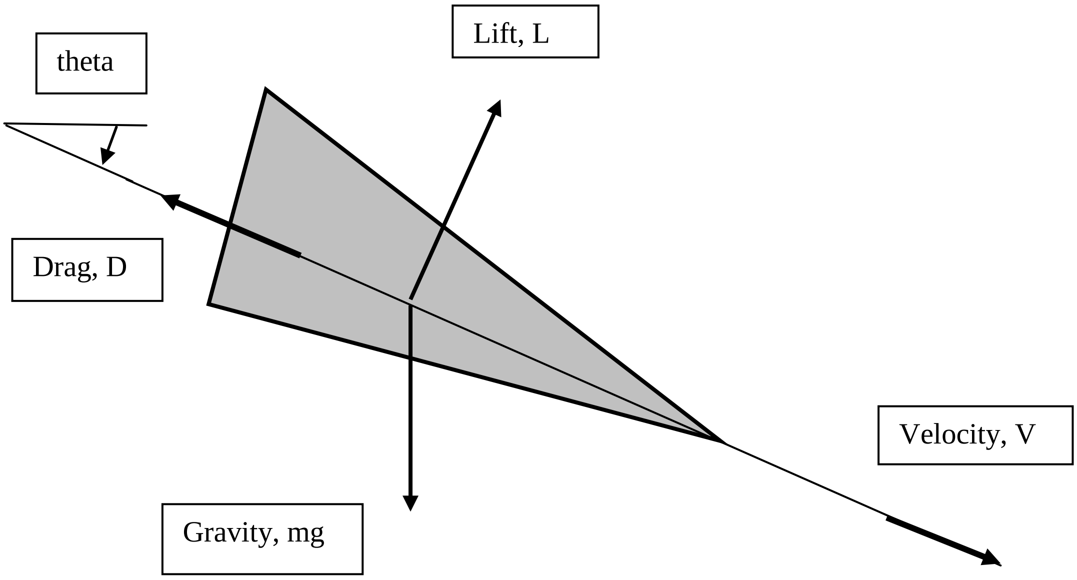


Figure 10. Simplified sketch of an aircraft at descent

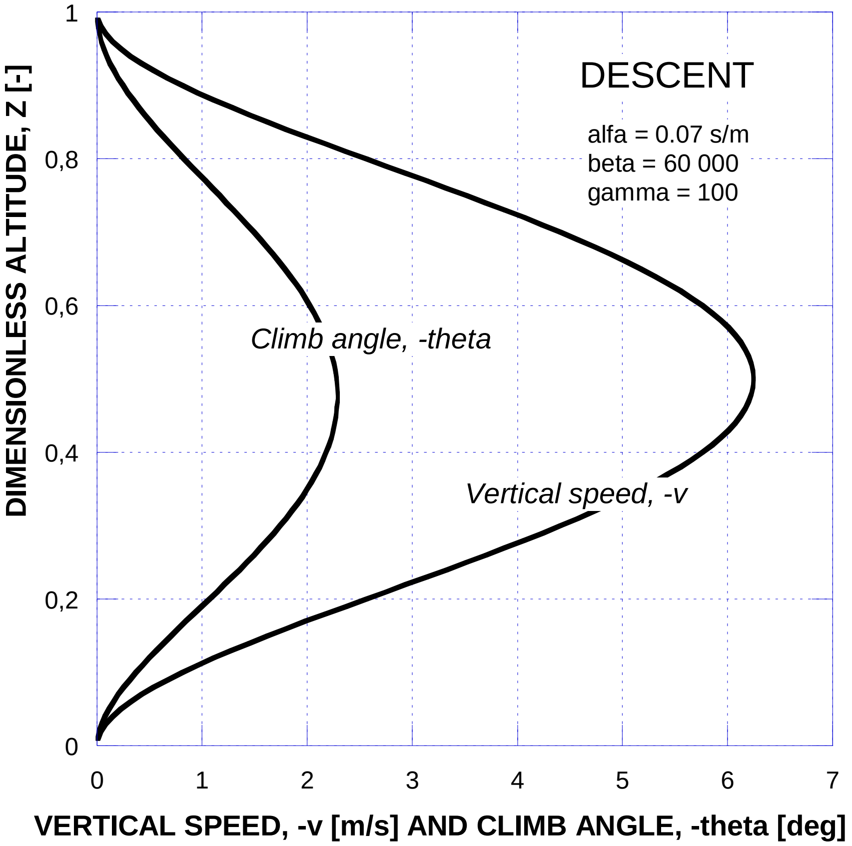


Figure 11. Vertical speed and climb angle as function of Z

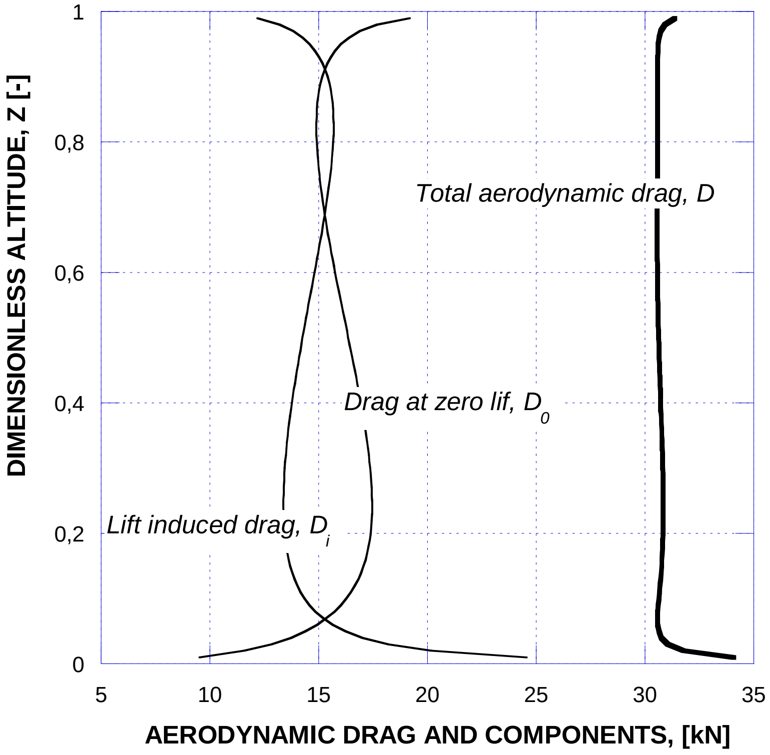


Figure 12. Aerodynamic drag and its components at descent

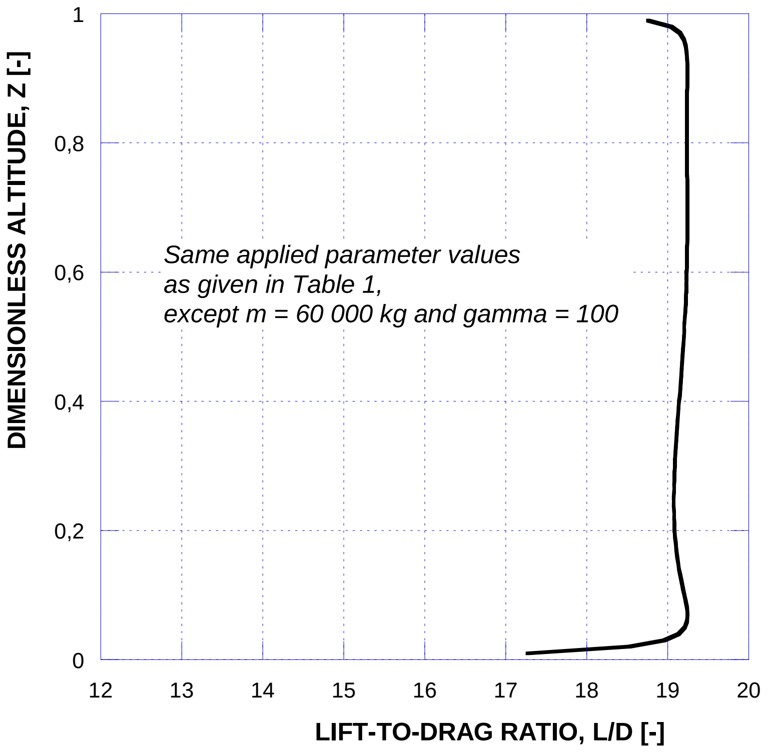


Figure 13. Lift-to-drag ratio at descent for constant aircraft mass

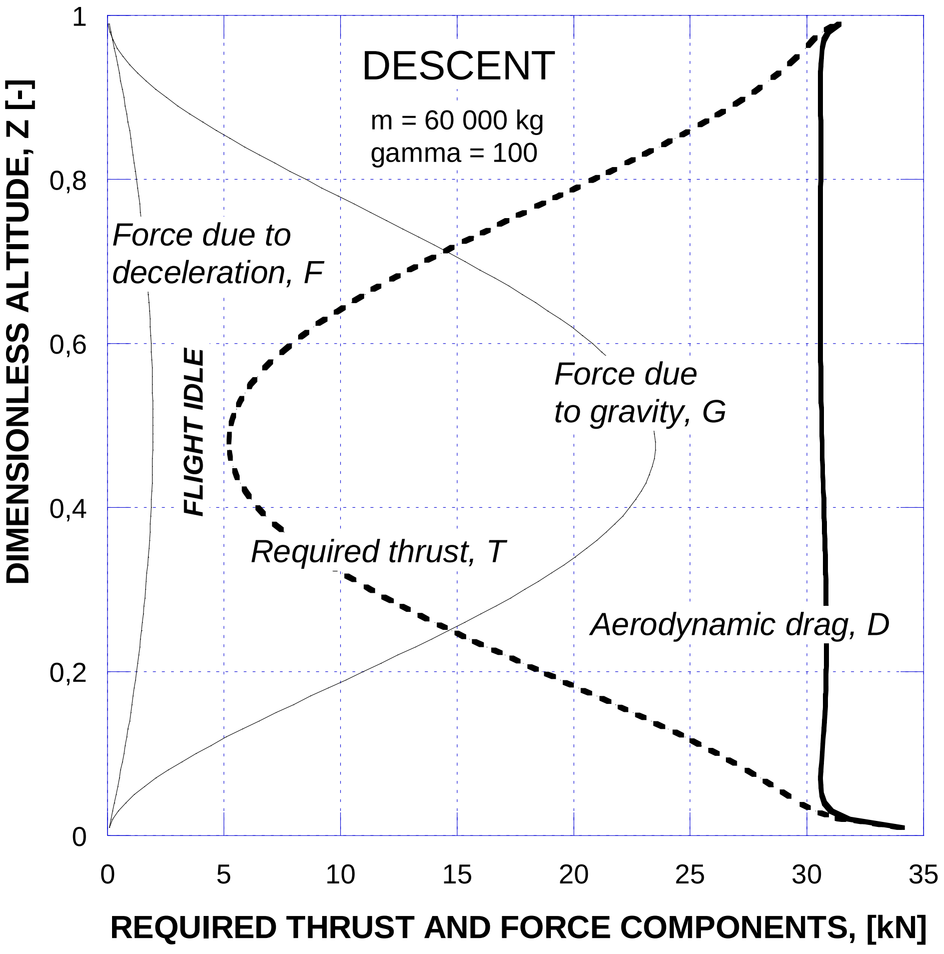


Figure 14. Force balance on the aircraft at descent

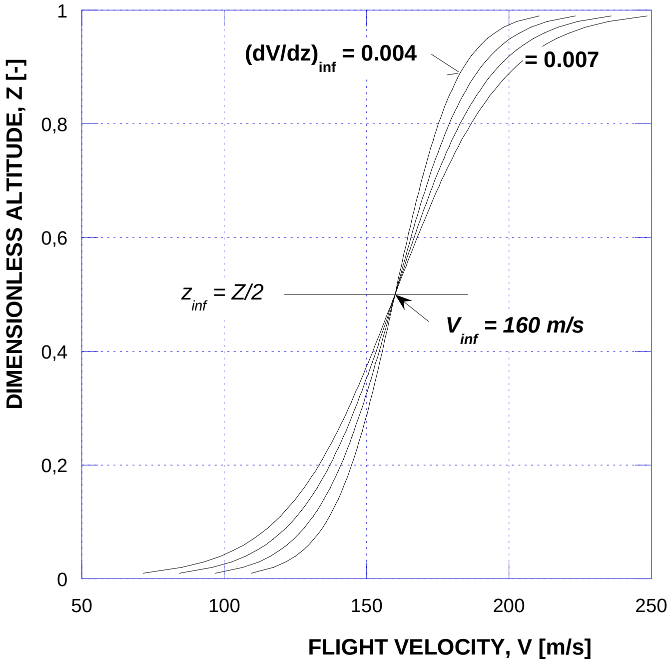


Figure 15. Sample trajectory input parameters for the climb or descent phase

Appendix B – Computer code

A computer program has been written in Matlab to calculate the times, the horizontal distance and the fuel consumption in the different phases of the flight trajectory and to reduce the fuel burn. The program is divided in three parts corresponding of the different phases of the flight trajectory: climb, cruise and descent. It applies to three different flight trajectories: Trondheim – Oslo, Trondheim – Nice and Paris – New York. For each trajectory, the user has the choice between two aircrafts. Two cruise trajectories are defined and depend on the flight trajectory. Two profiles of the vertical speed are defined and the user can choose one or the other profile.

```
%% STEP 1: Choice of the trajectory and the aircraft
clear all, clc


desti_number=menu('Choose the trajectory','Trondheim-Oslo','Trondheim-
Nice','Paris-New York');

% Two cruise trajectory configurations possible, depend on the trajectory
chosen



% STEP 2: Choice of the aircraft in function of the travel chosen

% Configuration 1: the cruise trajectory is horizontal

if desti_number==1 % Destination: Trondheim-Oslo
    plane_number = menu('Plane','737-300','737-800');
    d=390000; % Distance Trondheim-Oslo [m]
    if plane_number==1
        disp('Trondheim-Oslo and plane 737-300');
        % Characteristics of the aircraft 737-300
        S=105.4; % Wing area [m²]
        m0=62800; % Maximum takeoff weight [kg]
        lf=20100; % Maximum fuel capacity [l]
        alpha=0.0735; % Parameter defining the dimensional velocity
        SFC=0.03977; % Coefficient fuel burn
    else
        disp('Trondheim-Oslo and plane 737-800');
        % Characteristics of the aircraft 737-800
        S=125.58; % Wing area [m²]
        m0=79100; % Maximum takeoff weight [kg]
        lf=26020; % Maximum fuel capacity [l]
        alpha=0.0697; % Parameter defining the dimensional velocity
        SFC=0.03875; % Coefficient fuel burn
    end
end
```

```
if desti_number==2 % Destination Trondheim-Nice
    plane_number = menu('Plane','A320','737-800');
    d=2204000; % Distance Trondheim-Nice [m]
    if plane_number==1
        disp('Trondheim-Nice and A320');
        % Characteristics of the aircraft A320
        S=125; % Wing area [m²]
        m0=70000; % Maximum takeoff weight [kg]
        lf=24050; % Maximum fuel capacity [l]
        alpha=0.07; % Parameter defining the dimensional velocity
        SFC=0.03467; % Coefficient fuel burn
    else
        disp('Trondheim-Nice and plane 737-800');
        % Characteristics of the aircraft 737-800
        S=125.58; % Wing area [m²]
        m0=79100; % Maximum takeoff weight [kg]
        lf=26020; % Maximum fuel capacity [l]
        alpha=0.0697; % Parameter defining the dimensional velocity
        SFC=0.03875; % Coefficient fuel burn
    end
end


% Configuration 2: the cruise trajectory is not horizontal
% theta cruise and delta altitude

if desti_number==3
    plane_number = menu('Plane','A340','777-200');
    d=5851000; % Distance Paris-New York
    if plane_number==1
         disp('Paris-New York and plane A340');
         % Characteristics of the aircraft A340
         S=361.6; % Wing area [m²]
         m0=275000; % Maximum takeoff weight [kg]
         lf=155040; % Maximum fuel capacity [l]
         alpha=0.0663; % Parameter defining the dimensional velocity
         SFC=0.03263; % Coefficient fuel burn
    else
         disp('Paris-New York and plane 777-200');
         % Characteristics of the aircraft 777-200
         S=427.8; % Wing area [m²]
         m0=247200; % Maximum takeoff weight [kg]
         lf=117348; % Maximum fuel capacity [l]
         alpha=0.0648; % Parameter defining the dimensional velocity
         SFC=0.03365; % Coefficient fuel burn
    end
end


%% Parameters and constants

beta=60000; % Velocity/altitude parameter
gamma=input('Parameter for climb time determination (between 150 and 300)')
zc=input('The cruise altitude [m]') % Cruise altitude [m]
rho0=1.225; % Air density at sea level [kg/m3]
epsilon=0.0001; % Constant in the density function [1/m]
k=0.045; % Constant selected for the included drag coefficient
CDo=0.015; % Constant selected for the zero-lift drag coefficient
```

```
%% STEP 3: CLIMB

n=length(0.01:0.001:0.99); % Number of values in the vector

% Initialization at t=0 (takeoff)
% The index 1 is for the climb values
V1=zeros(nz,1); v1=zeros(nz,1); theta1=zeros(nz,1); c1=zeros(nz,1);
M1=zeros(nz,1); rho1=zeros(nz,1); CL1=zeros(nz,1); F1=zeros(nz,1);
G1=zeros(nz,1); D1=zeros(nz,1); T1=zeros(nz,1);
fuelburnCL=zeros(nz+1,1); fuelburnCL(1)=0;

% Fuel capacity in the aircraft at takeoff
rhof=817.15; % Fuel density [kg/m3]
fuelCL=zeros(nz,1); % Fuel burn vector [kg]
mf1=zeros(nz+1,1); % Weight of fuel vector
mf1(1)=lf*10^-3*rhof; % Weight of fuel at t=0 [kg]

%Maximum takeoff weight [kg]
m1=zeros(nz+1,1);% Weight of the aircraft vector
m1(1)=M0; % Maximum takeoff weight [kg]

% Calculation of the time of climb
vmean1=0.0333*gamma; % Mean vertical speed [m/s]
Timeclimb=(0.99*zc)/vmean1/3600; % Time of climb [hours]
tclimb=Timeclimb/n; % Time increment [hours]


% Climb trajectory
% Calculate the velocities, forces, weight of the fuel burn and aircraft
% for each increment until the cruise altitude

i=1;
for Z=0.01:0.001:0.99;
    V1(i)=(-1/alpha)*log((1/beta)*((1/Z)-1));%Velocity along flight path
    v1(i)=gamma*Z^2*(1-Z)^2; % Vertical velocity (m/s)
     v1(i)=-6*vm1*Z^2+6*vm1*Z; % the parabola vertical speed
    theta1(i)=asin(v1(i)/V1(i)); % Climb angle [rad]
    c1(i)=340.245-54.15*Z; % Speed of sound [m/s]
    M1(i)=V1(i)/c1(i); % Mach number [-]
    rho1(i)=rho0*exp(-epsilon*zc*Z); % Air density [kg/m3]
    CL1(i)=(m1(i)*9.81*cos(theta1(i)))/(0.5*S*rho1(i)*V1(i)^2); %Lift coef.
    F1(i)=(m1(i)*gamma*Z*(1-Z))/(alpha*zc); % Acceleration force F [N]
    G1(i)=m1(i)*9.81*sin(theta1(i)); % Drag due to gravity G [N]
    D1(i)=0.5*S*rho1(i)*V1(i)^2*(CDo+k*CL1(i)^2); % Aerodynamic drag D [N]
    T1(i)=F1(i)+G1(i)+D1(i); % Thrust force T [N]
    fuelCL(i)=SFC*T1(i)*tclimb; % Fuel burn at each increment [kg]
    fuelburnCL(i+1)=fuelburnCL(i)+fuelCL(i); % Total fuel burn [kg]
    mf1(i+1)=mf1(i)-fuelCL(i); % Weight of fuel [kg]
    m1(i+1)=m1(i)-fuelCL(i); % Weight of the aircraft [kg]
    i=i+1;
end


% Maximum thrust required during the climb
Tmax=max(T1)
```

```
% Distance climb
Vmean1=mean(V1); % Mean velocity along the flight path [m/s]
vhmean1=sqrt(Vmean1^2-vmean1^2); % Mean horizontal velocity [m/s]
dHclimb=vhmean1*Timeclimb*3600; % Horizontal climb distance [m]
% Plot

% Velocity along the flight path
figure(1);
subplot(2,1,1), plot(V1,0.01:0.001:0.99)
title('Climb velocity along the flight path')
xlabel('Velocity along the flight path V [m/s]')
ylabel('Dimensionless altitude Z [-]')

% Vertical velocity and climb angle
subplot(2,1,2), plot(v1,0.01:0.001:0.99,theta1*(180/pi),0.01:0.001:0.99)
title('Climb vertical velocity and climb angle')
legend('Vertical velocity','Climb angle')
xlabel('Vertical velocity v [m/s] and climb angle theta [°]')
ylabel('Dimensionless altitude Z [-]')

% Forces
%figure(2);
subplot(2,2,1),
plot(F1,0.01:0.001:0.99,G1,0.01:0.001:0.99,D1,0.01:0.001:0.99,T1,0.01:0.001
:0.99)
title('Climb forces')
legend('Acceleration force F','Drag due to gravity G','Aerodynamic drag
D','Thrust force T')
xlabel('Forces and required thrust [N]')
ylabel('Dimensionless altitude Z [-]')

% Fuel burn at each increment
subplot(2,2,2), plot(fuelCL,0.01:0.001:0.99)
xlabel('Fuel burn [kg]')
ylabel('Dimensionless altitude Z [-]')
title('Fuel consumption at each step')

% Fuel consumption
subplot(2,2,3), plot(fuelburnCL,0.009:0.001:0.99)
xlabel('Fuel burn [kg]')
ylabel('Dimensionless altitude Z [-]')
title('Fuel consumption during the climb')

% Values for the next step: the cruise

step1=('CLIMB')

disp(['Fuel burn after the climb [kg]: ', num2str(fuelburnCL(end))]);
disp(['Weight of the aircraft m [kg]: ', num2str(m1(end))]);
disp(['Weight of fuel mf [kg]: ', num2str(mf1(end))]);
disp(['Mach number M [-] (for the cruise phase): ', num2str(M1(end))]);
disp(['Speed of sound c [m/s]: ', num2str(c1(end))]);
disp(['Value of CL: ', num2str(CL1(end))]);
disp(['Time of climb [h]: ',num2str(Timeclimb)]);
disp(['Horizontal climb distance [m]: ',num2str(dHclimb)]);
```

```
%% STEP 4: CRUISE
% The cruise trajectory for the short and medium flight is horizontal
% Use for the flight Trondheim - Oslo and Trondheim - Nice
% The index c and 2 are for the cruise values

% Initialization for the cruise trajectory
step2=('CRUISE')

V2=M1(end)*c1(end); % Cruise velocity [m/s](constant)
disp(['Cruise velocity Vc [m/s]: ',num2str(V2)]);

% Calculate the air density and the aircraft weight at zc (after climb)
zc=Z(end)*zc;
rhoc=rho0*exp(-epsilon*zc);% Air density [kg/m3]
mc=((CL1(end)*S*V2^2)/(2*9.81))*rhoc; % Weight of the aircraft [kg]
disp(['Weight of the aircraft at cruise altitude 1 [kg]: ',num2str(mc)]);


% Cruise time and distance
% Define the time and the distance for the descent
gamma=input('Parameter for descent time determination (between 100 and
200)')
vmean3=0.0333*gamma; % Mean vertical speed [m/s]
vhmean3=sqrt(Vmean1^2-vmean3^2); % Mean horizontal velocity [m/s]
Timedescent=(zc*0.99)/vmean3/3600; % Time of descent[hours]
dHdescent=vhmean3*Timedescent*3600; % Horizontal descent distance [m]

% Time and distance for the cruise phase
dHcruise=d-dHdescent-dHclimb; % Horizontal cruise distance [m]
Timecruise=dHcruise/V2/3600; % Time of cruise [hours]

% Thrust force [N]
T2=D1(end);% Thrust force is constant [N]

% Fuel burn
fuelburnCR=SFC*Timecruise*T2;% Fuel burn during the cruise [kg]
m2=mc-fuelburnCR; % Weight of aircraft after the cruise [kg]
mf2=mf1(end)-fuelburnCR;% Weight of fuel [kg]

disp(['Fuel burn during the cruise [kg]: ', num2str(fuelburnCR)]);
disp(['Weight of the aircraft m after the cruise[kg]: ', num2str(m2)]);
disp(['Weight of fuel mf after the cruise[kg]: ', num2str(mf2)]);
disp(['Time of cruise [h]: ',num2str(Timecruise)]);
disp(['Horizontal cruise distance [m]: ',num2str(dHcruise)]);
```

```
%% STEP 4-bis: CRUISE
% The cruise trajectory for the long flight is linear between zc1 and zc2
% Use for the flight Paris - New York

% Calculate the air density and the aircraft weight at zc (after climb)
zc1=Z(end)*zc1;
rhoc=rho0*exp(-epsilon*zc1);% air density [kg/m3]
mc=((CL1(end)*S*V2^2)/(2*9.81))*rhoc;% Weight of the aircraft [kg]
disp(['Weight of the aircraft at cruise altitude 1 [kg]: ',num2str(mc)]);

% Delta cruise altitude [m]
deltazc=input('Value of the delta cruise altitude [m]')
% Altitude at the end of the cruise
zc2=zc1+deltazc;

ncr=length(zc1:1:zc1+deltazc); % Number of values in the vector

% Cruise time and distance
% Define the time and the distance for the descent
gamma=input('Parameter descent time determination (between 100 and 200)')
vmean3=0.0333*gamma; % Mean vertical velocity [m/s]
vhmean3=sqrt(Vmean1^2-vmean3^2); % Mean horizontal descent velocity [m/s]
Timedescent=(zc2*0.99)/vmean3/3600; % Time of descent[hours]
dHdescent=vhmean3*Timedescent*3600; % Horizontal descent distance [m]
% Time and distance for the cruise phase
dHcruise=d-dHdescent-dHclimb;% Horizontal cruise distance [m]
Timecruise=dHcruise/V2/3600;% Time of cruise [hours]
tcruise=Timecruise/ncr;% Increment time [hours]

% Initialisation at cruise altitude z1
v2=zeros(ncr,1); theta2=zeros(ncr,1); rho2=zeros(ncr,1);
D2=zeros(ncr,1); T2=zeros(nz,1); fuelburnCR=zeros(nz,1);

% Fuel capacity in the airplane after the climb
fuelCR=zeros(nz,1); % Fuel burn during the cruise[kg]
mf2=zeros(ncr+1,1);% Weight of fuel vector
mf2(1)=mf1(end); % Weight of fuel during the cruise[kg]

%Maximum take off weight [kg]
m2=zeros(nz+1,1);% Weight of the aircraft vector
m2(1)=mc;% Weight of the aircraft [kg]

% Cruise trajectory

i=1;
for z=zc1:1:zc2;
    rho2(i)=rho0*exp(-epsilon*z); % Air density [kg/m3]
    D2(i)=0.5*S*rho2(i)*V2^2*(CDo+k*CL1(end)^2); % Aerodynamic drag D [N]
    T2(i)=D2(i); % Thrust force [N]
    fuelCR(i)=SFC*T2(i)*tcruise; % Fuel burn at each increment [kg]
    fuelburnCR(i+1)=fuelburnCR(i)+fuelCR(i); % Total fuel burn [kg]
    mf2(i+1)=mf2(i)-fuelCR(i); % Weight of fuel [kg]
    m2(i+1)=m2(i)-fuelCR(i); % Weight of the airplane [kg]
    i=i+1;
end
```

```
%% STEP 5: DESCENT
% The index 3 is for the descent values

% Initialization
% Fuel burn before the descent [kg]
fuelD=zeros(nz,1);
fuelburnD=zeros(nz+1,1); fuelburnD(1)=0;

% Weight aircraft and fuel before the descent [kg]
m3=zeros(nz+1,1); m3(1)=m2;
mf3=zeros(nz+1,1); mf3(1)=mf2;

V3=zeros(nz,1); v3=zeros(nz,1); theta3=zeros(nz,1); rho3=zeros(nz,1);
c3=zeros(nz,1); M3=zeros(nz,1);
CL3=zeros(nz,1); F3=zeros(nz,1); G3=zeros(nz,1); D3=zeros(nz,1);
T3=zeros(nz,1);

% Calculation of the time increment
tdescent=Timedescent/n; % Time increment [hours]


% Descent trajectory
% Calculate the velocities, forces, weight of the fuel burn and airplane
% for each increment until the landing

i=1;
for Z=0.01:0.001:0.99;
    Zd=1-Z; % dimensionless altitude
    V3(i)=(-1/alpha)*log((1/beta)*((1/Zd)-1));%Velocity along flight path
    v3(i)=-gamma*Zd^2*(1-Zd)^2; % Vertical speed (m/s)
     v3(i)=-(-6*vm1*Z^2+6*vm1*Z); % the parabola vertical speed
    theta3(i)=asin(v3(i)/V3(i)); % Climb angle [rad]
    c3(i)=340.254-45.15*Zd; % Speed of sound [m/s]
    M3(i)=V3(i)/c3(i); % Mach number [-]
    rho3(i)=rho0*exp(-epsilon*zc*Zd); % Air density [kg/m3]
    CL3(i)=(m3(i)*9.81*cos(-theta3(i)))/(0.5*S*rho3(i)*V3(i)^2);%Lift coef.
    F3(i)=(m3(i)*gamma*Zd*(1-Zd))/(alpha*zc); % Acceleration force F [N]
    G3(i)=m3(i)*9.81*sin(-theta3(i)); % Drag due to gravity Gd [N]
    D3(i)=0.5*S*rho3(i)*V3(i)^2*(CDo+k*CL3(i)^2); % Aerodynamic drag D [N]
    T3(i)=D3(i)-G3(i)-F3(i); % Thrust force [N] can't be negative
    if T3(i)<=(10/100)*Tmax
        T3(i)=(10/100)*Tmax;
    elseif T3(i)>(10/100)*Tmax
        T3(i)=T3(i);
    end
    fuelD(i)=SFC*T3(i)*tdescent; % Fuel burn at each increment [kg]
    fuelburnD(i+1)=fuelburnD(i)+fuelD(i); % Total fuel burn [kg]
    mf3(i+1)=mf3(i)-fuelD(i); % Weight of fuel [kg]
    m3(i+1)=m3(i)-fuelD(i); % Weight of the aircraft [kg]
    i=i+1;
end

Tmean=mean(T3)

% Horizontal descent distance
Vmean3=mean(V3); % Mean velocity along the flight path [m/s]
vhmean3=sqrt(Vmean3^2-vmean3^2); % Mean horizontal descent velocity [m/s]
dHdescent=vhmean3*Timedescent*3600; % Horizontal descent distance [m]
```

```
% Plot

% Forces
figure(3);
subplot(2,2,1),
plot(F3,0.01:0.001:0.99,G3,0.01:0.001:0.99,D3,0.01:0.001:0.99,T3,0.01:0.001
:0.99)
title('Descent forces')
legend('Acceleration force F','Drag due to gravity G','Aerodynamic drag
D','Thrust force T')
xlabel('Forces and required thrust [N]')
ylabel('Dimensionless altitude Z [-]')

% Vertical velocity and descent angle
subplot(2,2,2), plot(v3,0.01:0.001:0.99,theta3*(180/pi),0.01:0.001:0.99)
title('Descent vertical velocity and descent angle')
legend('Vertical velocity','Descent angle')
xlabel('Vertical velocity v [m/s] and descent angle theta [°]')
ylabel('Dimensionless altitude Z [-]')

% Fuel burn at each increment
subplot(2,2,3), plot(fuelD,0.01:0.001:0.99)
xlabel('Fuel burn [kg]')
ylabel('Dimensionless altitude Z [-]')
title('Fuel consumption at each step')

% Fuel burn
subplot(2,2,4), plot(fuelburnD,0.99:-0.001:0.009)
xlabel('Fuel consumption [kg]')
ylabel('Dimensionless altitude Z [-]')
title('Fuel consumption during the descent')

% Values of the descent
step3=('DESCENT')
disp(['Value of fuel burn during the descent [kg]: ',
num2str(fuelburnD(end))]);
disp(['Time of descent [hr]: ',num2str(Timedescent)]);
disp(['Horizontal descent distance [m]: ',num2str(dHdescent)]);



%% Results

('RESULTS')

% Time of the trajectory [hours]
TIME=Timeclimb+Timecruise+Timedescent;
disp(['Time trajectory [h]: ',num2str(TIME)]);

% The fuel burn [kg]
fuelburn=fuelburnCL(end)+fuelburnCR+fuelburnD(end);
disp(['Fuel burn during the trajectory [kg]: ',num2str(fuelburn)]);

% The remaining fuel [kg]
disp(['Weight of remaining fuel [kg]: ',num2str(mf3(end))]);
% The weight of the airplane [kg]
disp(['Weight of airplane after the trajectory[kg]: ',num2str(m3(end))]);
```

Appendix C – Results tables

Trondheim – Oslo

737-800 Next generation

	γ		Tmax	Horizontal distance			Time				Fuelburn				
	cl.	d.		climb	cruise	descent	climb	cruise	descent	total	climb	cruise	descent	total	diff.
	-	-	N	m			h				kg				
zc (m) 5000	**200**	100	1,13E+05	117216,0	40541,2	232242,8	0,206	0,050	0,409	**0,666**	656,5	90,8	346,5	**1093,8**	2,0
		150			117998,6	154785,5		0,146	0,273	**0,625**		264,3	200,9	**1121,8**	30,0
		200			156740,2	116043,8		0,195	0,204	**0,605**		351,1	139,2	**1146,8**	55,0
	250	100	1,31E+05	93725,7	64031,4	232242,8	0,165	0,079	0,409	**0,653**	591,1	143,5	357,2	**1091,8**	-
		150			141488,8	154785,5		0,176	0,273	**0,613**		317,1	211,8	**1120,0**	28,2
		200			180230,4	116043,8		0,224	0,204	**0,593**		403,9	148,6	**1143,6**	51,8
	300	100	1,49E+05	78056,8	79700,3	232242,8	0,138	0,099	0,409	**0,645**	547,5	178,6	369,0	**1095,2**	3,4
		150			157157,7	154785,5		0,195	0,273	**0,605**		352,3	223,0	**1122,8**	31,0
		200			195899,3	116043,8		0,243	0,204	**0,585**		439,1	158,2	**1144,8**	53,0
zc (m) 6000	**200**	150	1,11E+05	140659,2	63598,3	185742,5	0,248	0,079	0,327	**0,654**	772,2	135,8	240,4	**1148,4**	27,1
		200			110088,2	139252,6		0,137	0,245	**0,630**		235,1	166,2	**1173,5**	52,2
	250	150	1,29E+05	112470,9	91786,6	185742,5	0,198	0,114	0,327	**0,639**	694,4	196,1	252,8	**1143,3**	22,0
		200			138276,5	139252,6		0,172	0,245	**0,615**		295,4	177,0	**1166,8**	45,6
	300	100	1,46E+05	**93668,2**	**17640,4**	**278691,4**	**0,165**	**0,022**	**0,491**	**0,678**	**642,4**	**37,7**	**441,1**	**1121,3**	-
		150			110589,2	185742,5		0,137	0,327	**0,629**		236,4	265,7	**1144,5**	23,2
		200			157079,2	139252,6		0,195	0,245	**0,605**		335,7	188,1	**1166,3**	45,0
zc (m) 7000	**200**	150	1,09E+05	164102,4	9198,0	216699,6	0,289	0,011	0,382	**0,682**	888,2	18,9	280,2	**1187,3**	9,6
		200			63436,2	162461,4		0,079	0,286	**0,654**		130,1	193,5	**1211,8**	34,1
	250	150	1,27E+05	131216,0	42084,3	216699,6	0,231	0,052	0,382	**0,665**	797,9	86,4	294,1	**1178,5**	0,7
		200			96322,6	162461,4		0,120	0,286	**0,637**		197,7	205,7	**1201,4**	23,6
	300	150	1,44E+05	109279,6	64020,8	216699,6	0,193	0,079	0,382	**0,654**	737,7	131,5	308,6	**1177,8**	-
		200			118259,0	162461,4		0,147	0,286	**0,626**		242,9	218,2	**1198,7**	21,0

🟦 less fuel burn
🟧 less time
🟨 the best ratio fuelburn/h

	γ		737-300				737-800			
			Fuelburn per hour				Fuelburn per hour			
	cl.	d.	climb	cruise	descent	total	climb	cruise	descent	total
	-	-	kg/h				kg/h			
zc (m) 5000	200	100	2631,6	1433,8	682,7	1347,5	3179,9	1804,5	847,6	1643,4
		150		1433,9	599,1	1464,2		1804,6	737,3	1793,5
		200		1433,8	555,7	1541,1		1804,6	680,9	1894,2
	250	100	2967,9	1434,5	706,7	1369,5	3579,0	1805,3	873,8	1670,9
		150		1434,5	633,7	1489,3		1805,3	777,1	1826,1
		200		1434,5	595,2	1566,4		1805,3	726,8	1927,5
	300	100	3304,0	1434,9	733,1	1390,8	3977,7	1805,8	902,8	1697,0
		150		1434,9	669,6	1512,0		1805,8	818,4	1855,1
		200		1434,9	635,7	1588,5		1805,8	774,0	1956,2
zc (m) 6000	200	150	2585,4	1372,1	598,0	1441,9	3116,8	1720,2	735,0	1756,7
		200		1372,1	553,8	1524,3		1720,3	677,6	1863,6
	250	150	2912,1	1373,0	631,1	1466,8	3503,5	1721,1	772,9	1788,7
		200		1372,9	591,9	1549,8		1721,1	721,7	1897,0
	300	100	3238,3	1373,5	731,0	1363,4	3889,6	1721,9	899,2	1654,8
		150		1373,5	665,5	1490,1		1721,8	812,3	1818,1
		200		1373,5	630,8	1572,7		1721,8	766,9	1926,4
zc (m) 7000	200	150	2554,2	1323,6	598,5	1435,2	3073,1	1652,5	734,5	1741,0
		200		1323,5	553,4	1522,5		1652,5	676,1	1853,1
	250	150	2874,1	1324,7	630,5	1459,8	3450,9	1653,7	771,0	1772,2
		200		1324,6	590,4	1548,0		1653,7	718,8	1886,1
	300	150	3193,5	1325,4	663,7	1483,5	3828,2	1654,6	808,9	1801,7
		200		1325,4	628,2	1571,5		1654,5	762,6	1916,0

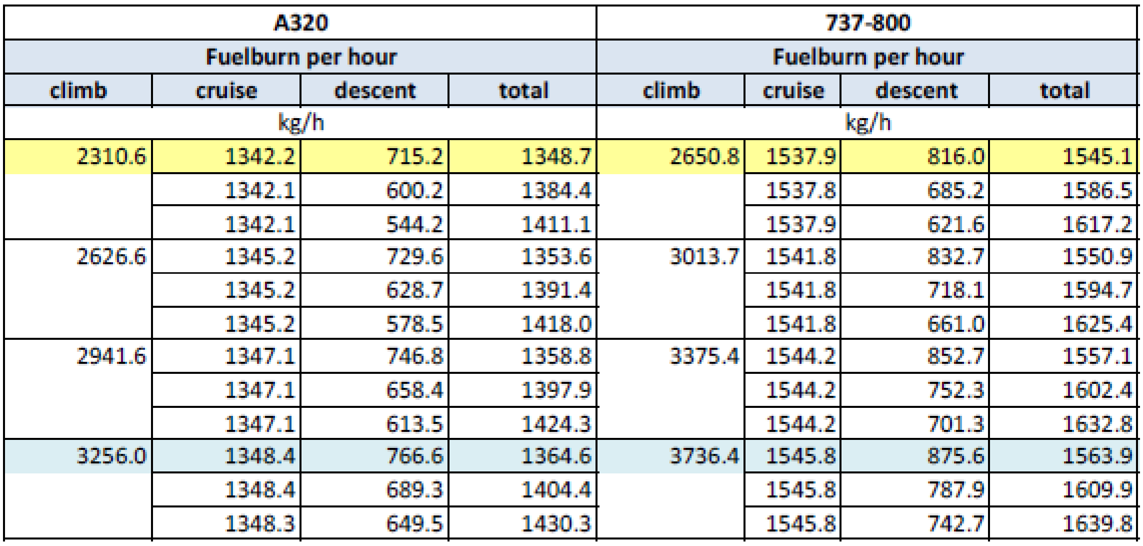
Trondheim – Nice

A320

	γ		Tmax	Horizontal distance			Time				Fuelburn				
	cl.	d.		climb	cruise	descent	climb	cruise	descent	total	climb	cruise	descent	total	diff.
	-	-	N	m			h				kg				
zc (m) 10058.4	**150**	100	8.80E+04	314511.5	1422308.8	467179.7	0.6	1.766	0.822	**3.142**	1279.4	2369.6	588.1	**4237.2**	61.1
		150			1578122.0	311366.4		1.959	0.548	**3.061**		2629.2	329.0	**4237.7**	61.6
		200			1656054.7	233433.8		2.056	0.411	**3.021**		2759.0	223.7	**4262.2**	86.2
	200	100	1.04E+05	235791.7	1501028.6	467179.7	0.415	1.863	0.822	**3.101**	1090.8	2506.5	599.9	**4197.3**	21.2
		150			1656841.9	311366.4		2.057	0.548	**3.020**		2766.7	344.7	**4202.2**	26.2
		200			1734774.5	233433.8		2.153	0.411	**2.980**		2896.8	237.8	**4225.5**	49.5
	250	100	1.21E+05	188538.7	1548281.6	467179.7	0.332	1.922	0.822	**3.077**	977.3	2589.0	614.1	**4180.5**	4.4
		150			1704094.9	311366.4		2.115	0.548	**2.996**		2849.5	361.0	**4187.8**	11.8
		200			1782027.5	233433.8		2.212	0.411	**2.956**		2979.9	252.3	**4209.5**	33.4
	300	100	1.37E+05	157019.2	1579801.2	467179.7	0.277	1.961	0.822	**3.1**	901.5	2644.2	630.4125	**4176.1**	-
		150			1735614.4	311366.4		2.154	0.548	**2.980**		2904.9	377.9	**4184.3**	8.3
		200			1813547.1	233433.8		2.251	0.411	**2.939**		3035.4	267.0	**4203.9**	27.8

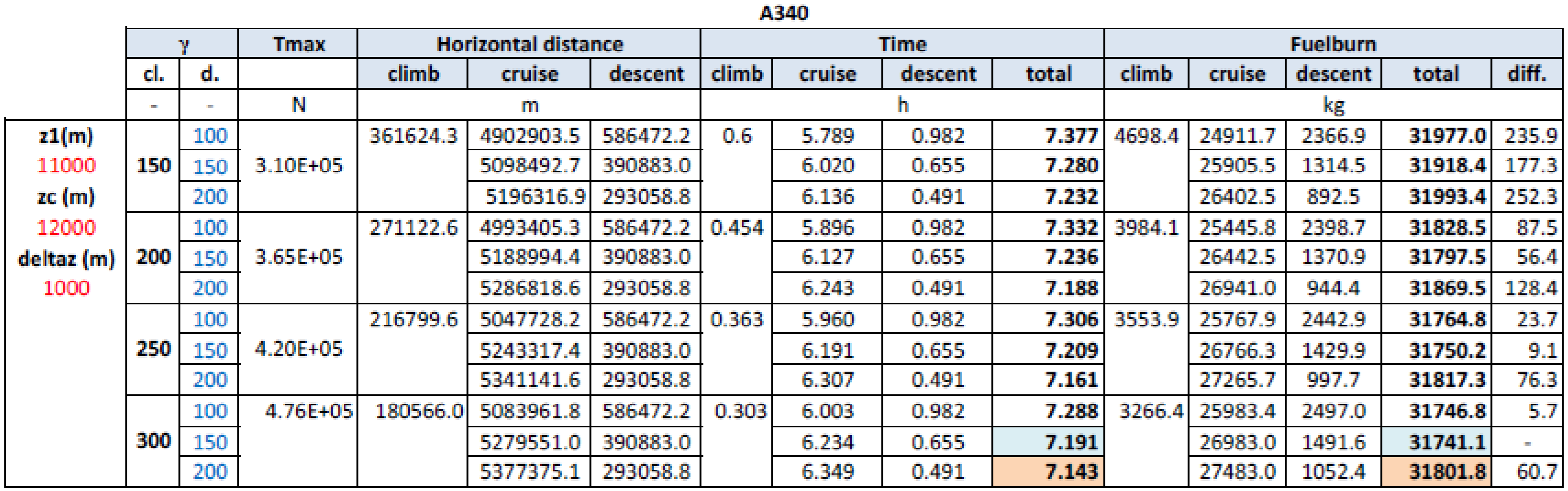
737-800

	γ		Tmax	Horizontal distance			Time				Fuelburn				
	cl.	d.		climb	cruise	descent	climb	cruise	descent	total	climb	cruise	descent	total	diff.
	-	-	N	m			h				kg				
zc (m) 10058.4	**150**	100	9.03E+04	314511.5	1422308.8	467179.7	0.6	1.766	0.822	**3.142**	1467.8	2715.1	671.0	**4854.0**	68.1
		150			1578122.0	311366.4		1.959	0.548	**3.061**		3012.6	375.6	**4856.1**	70.2
		200			1656054.7	233433.8		2.056	0.411	**3.021**		3161.4	255.6	**4884.8**	98.9
	200	100	1.07E+05	235791.7	1501028.6	467179.7	0.415	1.863	0.822	**3.101**	1251.6	2872.8	684.7	**4809.2**	23.3
		150			1656841.9	311366.4		2.057	0.548	**3.020**		3171.0	393.7	**4816.3**	30.4
		200			1734774.5	233433.8		2.153	0.411	**2.980**		3320.2	271.8	**4843.6**	57.7
	250	100	1.24E+05	188538.7	1548281.6	467179.7	0.332	1.922	0.822	**3.077**	1121.5	2967.8	701.2	**4790.5**	4.6
		150			1704094.9	311366.4		2.115	0.548	**2.996**		3266.5	412.4	**4800.4**	14.5
		200			1782027.5	233433.8		2.212	0.411	**2.956**		3415.9	288.4	**4825.7**	39.9
	300	100	1.40E+05	157019.2	1579801.2	467179.7	0.277	1.961	0.822	**3.1**	1034.5	3031.4	719.9938	**4785.9**	-
		150			1735614.4	311366.4		2.154	0.548	**2.980**		3330.4	431.9	**4796.8**	10.9
		200			1813547.1	233433.8		2.251	0.411	**2.939**		3479.9	305.4	**4819.8**	33.9



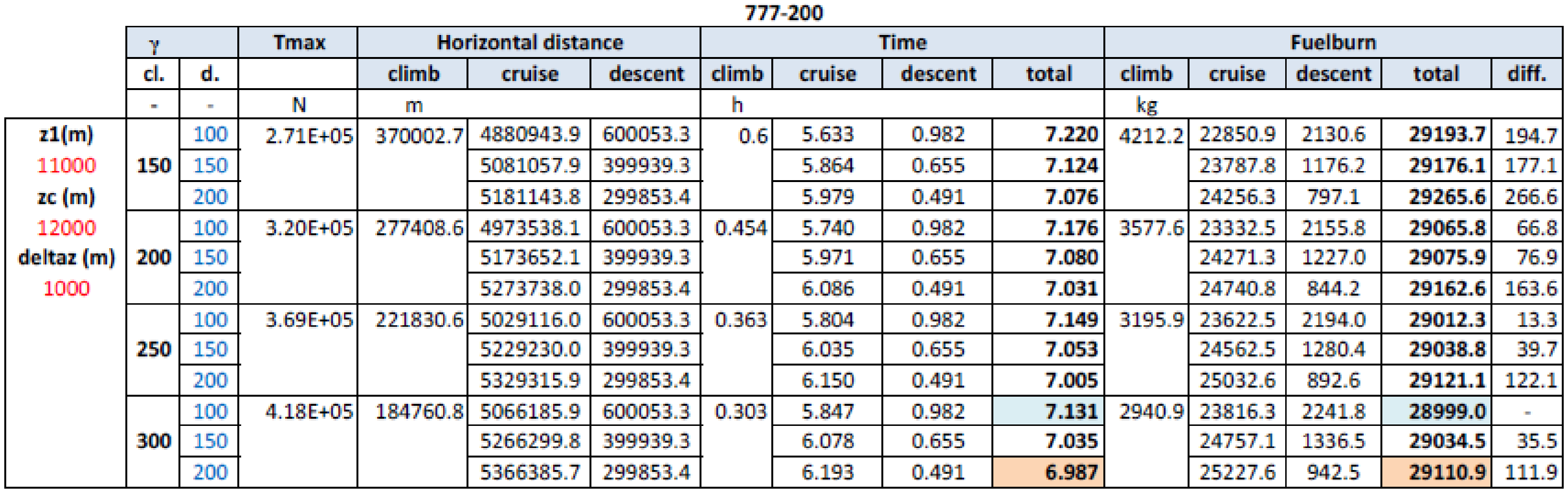
A320				737-800			
Fuelburn per hour				Fuelburn per hour			
climb	cruise	descent	total	climb	cruise	descent	total
kg/h				kg/h			
2310.6	1342.2	715.2	1348.7	2650.8	1537.9	816.0	1545.1
	1342.1	600.2	1384.4		1537.8	685.2	1586.5
	1342.1	544.2	1411.1		1537.9	621.6	1617.2
2626.6	1345.2	729.6	1353.6	3013.7	1541.8	832.7	1550.9
	1345.2	628.7	1391.4		1541.8	718.1	1594.7
	1345.2	578.5	1418.0		1541.8	661.0	1625.4
2941.6	1347.1	746.8	1358.8	3375.4	1544.2	852.7	1557.1
	1347.1	658.4	1397.9		1544.2	752.3	1602.4
	1347.1	613.5	1424.3		1544.2	701.3	1632.8
3256.0	1348.4	766.6	1364.6	3736.4	1545.8	875.6	1563.9
	1348.4	689.3	1404.4		1545.8	787.9	1609.9
	1348.3	649.5	1430.3		1545.8	742.7	1639.8

Paris – New York



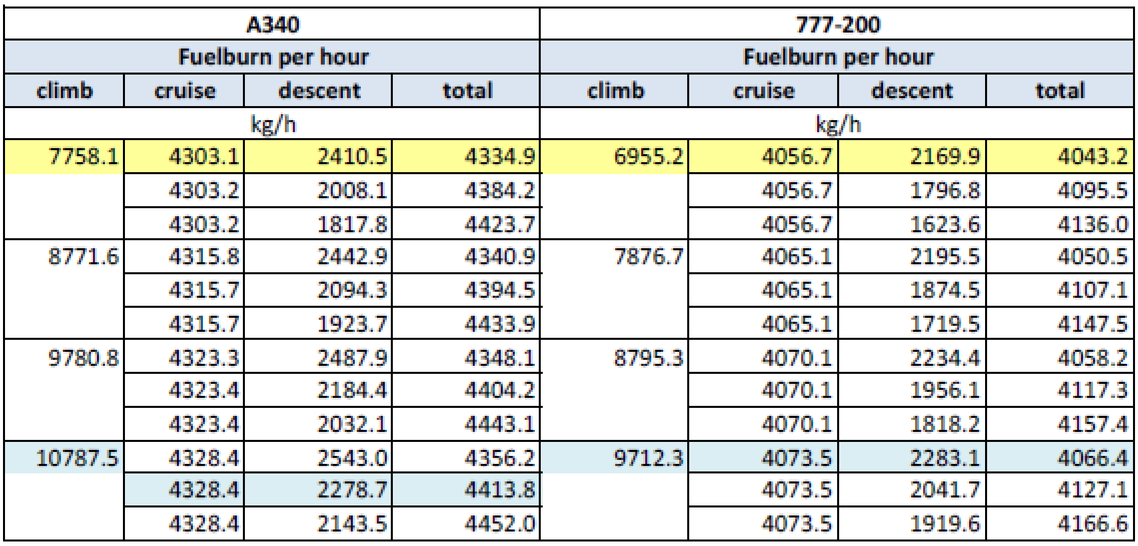
A340

	γ		Tmax	Horizontal distance			Time				Fuelburn				
	cl.	d.		climb	cruise	descent	climb	cruise	descent	total	climb	cruise	descent	total	diff.
	-	-	N	m			h				kg				
z1(m)	**150**	100	3.10E+05	361624.3	4902903.5	586472.2	0.6	5.789	0.982	**7.377**	4698.4	24911.7	2366.9	**31977.0**	235.9
11000		150			5098492.7	390883.0		6.020	0.655	**7.280**		25905.5	1314.5	**31918.4**	177.3
zc (m)		200			5196316.9	293058.8		6.136	0.491	**7.232**		26402.5	892.5	**31993.4**	252.3
12000	**200**	100	3.65E+05	271122.6	4993405.3	586472.2	0.454	5.896	0.982	**7.332**	3984.1	25445.8	2398.7	**31828.5**	87.5
deltaz (m)		150			5188994.4	390883.0		6.127	0.655	**7.236**		26442.5	1370.9	**31797.5**	56.4
1000		200			5286818.6	293058.8		6.243	0.491	**7.188**		26941.0	944.4	**31869.5**	128.4
	250	100	4.20E+05	216799.6	5047728.2	586472.2	0.363	5.960	0.982	**7.306**	3553.9	25767.9	2442.9	**31764.8**	23.7
		150			5243317.4	390883.0		6.191	0.655	**7.209**		26766.3	1429.9	**31750.2**	9.1
		200			5341141.6	293058.8		6.307	0.491	**7.161**		27265.7	997.7	**31817.3**	76.3
	300	100	4.76E+05	180566.0	5083961.8	586472.2	0.303	6.003	0.982	**7.288**	3266.4	25983.4	2497.0	**31746.8**	5.7
		150			5279551.0	390883.0		6.234	0.655	**7.191**		26983.0	1491.6	**31741.1**	-
		200			5377375.1	293058.8		6.349	0.491	**7.143**		27483.0	1052.4	**31801.8**	60.7



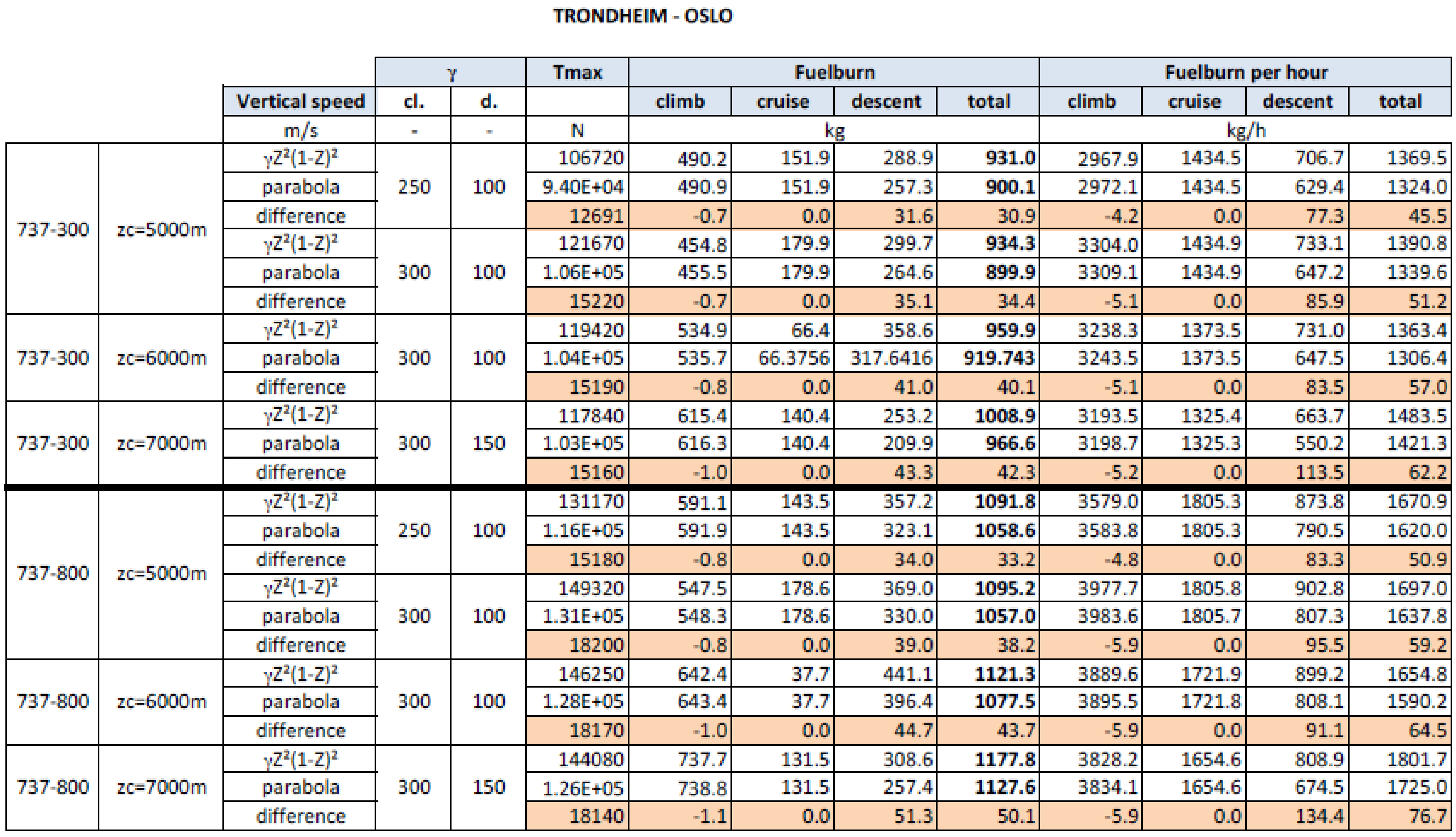
777-200

	γ		Tmax	Horizontal distance			Time				Fuelburn				
	cl.	d.		climb	cruise	descent	climb	cruise	descent	total	climb	cruise	descent	total	diff.
	-	-	N	m			h				kg				
z1(m)	**150**	100	2.71E+05	370002.7	4880943.9	600053.3	0.6	5.633	0.982	**7.220**	4212.2	22850.9	2130.6	**29193.7**	194.7
11000		150			5081057.9	399939.3		5.864	0.655	**7.124**		23787.8	1176.2	**29176.1**	177.1
zc (m)		200			5181143.8	299853.4		5.979	0.491	**7.076**		24256.3	797.1	**29265.6**	266.6
12000	**200**	100	3.20E+05	277408.6	4973538.1	600053.3	0.454	5.740	0.982	**7.176**	3577.6	23332.5	2155.8	**29065.8**	66.8
deltaz (m)		150			5173652.1	399939.3		5.971	0.655	**7.080**		24271.3	1227.0	**29075.9**	76.9
1000		200			5273738.0	299853.4		6.086	0.491	**7.031**		24740.8	844.2	**29162.6**	163.6
	250	100	3.69E+05	221830.6	5029116.0	600053.3	0.363	5.804	0.982	**7.149**	3195.9	23622.5	2194.0	**29012.3**	13.3
		150			5229230.0	399939.3		6.035	0.655	**7.053**		24562.5	1280.4	**29038.8**	39.7
		200			5329315.9	299853.4		6.150	0.491	**7.005**		25032.6	892.6	**29121.1**	122.1
	300	100	4.18E+05	184760.8	5066185.9	600053.3	0.303	5.847	0.982	**7.131**	2940.9	23816.3	2241.8	**28999.0**	-
		150			5266299.8	399939.3		6.078	0.655	**7.035**		24757.1	1336.5	**29034.5**	35.5
		200			5366385.7	299853.4		6.193	0.491	**6.987**		25227.6	942.5	**29110.9**	111.9



A340				777-200			
Fuelburn per hour				Fuelburn per hour			
climb	cruise	descent	total	climb	cruise	descent	total
kg/h				kg/h			
7758.1	4303.1	2410.5	4334.9	6955.2	4056.7	2169.9	4043.2
	4303.2	2008.1	4384.2		4056.7	1796.8	4095.5
	4303.2	1817.8	4423.7		4056.7	1623.6	4136.0
8771.6	4315.8	2442.9	4340.9	7876.7	4065.1	2195.5	4050.5
	4315.7	2094.3	4394.5		4065.1	1874.5	4107.1
	4315.7	1923.7	4433.9		4065.1	1719.5	4147.5
9780.8	4323.3	2487.9	4348.1	8795.3	4070.1	2234.4	4058.2
	4323.4	2184.4	4404.2		4070.1	1956.1	4117.3
	4323.4	2032.1	4443.1		4070.1	1818.2	4157.4
10787.5	4328.4	2543.0	4356.2	9712.3	4073.5	2283.1	4066.4
	4328.4	2278.7	4413.8		4073.5	2041.7	4127.1
	4328.4	2143.5	4452.0		4073.5	1919.6	4166.6

Comparison between the vertical speed profiles



TRONDHEIM - OSLO

			γ		Tmax	Fuelburn				Fuelburn per hour			
		Vertical speed	**cl.**	**d.**		**climb**	**cruise**	**descent**	**total**	**climb**	**cruise**	**descent**	**total**
		m/s	-	-	N	kg				kg/h			
737-300	zc=5000m	$\gamma Z^2(1-Z)^2$	250	100	106720	490.2	151.9	288.9	**931.0**	2967.9	1434.5	706.7	1369.5
		parabola			9.40E+04	490.9	151.9	257.3	**900.1**	2972.1	1434.5	629.4	1324.0
		difference			12691	-0.7	0.0	31.6	30.9	-4.2	0.0	77.3	45.5
		$\gamma Z^2(1-Z)^2$	300	100	121670	454.8	179.9	299.7	**934.3**	3304.0	1434.9	733.1	1390.8
		parabola			1.06E+05	455.5	179.9	264.6	**899.9**	3309.1	1434.9	647.2	1339.6
		difference			15220	-0.7	0.0	35.1	34.4	-5.1	0.0	85.9	51.2
737-300	zc=6000m	$\gamma Z^2(1-Z)^2$	300	100	119420	534.9	66.4	358.6	**959.9**	3238.3	1373.5	731.0	1363.4
		parabola			1.04E+05	535.7	66.3756	317.6416	**919.743**	3243.5	1373.5	647.5	1306.4
		difference			15190	-0.8	0.0	41.0	40.1	-5.1	0.0	83.5	57.0
737-300	zc=7000m	$\gamma Z^2(1-Z)^2$	300	150	117840	615.4	140.4	253.2	**1008.9**	3193.5	1325.4	663.7	1483.5
		parabola			1.03E+05	616.3	140.4	209.9	**966.6**	3198.7	1325.3	550.2	1421.3
		difference			15160	-1.0	0.0	43.3	42.3	-5.2	0.0	113.5	62.2
737-800	zc=5000m	$\gamma Z^2(1-Z)^2$	250	100	131170	591.1	143.5	357.2	**1091.8**	3579.0	1805.3	873.8	1670.9
		parabola			1.16E+05	591.9	143.5	323.1	**1058.6**	3583.8	1805.3	790.5	1620.0
		difference			15180	-0.8	0.0	34.0	33.2	-4.8	0.0	83.3	50.9
		$\gamma Z^2(1-Z)^2$	300	100	149320	547.5	178.6	369.0	**1095.2**	3977.7	1805.8	902.8	1697.0
		parabola			1.31E+05	548.3	178.6	330.0	**1057.0**	3983.6	1805.7	807.3	1637.8
		difference			18200	-0.8	0.0	39.0	38.2	-5.9	0.0	95.5	59.2
737-800	zc=6000m	$\gamma Z^2(1-Z)^2$	300	100	146250	642.4	37.7	441.1	**1121.3**	3889.6	1721.9	899.2	1654.8
		parabola			1.28E+05	643.4	37.7	396.4	**1077.5**	3895.5	1721.8	808.1	1590.2
		difference			18170	-1.0	0.0	44.7	43.7	-5.9	0.0	91.1	64.5
737-800	zc=7000m	$\gamma Z^2(1-Z)^2$	300	150	144080	737.7	131.5	308.6	**1177.8**	3828.2	1654.6	808.9	1801.7
		parabola			1.26E+05	738.8	131.5	257.4	**1127.6**	3834.1	1654.6	674.5	1725.0
		difference			18140	-1.1	0.0	51.3	50.1	-5.9	0.0	134.4	76.7

TRONDHEIM - NICE

			γ		Tmax	Fuelburn				Fuelburn per hour			
		Vertical speed	cl.	d.		climb	cruise	descent	total	climb	cruise	descent	total
		m/s	-	-	N	kg				kg/h			
A320	zc=10058m	γZ²(1-Z)²	300	100	1.37E+05	901.5	2644.2	630.4	**4176.1**	3256.0	1348.4	766.6	1364.6
		parabola			1.19E+05	903.0	2644.1	577.5	**4124.6**	3261.3	1348.3	702.3	1347.8
		difference			1.76E+04	-1.5	0.1	52.9	51.5	-5.3	0.0	64.3	16.8
		γZ²(1-Z)²	300	150	1.37E+05	901.5	2904.9	377.9	**4184.3**	3256.0	1348.4	689.3	1404.4
		parabola			1.19E+05	903.0	2904.9	314.9	**4122.8**	3261.3	1348.3	574.4	1383.7
		difference			1.76E+04	-1.5	0.1	63.0	61.6	-5.3	0.0	114.9	20.7
737-300	zc=10058m	γZ²(1-Z)²	300	100	1.40E+05	1034.5	3031.4	720.0	**4785.9**	3736.4	1545.8	875.6	1563.9
		parabola			1.22E+05	1036.2	3031.3	658.8	**4726.3**	3742.5	1545.8	801.1	1544.4
		difference			1.81E+04	-1.7	0.1	61.2	59.6	-6.0	0.0	74.4	19.5

PARIS - NEW YORK

			γ		Tmax	Fuelburn				Fuelburn per hour			
		Vertical speed	cl.	d.		climb	cruise	descent	total	climb	cruise	descent	total
		m/s	-	-	N	kg				kg/h			
777-200	zc=11000m	γZ²(1-Z)²	300	100	4.18E+05	2940.9	23816.3	2241.8	**28999.0**	9712.3	4073.5	2283.1	4066.4
		parabola			3.65E+05	2945.4	23815.9	2127.0	**28888.4**	9727.2	4073.5	2166.2	4050.9
		difference			5.25E+04	-4.5	0.4	114.7	110.6	-14.9	0.1	116.8	15.5
		γZ²(1-Z)²	300	150	4.18E+05	2940.9	24757.1	1336.5	**29034.5**	9712.3	4073.5	2041.7	4127.1
		parabola			3.65E+05	2945.4	24756.7	1125.3	**28827.4**	9727.2	4073.4	1719.1	4097.7
		difference			5.25E+04	-4.5	0.4	211.1	207.1	-14.9	0.1	322.6	29.4
A340	zc=11000m	γZ²(1-Z)²	300	100	4.76E+05	3266.4	26983.0	1491.6	**31741.1**	10787.5	4328.4	2278.7	4413.8
		parabola			4.16E+05	3271.6	26982.3	1261.1	**31515.0**	10804.5	4328.3	1926.5	4382.4
		difference			6.00E+04	-5.2	0.7	230.6	226.1	-17.1	0.1	352.2	31.4