

Porous materials

Permeability

Background. Following the original work of Henry Darcy, mathematical descriptions of liquid flow in porous media are based on Darcy's law. This states that the volumetric flow rate Q of liquid through a specimen of porous material is proportional to the hydrostatic pressure difference Δp across the specimen, inversely proportional to the length L of the specimen, and proportional to the cross sectional area A . Darcy's law is expressed simply as $Q = kA\Delta p/L$. We call the constant of proportionality defined by Darcy's law the Darcy permeability of the material. The quantity Q/A has dimensions LT^{-1} and is the flow rate, flux or Darcy velocity u , so that Darcy's law is more commonly written as $u = k\Delta p/L$. It is clear that Darcy's law is a simple linear transport law and has exactly the same form as Ohm's law, Fick's law and the heat conduction equation. Each of these laws defines a transport property, a *conductivity*, that relates to the flow process that each describes, and we might envisage a unifying theoretical framework which could have application to heat, diffusion and electrical current and to water flow. In fact valuable insights into water flow processes may be gained by comparing these with heat flow problems.

Definition of permeability. If AB in Fig 1 is fully saturated with liquid, then imposing a difference in hydrostatic pressure $\Delta p = p_A - p_B$ between A and B leads to a steady Darcian flow $u = k\Delta p/L$. u is the (scalar) flow rate (dimensions LT^{-1}). Darcy's law may be expressed locally as $\mathbf{u} = -k\nabla p$, with \mathbf{u} the vector flow velocity. We do not assume that the Darcy permeability k is necessarily constant in time and indeed for water in concrete there is evidence that this is not always the case.

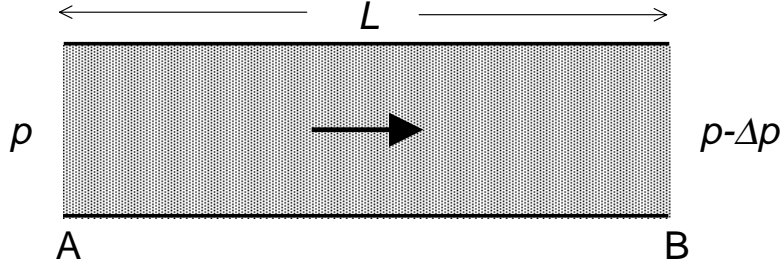


Figure 1: Simple Darcy flow through a liquid-saturated homogeneous medium under the action of a pressure gradient.

The permeability k as defined in this equation has dimensions $M^{-1}L^3T$. However it is common to express hydrostatic pressures in terms of the pressure potential $P = p/\rho g$, where ρ is the liquid density. P has dimension L and is entirely equivalent to the hydrostatic head. We then express Darcy's law as

$$\mathbf{u} = -K_s \nabla P \quad (1)$$

where $K_s = k\rho g$. The quantity K_s is the conventional saturated permeability of the material, with dimensions LT^{-1} .

Intrinsic permeability. We note that the permeability so defined depends both on the material and on the fluid. For permeation flows which are geometrically similar (in practice, newtonian liquids in laminar flow in inert non-swelling media) the permeability k (and K_s also) varies inversely as the fluid viscosity η . We can therefore define an intrinsic permeability $k' = k\eta$ such that $\mathbf{u} = -(k'/\eta)\nabla p$, in which k' is a material property independent of the fluid used to measure it. k' has the dimensions L^2 . Both definitions of permeability are used in the materials literature, although K_s more widely than k' . The conversion between the two depends not only on the viscosity but also on the density of the fluid at the temperature of measurement since $k' = K_s\eta/\rho g$. For water at 25 deg C, $k'/K_s = 9.103 \times 10^{-8}$ m s.

It follows from the definition of K_s that a series of measurements of K_s on a single material using different fluids should scale as ρ/η if the material is truly inert to the fluids. Failures of this scaling are an indication of specific interactions between the material and the test fluid.

Temperature dependence of the permeability. We note that the variation of K_s (and similarly k) with temperature is controlled mainly by the change of viscosity, so that $dK_s/dT = -K_s(d\ln\eta/dT)$. For most liquids, the temperature coefficient of permeability is in the range $+0.01$ to $+0.03 K_s/\text{deg C}$.

Another permeability unit. A further unit of permeability, the darcy, denoted D, is widely used in petroleum engineering and petrophysics. The darcy is a non-SI metric unit of intrinsic permeability k' , equal to $\text{cm}^2 \text{ cP}/(\text{atm s})$ where cP denotes the centipoise. Since the unit group $\text{cP}/(\text{atm s})$ is dimensionless and has the value 9.8692×10^{-9} , it follows that $\text{D} = 9.8692 \times 10^{-13} \text{ m}^2$. It may be helpful to remember that $1 \text{ D} \approx 1 \mu\text{m}^2$.

Values of the permeability. Some indicative values of permeability for construction materials are given in Table 1. In this small group of unexceptional materials, the lowest and highest permeability differ by a factor of 10^6 . This reflects the underlying dependence of the intrinsic permeability k' on l_c^2 where l_c is some characteristic length scale of the pore system. Of course these materials are not geometrically similar, but nonetheless the sensitivity of the permeability to the fine-grainedness of the porosity is apparent.

There are some examples of the permeability of cement-based materials in Table 1. The wide range of these values is notable, extending over at least four orders of magnitude. This of course is hugely greater than the range of porosity and reflects the connectivity and the length scale of the percolating porosity. In dense well-compacted cement-based materials, the pore space is extremely fine-grained. This is the case with hardened cement paste and high performance concretes made at low water-cement ratios. The wide range of permeabilities depending on free water content, water/cement ratios, curing and drying conditions is clear from recent research.

Reference. From C Hall and W D Hoff: *Transport in brick, stone and concrete*, 2002 [in press].

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Material	Fluid	Permeability K_s/ms^{-1}	Intrinsic permeability k'/m^2	Note
Clashach sandstone	Gas	–	3.0×10^{-13}	<i>a</i>
Lépine limestone	Water	2.5×10^{-9}	2.3×10^{-16}	<i>b</i>
St Maximin fine limestone	Water	2.5×10^{-6}	2.3×10^{-13}	<i>c</i>
Clay brick ceramic	Water	3.2×10^{-8}	2.9×10^{-15}	<i>d</i>
Clay brick ceramic	Water	3.8×10^{-9}	3.4×10^{-16}	<i>e</i>
Autoclaved aerated concrete	Gas	–	2.8×10^{-14}	<i>f</i>
Hardened cement paste w/c 0.5	Water	3.8×10^{-13}	3.5×10^{-20}	<i>g</i>
Hardened cement paste w/c 0.8	Water	4.6×10^{-11}	4.2×10^{-18}	<i>g</i>
Cement-sand mortars	Water	4.6×10^{-11}	4.2×10^{-18}	<i>h</i>
High strength concrete, moist cured	Water	7.7×10^{-13}	7.0×10^{-20}	<i>i</i>
High strength concrete, air cured	Water	1.9×10^{-11}	1.7×10^{-18}	<i>i</i>

a Clashach sandstone, porosity f 0.145, nitrogen gas permeability. *b* Lépine limestone, porosity f 0.245. *c* St Maximin fine limestone, porosity f 0.23. *d* Clay brick ceramic, f 0.40. *e* Clay brick ceramic, $f=0.31$. *f* Autoclaved aerated concrete, bulk density 390 kg/m^3 , hydrogen gas permeability. *g* Fully hydrated cement paste. *h* 1:3 cement:sand mortar. *i* Portland cement concrete w/c 0.45.

Table 1: Selected permeability data on construction and similar materials by several methods.