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Geoid determination over Norway using global Earth gravity Models

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The only global gravity model that was close to the accuracy of the national geoid model is EGM2008 which has an accuracy of 6.9 centimeters while the national geoid models, NMA2013v22 has an accuracy of 5.6 centimeters. The accuracy increases with the degree and order of the model. It was also found that the indirect geoid determination had a small negative effect on the results.

Keywords:

1. Geoid
2. GPS/leveling
3. Global gravity models
4. Gravity

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Page 1 of 4 pages

MASTER DEGREE THESIS
Spring 2013
for

Student: Aina Borge

Geoid determination over Norway using global Earth gravity models

BACKGROUND

Geoid is one of the Earth's shapes. Geoid surface is used to approximate the physical shape of the Earth. Geoid is the equipotential surface of the Earth's gravity field which more or less coincides with mean sea level. Civil engineers use it as the reference surface for elevations while oceanographers use it for studies of ocean circulation, currents and tides. It is also valuable to geophysicists for displacement studies, geophysical interpretation of the Earth's crust, and prospecting. The geoid is not a simple mathematical surface (although it can be modelled), but deviates by up to ±100m from an ellipsoid (mathematical shape of the Earth), largely due to variations in gravity around the globe. Over the last decade, there has been an increased interest in the determination of the geoid. This is mainly due to the demands for height transformation from users of GPS (Global Positioning System) (This transformation is called GPS-levelling). GPS-computed heights cannot be used in civil projects and it should be transformed to the heights referred to the geoid. GPS-levelling replaces costly conventional levelling operations with quicker and cheaper GPS surveys, as long as the geoidal height has been computed to a high accuracy. (Levelling operations is the classical method to measure heights referred to geoid.)

An accurate solution of the geoid in physical geodesy has usually been found using Stokes's well-known formula (integral). Stokes's formula stipulates the relation between the geoidal height (or gravitational potential) at a single point on the geoid and the gravity anomalies on the entire geoid. It can be seen that Stokes's formula is a rigorous formula for computing the geoidal height from globally and continuously distributed gravity anomalies. At present, a homogenous coverage of high resolution gravity all over the Earth is hard to come by and, at the same time, the gravity data are available at discrete points. This promotes the restriction of the integration area of Stokes's formula (integral), where a homogenous and relatively high resolution gravity anomaly data set can be found at our disposal in conjunction to the Earth gravity models. However, the computation of geoid from Earth gravity models alone (disregarding the Stokes's integral) has been an issue of increasing importance in the geodetic community, as the accuracy of the global gravity models increases with newer satellite missions and the maximum degree of expansion goes to higher degrees.

In determining the geoid from Earth gravity models, one must expect a bias from the external harmonic series when applied at the geoid within the topographic masses. The topographic corrections are needed to handle this bias in geoid computations.

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TASK DESCRIPTION

The purpose of this thesis is to demonstrate the efficiency of geoid determination from Earth gravity models. The Student should demonstrate whether newer satellite gravity missions provide models with sufficient accuracy for geoid computations. GNSS-levelling data sets and available regional geoid models could be used to demonstrate the accuracy and efficiency of the estimated geoid models.

The thesis should contain a description of the theory, satellite gravity models used in geoid computations, and an explanation of the accuracy targets one might expect from the global models. As a result of the thesis, the student should comment and recommend the best available gravity model in the study area. To achieve these objectives, the student likely requires developing knowledge about geoid computation such as:

- Classification of different global Earth gravity models
- Geoid determination from geopotential coefficients- different scenarios should be considered
- Topographic corrections computation
- Comparison of the estimated models with GNSS-levelling data
- Comparison of the estimated models with available regional geoid models
- Recommendation on the Earth gravity models and their accuracy

General about content, work and presentation

The text for the master thesis is meant as a framework for the work of the candidate. Adjustments might be done as the work progresses. Tentative changes must be done in cooperation and agreement with the professor in charge at the Department.

In the evaluation thoroughness in the work will be emphasized, as will be documentation of independence in assessments and conclusions. Furthermore the presentation (report) should be well organized and edited; providing clear, precise and orderly descriptions without being unnecessary voluminous.

The report shall include:

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- Preface
- Summary and acknowledgement. The summary shall include the objectives of the work, explain how the work has been conducted, present the main results achieved and give the main conclusions of the work.
- Table of content including list of figures, tables, enclosures and appendices.
- If useful and applicable a list explaining important terms and abbreviations should be included.
- The main text.
- Clear and complete references to material used, both in text and figures/tables. This also applies for personal and/or oral communication and information.
- Text of the Thesis (these pages) signed by professor in charge as Attachment 1..
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Advice and guidelines for writing of the report is given in: "Writing Reports" by Øivind Arntsen. Additional information on report writing is found in "Råd og retningslinjer for rapportskriving ved

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Submission procedure

Procedures relating to the submission of the thesis are described in DAIM (http://daim.idi.ntnu.no/). Printing of the thesis is ordered through DAIM directly to Skipnes Printing delivering the printed paper to the department office 2-4 days later. The department will pay for 3 copies, of which the institute retains two copies. Additional copies must be paid for by the candidate / external partner.

On submission of the thesis the candidate shall submit a CD with the paper in digital form in pdf and Word version, the underlying material (such as data collection) in digital form (eg. Excel). Students must submit the submission form (from DAIM) where both the Ark-Bibl in SBI and Public Services (Building Safety) of SB II has signed the form. The submission form including the appropriate signatures must be signed by the department office before the form is delivered Faculty Office.

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Start and submission deadlines
The work on the Master Thesis starts on January 14, 2013

The thesis report as described above shall be submitted digitally in DAIM at the latest at
3pm June 10, 2013

Professor in charge: Hossein Nahavandchi

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Trondheim, January 11, 2013. (revised:)

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Professor in charge (sign)

Abstract

Due to increasing accuracy in measurements of the earth's gravity potential from satellite missions the goeid can be computed with higher resolution and accuracy than earlier. In the thesis the accuracy of geoid heights derived from global gravity models are investigated. GPS/leveling data are used to demonstrate the accuracy of global gravity models. In addition, national geoid models of Norway are compared at the same GPS/leveling stations. Geoidal heights from different global gravity models are computed over Norway. The geoid is computed in two different ways. In the first method, geoidal heights are computed directly from geopotential coefficients and then corrected for topographical masses. Second method focuses on geoidal height computation through height anomaly. Proper corrections are added. Accuracy of the models was investigated by comparing the geoidal heights from the models with geoidal heights found from 2329 GPS/leveling points distributed over whole Norway. It was also checked if the error was dependent of elevation.

The only global gravity model that was close to the accuracy of the national geoid model is EGM2008 which has an accuracy of 6.9 centimeters while the national geoid models, NMA2013v22 has an accuracy of 5.6 centimeters. The accuracy increases with the degree and order of the model. It was also found that the indirect geoid determination had a small negative effect on the results.

Sammendrag

På grunn av økende nøyaktighet i satellit prosjekters målinger av jordens gravitasjonspotensial kan geoiden bli beregnet med høyere oppløsning og nøyaktighet enn tidligere. I denne oppgaven undersøkes nøyaktigheten til geoide høyder bregnet fra globale geoide modeller. Data fra GPS/leveling ble brukt for å undersøke nøyaktigheten til de globale gravitasjons modellene. To nasjonale geoide modeller over Norge ble også samenlignet med GPS/levelig stasjonene. Geoidehøydene ble funnet fra de ulike modellene, to ulike beregningsmetoder ble brukt. Den første metoden beregner geoidehøyden direkte fra geopotensial koeffisienter og korrigerer deretter for topografiske masser. Den andre medtoden beregner groidehøydne indirekte ved først å beregne høyde anomalien i punktet for deretter å legge til konvensjonstermer for å få geoide høyden. Nøyaktigheten til modellene ble undersøkt ved å sammenligne den beregnede geoide høyden med geoidehøyden funnet i de 2329 GPS/leveling punktene over Norge. Det ble også undersøkt om det var en sammenheng mellom høyde og feil i enkelte av modellene.

Den eneste globale geoide modellen som var i nærehheten av nøyaktigheten til de nasjonale geoide modellene var EGM2008 med en nøyaktighet på 6.9 centimeter mens den nasjonale geoide modellen har en nøyaktighet på 5.6 centimeter. Nøyaktigheten til de globale geoide modellene øker med høyere grad og orden av modellen. Den indirekte metoden ga dårligere resultat på GRACE modellene enn den direkte.

Contents

List of Figures

List of Tables

Glossaries

DTM2006.0	A digital topographic model
DTU10	A global ocean tide model
EIGEN06C	Global geoid based on satellite data from GOCE, GRACE and LAGEOS
ellipsoidal height	The height of the earth's surface above the reference ellipsoid
geoidal undulation	The distance between the geoid and ellipsoid, also called geoidal height
ITG-GRACE03S	A gravity model from satellites only, based on Satellite-to-Satellite tracking data from GRACE
NMA2012v30	National geoid over Norway, spanning from 5°W to 35°E and 55°N to 75°N
NMA2013v22	National geoid over Norway, spanning from 35°W to 40.16°E and 53°N to 84°N
orthometric height	The height above the geoid
plumb line	A line that orthogonally intersecting with the equipotential surfaces
telluroid	An imaginary surface replacing the geoid, determined from normal height and height anomaly(Hirvonen 1961)
vertical datum	The coordinate surface which is referred to by heights(Vaníček 2009)

Acronyms

CHAMP	Challenging Minisatellite Payload (Hofmann-Wellenhof & Moritz 2006)
CRS	Celestial Reference System
CTRS	Conventional Terrestrial Reference System
DTM	Digital Topographic Model
EGM2008	Earth Gravitational Model of 2008
EGM96	Earth Gravitational Model of 1996
FFT	Fast Fourier Transform
GGM	Global Gravity Model
GNSS	Global Navigation Satellite System
GOCE	Gravity field and steady-state Ocean Circulation Explorer(Hofmann-Wellenhof & Moritz 2006)
GPS	Global Positioning System
GRACE	Gravity Recovery and Climate Experiment(Hofmann-Wellenhof & Moritz 2006)
GRS80	Geodetic Reference System 1980
LSC	Least-Squares Collocation
NIMA	National Imagery and Mapping Agency
NKG	Nordic Geodetic Commission
NMA	Norwegian Mapping Authority
RTM	Residual Terrain Method
SST	Satellite-to-Satellite Tracking
SST-ll	Satellite-to-Satellite Tracking in low-low mode

TEG-4   Texas Earth Gravity model 4

WGS 84   World Geodetic System of 1984

Symbols

F gravitational force
G the universal gravitational constant
H^* normal height
H orthometric height
N geoid undulation or geoidal height
T anomalous potential
U normal gravity potential
V gravitational potential
W_0 gravity potential at the geoid
W gravity potential
Δg gravity anomaly
γ_0 mean gravity, normal gravity at latitude 45°
γ normal gravity at the ellipsoid
ω angular velocity
ϱ mass density
ξ height anomaly
f centrifugal force
h ellipsoidal height

1 Introduction

The geoid approximates the earth's surface and are used as reference surface for height determinations. If the geoid model has good enough accuracy it can be used to give information about the earth's composition. Accurate orthometric heights is important in construction work and planning. It can also be used to investigate movement of oceans and other masses. This thesis investigate if existing geoid models are good enough to be used to height determination. If the Global Geoid Models(GGM) has high enough accuracy, GNSS measurements can be used for orthometric height determination. The goal for accuracy is in the area of centimeters(Lysaker et al. (2007), Sjöberg (2003)).

In Norway the Norwegian Mapping Authority(NMA) computes national geoids for Norway. If the geoids are good enough construction companies and other in need of accurate orthometric heights can use the global geoids. To investigate this the accuracy of orthometric heights computed from global geoids are compared with the accuracy from the national geoids. The accuracy is investigated by comparing the computed geoidal heights from global gravity models(GGMs) with heights obtained from GPS/leveling. Six different models are checked, two earth gravitational models and four models based on gravity data from the Gravity Recovery and Climate Experiment(GRACE).

The geoid is often computed with geopotential coefficients ans Stokes's formula. Stokes's formula requires no masses outside the geoid, so these are removed for the computation and restored after the computation. In this comparison the geoid is determined from Stokes formula and indirect geoid determination through height anomaly. For the GRACE gravity models a correction of second power of H was added to restore the masses outside the geoid.

The report starts with a theoretical part in Chapter 2, Gravity and geoid, where there is some definitions, a presentation of mathematical models and how the geoid can be obtained from measurements. Chapter 3 presents the models and data that are used and the results of the computations. In Chapter 4 the the results are discussed and there is a short conclusion and ideas for future work.

2 Gravity and Geoid

In the beginning of this chapter gravity, geoid, and other terms are defined. Then it is shown how to compute the gravity field of the earth and find geoidal undulations. At the end there is a presentation of how the geoid can be found by measurement.

2.1 Basic definitions

Gravity

The earth's gravity field is generated from a gravitational force and a centrifugal force. The centrifugal force is generated from the earth's rotation and are dependent on the angular velocity and radial distance to the spin axis. Gravitational force is due to the attraction between masses, Newton's law of gravitation.

$$F = G\frac{m_1 m_2}{l^2} \tag{2.1}$$

$$G = 6.6742 \cdot 10^{-11} m^3 kg^{-3} s^{-2} \tag{2.2}$$

Where F is the gravitational force, G is the gravitational constant m_1 and m_2 are the masses of two points with the distance l.(Hofmann-Wellenhof & Moritz 2006)

Geoid

The geoid is one of the Earth's equipotential surfaces and a vertical datum. This means that the gravity potential is constant on the geoid. The value for the gravity potential is chosen so that the geoid coincides with mean sea level(Hofmann-Wellenhof & Moritz (2006), Nahavandchi & Mahdavipour (2012)). Because of different tide effects on the geoid it is defined three types of geoids, 'The Development of the Joint NASA GSFC and the National Imagery and Mapping Agency(NIMA) Geopotential Model EGM 96' they are defined as follows:

1. **Tide-free (or nontidal):** This geoid would exist for a tide-free Earth with all (direct and indirect) effects of the Sun and Moon removed.

2. **Mean:** This geoid would exist in the presence of the Sun and the Moon (or, equivalently, if no permanent tidal effects are removed).

3. **Zero:** This geoid would exist if the permanent direct effects of the Sun and Moon are removed, but the indirect effect component related to the elastic deformation of the Earth is retained.

Equation (2.3) shows the classical equation for the geoid, where W(x,y,z) is the gravity potential as a function of the coordinates and W_0 is the potential of the geoid.

$$W(x, y, z) = W_0 = constant \tag{2.3}$$

The geoid can be measured gravimetric or geometric and it can be made geoid models from both types. These models will never coincide because of approximation in the gravimetric geoid and systematic errors in geometric geoid determination.(Lysaker et al. 2007)

Two methods are often used to determine the geoid from gravimetric measurements, Least-Squares Collocation(LSC) and the solution of stokes formula. The LSC can be used on different data types while Stokes integration with FFT use data in a grid. Remove-compute-restore can be used on both techniques.(Lysaker et al. 2007)

CHAMP, GRACE and GOCE are the first generation of satellite missions who mainly focus on measuring the gravity field.(Flury & Rummel 2005)

Orthometric height

Orthometric heights is used in most of the world. It is measured along the plumb line from the geoid to the point, as shown in Figure 2.1.(Hofmann-Wellenhof & Moritz 2006) If the orthometric height is precise defined it will not give any problems in orthometric height determinations even though different datum's are used, for example along a border.(Vaníček 2009)
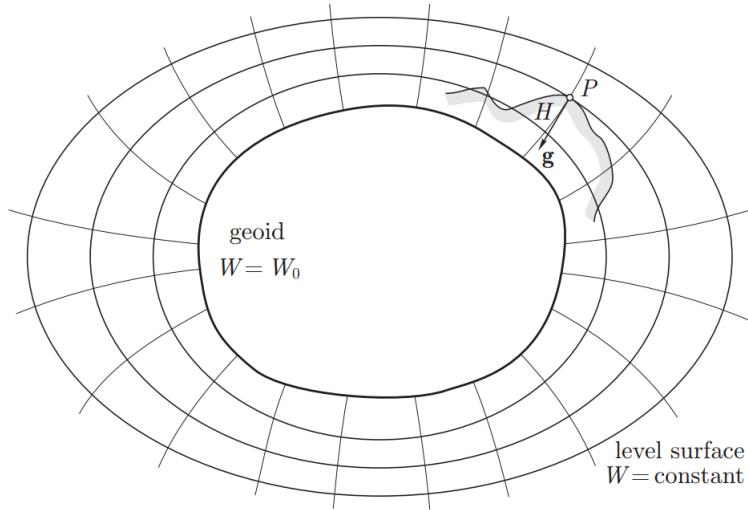
Figure 2.1: The orthometric height, from Hofmann-Wellenhof & Moritz (2006) page 47

Normal height

Normal heights is similar to orthometric height. The difference is that normal heights use the distance between the telluroid and the reference ellipsoid instead of the distance between the geoid and point of interest. The normal height is measured from the point Q at the plumb line where the normal gravity potential is the same as the gravity potential at the point P on the surface, see Figure 2.2 where H^* is the normal height, h is the ellipsoidal height and ξ is the height anomaly.
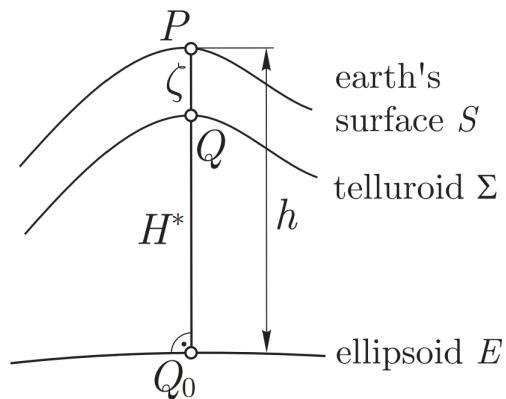

Figure 2.2: The normal height, from Hofmann-Wellenhof & Moritz (2006) page 297

Ellipsoidal height

Ellipsoidal height is the elevation above the ellipsoid and it is measured by GNSS. The satellite needs a reference for the measurements and use a reference ellipsoid to give coordinates and heights. A reference ellipsoid is used as reference for satellites so that they can give coordinates and heights in this system. The ellipsoid is used because it is a good approximation to the earth's shape and also a mathematical surface.
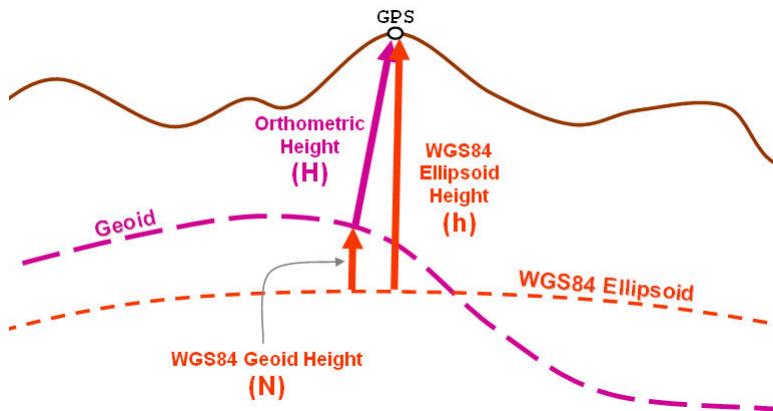

Figure 2.3: The orthometric height, geoid height and ellipsoidal height in WGS84, from `http://www.geod.nrcan.gc.ca/faq_e.php`

Geodetic Reference System 1980

Geodetic Reference System 1980(GRS80) is based on the theory of a geocentric equipotential ellipsoid and the parameters of it are defined exact. GRS80 is the basis for WGS 84.(Hofmann-Wellenhof & Moritz 2006)

World Geodetic System 1984

World Geodetic System 1984(WGS 84) is based on GRS80. The WGS 84 is defined as a Conventional Terrestrial Reference System(CTRS) by the National Imagery and Mapping Agency(NIMA), which means that it is geocentric, and the scale follows the relativistic theory of gravitation.The coordinate system is orthogonal, right-handed and earth-fixed. WGS 84 has its origin in the earth's center off mass and the Z-axis is the mean rotational axis of the earth. The X-axis is associated with the Greenwich meridian and the Y-axis completes the orthogonal right handed coordinate system.

The center of the reference ellipsoid of WGS 84 coincides with the center of the coordinate system of WGS 84. The GRACE gravity models in this paper uses the zero tide geoid(Tapley et al. (2007),Tapley et al. (2005)). For the EGMs a tide-free model is computed, and it uses WGS 84 as ellipsoid(Pavlis et al. (2008), Lemoine et al. (1998)).

2.2 Geoid determinations

The book "Physical Geodesy" by Hofmann-Wellenhof & Moritz (2006) is used to derive the mathematical formulas unless other is specified. To get a mathematical formula several modifications and assumptions are done.

2.2.1 Stokes theory

To compute the gravity potential of a solid body Newton's integral is used, where V is the total gravitational force for the solid body, v is the volume, dm is the mass element l is the distance between the mass element and the attracted point, ϱ is the density of the mass and dv is an volume element.

$$V = G \iiint\limits_{v} \frac{dm}{l} = G \iiint\limits_{v} \frac{\varrho}{l} dv \tag{2.4}$$

The earth's mass is not evenly distributed and the shape is uneven, so the computation is not as simple as in Equation (2.4). This is further explained in Hofmann-Wellenhof & Moritz (2006) and results in Stokes's formula. Some of the essential parts of the derivation is shown here.

The centrifugal force f at a given point at the surface of a rotating body is given by Equation (2.5) where ω is the angular velocity and p is the distance from the point to the rotation axis.

$$f = \omega^2 p \tag{2.5}$$

Derived from a potential the centrifugal force can be written as in Equation (2.6).

$$\Phi = \frac{1}{2}\omega^2(x^2 + y^2) \tag{2.6}$$

The potential of gravity can then be written as in Equation (2.7)

$$W = W(x, y, z) = G \iiint\limits_{v} \frac{\varrho}{l} dv + \frac{1}{2}\omega^2(x^2 + y^2) \tag{2.7}$$

By differentiating the potential of gravity force and centrifugal force the generalized Poisson equation is obtained:

$$\Delta W = -4\pi G \varrho + 2\omega^2 \tag{2.8}$$

7

The gravitational potential is the part of the potential gravity equation which is difficult to compute. It can be computed by looking at it outside the earth where it is harmonic. In Hofmann-Wellenhof & Moritz (2006), chapter 1 and 2, they derive the expression in Equation (2.9) to spherical harmonics and get Equation (2.10) for the Earth's gravitational potential.

$$V = G \iiint\limits_{earth} \frac{dM}{l} \tag{2.9}$$

$$V = \frac{GM}{r} \left[1 + \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^n [C_{nm}\mathcal{R}(\vartheta, \lambda) + S_{nm}\mathcal{S}(\vartheta, \lambda)] \right] \tag{2.10}$$

where

ϱ = polar distance
r = radius from the center of mass to point P
λ = geocentric longitude
a = radius of the earth at equator
C_{nm} = geopotential coefficients
S_{nm} = geopotential coefficients
$\mathcal{R}(\vartheta, \lambda) = P_{nm}(cos\ \vartheta)cos\ m\lambda$
$\mathcal{S}(\vartheta, \lambda) = P_{nm}(cos\ \vartheta)sin\ m\lambda$

The $P_{nm}(cos\vartheta)$, which is the Legendre functions, are explained in the next section. This is one of the versions of the formula, the fully normalized can be obtained by replacing the Legendre functions with fully normalized Legendre functions and C_{nm} and S_{nm} with $\overline{C}_{nm}$ and $\overline{S}_{nm}$, where the overline implies that the coefficients are fully normalized, are given by:

for $m = 0$:

$$\overline{C}_{n0} = \frac{1}{\sqrt{2n+1}}C_{n0} \tag{2.11}$$

for $m \neq 0$:

$$\overline{C}_{nm} = \sqrt{\frac{(n+m)!}{2(2n+1)(n-m)!}}C_{nm} \tag{2.12}$$

$$\overline{S}_{nm} = \sqrt{\frac{(n+m)!}{2(2n+1)(n-m)!}}S_{nm} \tag{2.13}$$

Anomalous gravity field

The anomalous potential(T) can be determined from the difference between the actual gravity potential and the normal gravity potential(U), shown in Figure 2.4.

$$W(x, y, z) = U(x, y, z) + T(x, y, z) \tag{2.14}$$
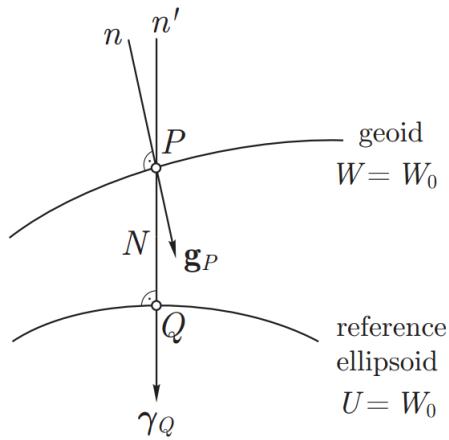
Figure 2.4: Relation between the geoid and reference ellipsoid, from Hofmann-Wellenhof & Moritz (2006) page 91

Where W is the potential of the geoid and U is a reference ellipsoid with the same potential W_0. The gravity anomaly Δg can be computed from the difference in the gravity in point P and the normal gravity γ at point Q as shown in Figure 2.4.

$$\Delta g = g_P - \gamma_Q \tag{2.15}$$

The gravity disturbance can be found by taking the difference of the gravity and the normal gravity in the point P

$$\delta g = g_P - \gamma_P \tag{2.16}$$

Hofmann-Wellenhof & Moritz (2006) derive *Bruns formula* from the relation between gravity potentials, normal gravity potentials, anomalous potential, geoidal undulation and normal gravity:

$$N = \frac{T}{\gamma} \tag{2.17}$$

By taking the gradient of the gravity disturbance and Bruns' formula they get the fundamental equation of physical geodesy:

$$\frac{\delta T}{\delta h} - \frac{1}{\gamma}\frac{\delta\gamma}{\delta h}T + \Delta g = 0 \tag{2.18}$$

Stokes's formula

By replacing γ by γ_0 Bruns' formula can be written as:

$$\delta g = -\frac{\delta T}{\delta r} - \frac{2}{r}T \tag{2.19}$$

where r is the distance to the point

which follows the boundary conditions when the gravity anomalies are known only at the earth's surface. When Equation (2.19) is multiplied by $-r^2$ and integrated Equation (2.20) is obtained.

$$r^2 T\big|_\infty^r = -\int_\infty^r r^2 \Delta g(r) dr \tag{2.20}$$

where $\Delta g(r)$ is a function of r. Expanding T to a summation from 2 to ∞ gives:

$$T = \sum_{n=2}^\infty \frac{R^{n+1}}{r} T_n = \frac{R^3}{r^3}T2 + \frac{R^4}{r^4}T3 + \dots \tag{2.21}$$

Where R is the mean radius of the earth.

By integration and substitution described in Hofmann-Wellenhof & Moritz (2006) Pizzetti's formula is obtained:

$$T(r, \varrho, \lambda) = \frac{R}{4\pi}\iint_\sigma S(r, \psi)\Delta g d\sigma \tag{2.22}$$

where

$$S(r, \psi) = \frac{2R}{l} + \frac{R}{r} - 3\frac{Rl}{r^2} - \frac{R^2}{r^2}cos\psi\left(5 + 3ln\frac{r - Rcos\psi + l}{2r}\right) \tag{2.23}$$

by computing on the geoid, where $r = R$ Stokes' formula is obtained:

10

$$N = \frac{R}{4\pi\gamma_0} \iint\limits_{\sigma} \Delta g S(\psi) d\sigma \qquad (2.24)$$

where

$$S(\psi) = \frac{1}{sin(\psi/2)} - 6sin\frac{\psi}{2} + 1 - 5cos\psi - 3cos\psi ln\left(sin\frac{\psi}{2} + sin^2\frac{\psi}{2}\right) \qquad (2.25)$$

where r is replaced by R and l by $2Rsin\frac{\psi}{2}$

Equation (2.24) is Stokes' formula which can be used to determine the geoid from gravity data. $S(\psi)$ is called Stokes' function.

Legendre functions

$P_{nm}(cos\,\vartheta)$ is a solution of Legendre's differential equation and are used to compute the earth's gravity potential. The derivation of the solution of Legendre's differential equation is described in 'Physical geodesy', Section 1.7 Legendre's functions. After substituting $cos\,\vartheta$ with t Equation (2.26) is obtained.

$$P_{nm}(t) = \frac{1}{2^n n!}(1-t^2)^{m/2}\frac{d^{n+m}}{dt^{n+m}}(t^2-1)^n \qquad (2.26)$$

The special case where m is zero, is called Legendre's polynomials and it is expressed by:

$$P_{n0}(t) = P_n(t) = \frac{1}{2^n n!}\frac{d^n}{dt^n}(t^2-1)^n \qquad (2.27)$$

The polynomials can be computed by this recursive formula:

$$P_n(t) = -\frac{n-1}{n}P_{n-2}(t) + \frac{2n-1}{n}tP_{n-1}(t) \qquad (2.28)$$

To compute the polynomials the first two has to be known, and they are:

$P_0 = 1$
$P_1 = t$

Equation (2.29) is used for the associated function.

$$P_{nm}(t) = 2^{-n}(1-t^2)^{m/2}\sum_{k=0}^{r}(-1)^k\frac{(2n-2k)!}{k!(n-k)!(n-m-2k)!}t^{n-m-2k} \qquad (2.29)$$

where r is the highest integer number smaller or equal to:

$$\frac{n-m}{2} \tag{2.30}$$

Spherical harmonics are accomplished when the Legendre functions are multiplied by $cos(m\lambda)$ and $sin(m\lambda)$. When the Legendre polynomials($m = 0$) are visualized we obtain zonal harmonics with n points of zero. If n and m are equal sectorial harmonics are obtained. Tesseral harmonics is the result in the rest of the cases($m \neq 0$ and $m \neq n$) . All these cases are shown in Figure 2.5, where (a) is zonal harmonics with $n = 6$, (b) is tesseral harmonics with $n = 12$ and $m = 6$ and (c) is sectorial harmonics with $n = m = 6$.
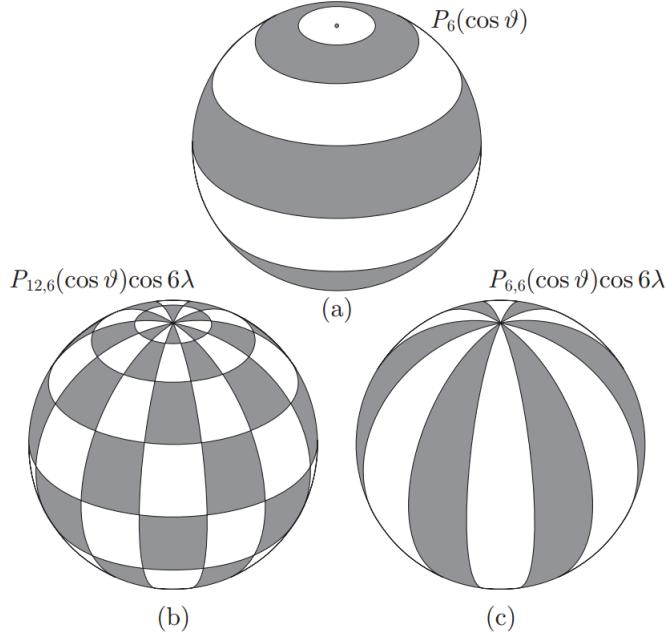

Figure 2.5: The spherical harmonics, from Hofmann-Wellenhof & Moritz (2006) page 18

2.2.2   Geoid determination using geopotential coefficients

The geoid height can be computed from geopotential coefficients given in the global gravity models as below:

$$N(R, \phi, \lambda) = \frac{GM}{R\gamma} \sum_{n=2}^{N_{max}} \left(\frac{a}{R}\right)^n \sum_{m=0}^{n} (\overline{R}_{nm}cos\ m\lambda + \overline{q}_{nm}sin\ m\lambda)\overline{P}_{nm}(sin\ \phi) \tag{2.31}$$

where

ϕ = geocentric latitude
λ = geocentric longitude
R = mean earth radius
a = radius of the earth at equator
γ = average normal gravity

$\overline{R}_{nm}$ and $\overline{q}_{nm}$ are given by:

$$\overline{R}_{nm} = \left\{ \begin{matrix} \overline{J}_{nm} - \overline{J}_n^{(N)} & for \ m = 0 \\ \overline{J}_{nm} & for \ m \neq 0 \end{matrix} \right\} \tag{2.32}$$

$$\overline{q}_{nm} = \overline{k}_{nm}$$

$$\overline{J}_n^{(N)} = \left\{ \begin{matrix} 0 & for \ odd \ n \\ (-1)^{\frac{n}{2}} \dfrac{3e^n + \left(1 - \frac{n}{2} + \frac{5}{2}\frac{J_2^{(N)}}{e^2}n\right)}{(n+1)(n+3)\sqrt{2n+1}} & for \ even \ n \end{matrix} \right\} \tag{2.33}$$

$$\overline{J}_2^{(N)} = -\frac{J_2^{(N)}}{\sqrt{5}} \tag{2.34}$$

$J_2^{(N)} = 0.108263 \cdot 10^{-2}$

2.2.3 Geoid determination using GPS/leveling

The connection between the geoid, ellipsoid and the orthometric height is shown in Equation (2.35) and Figure 2.3

$$H = h - N \tag{2.35}$$

Where H is the orthometric height, h is it's ellipsoidal height and N is the geoidal height.(Vaníček (2009)Hofmann-Wellenhof & Moritz (2006))

The telluroid is defined by the ellipsoid and the height anomalies ξ as shown in equation (2.36), where H^* is the normal height. It is not a physical surface like the geoid with constant potential but it coincides with the geoid over oceans and are close to the geoid elsewhere.(Hofmann-Wellenhof & Moritz 2006)

$$H^* = h - \xi \tag{2.36}$$

GPS/leveling can be used to determinate the orthometric height H, if the geoid is known in the area by Equation (2.35) and the ellipsoidal height is known from the GPS. It can also be used to determine the geoidal undulation if the orthometric height is known by leveling and the ellipsoidal height is determined by the GPS. By taking the difference of two points A and B Equation (2.37) can be used to find geoidal undulation and orthometric height in one of the points if the other point is known. The principal of GPS/leveling is shown in Figure 2.6.

$$H_B - H_A = h_B - h_A - N_B + N_A \tag{2.37}$$
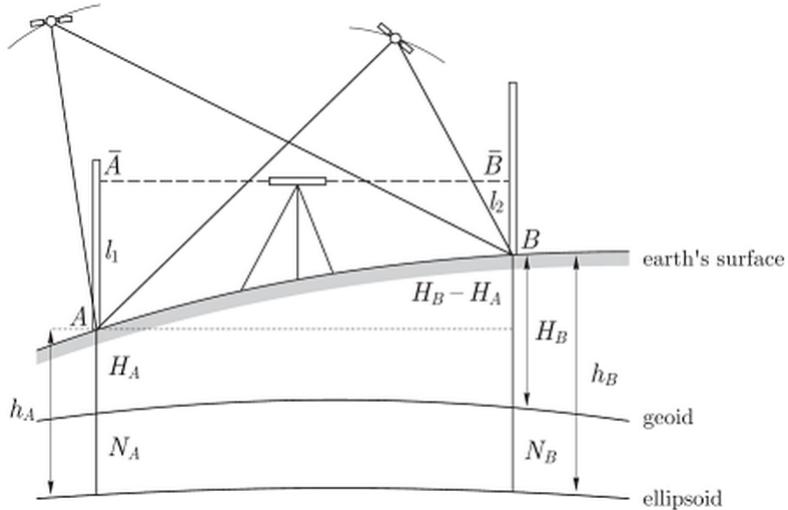
Figure 2.6: GPS/leveling, from Hofmann-Wellenhof & Moritz (2006) page 172

2.3 Corrections

Two methods are used to compute the geoidal heights in this study. In the first method we compute the geoidal heights directly at the geoid from geopotential coefficients, then the geoid are corrected for the effects of topographic masses. In the second method, we compute the height anomaly first i.e., the computation point is at the earths surface. Geopotential coefficients are also used in this method. Thereafter, we convert the height anomalies to geoid height using proper corrections. In Section 2.3.1 the theory is taken from Nahavandchi & Sjöberg (1998) and in Section 2.3.2 from Nahavandchi & Mahdavipour (2012) unless others are specified.

2.3.1 Topographic correction to the geoid derived satellite gravity model

When the computation point is at the geoid, theoretically, it causes a bias as Stokes's theory assumes that the gravity field is harmonic outside the geoid. This is not the case here, as there are topographic masses above the geoid. The correction in this section takes into account the effect of topographic masses. The method is called total topographic effect for geoid computation and uses Helmert's second condensation method to derive the sum of the effects. In "The total topographic effect in gravimetric geoid determinations" Sjöberg (1997) it is shown that the topographic effect of power two can be expressed by Equation (2.38). Power three is not computed because, as shown in Nahavandchi & Sjöberg (1998) these effects are small and can be neglected. The corrections in Equation (2.38) and (2.39) are the sum of both direct and indirect topographic effect and are further explained in Nahavandchi & Sjöberg (1998).

$$\delta N_{tot} = -\frac{2\pi\mu}{\gamma}H^2 \tag{2.38}$$

$$\delta N_{tot} = -\frac{2\pi\mu}{\gamma}\sum_{n=0}^{n_{max}}\sum_{m=0}^{n}(J_{nm}cosm\lambda + K_{nm}sinm\lambda)P_{nm}sin(\varphi) \tag{2.39}$$

$\gamma = 9.81[ms^{-2}]$
$G = 6.67384 \cdot 10^{-11}[m^3kg^{-1}s^{-2}]$
$\rho = 2670[kgm^3]$

where H topographical height, J_{nm} and K_{nm} are harmonic coefficients for the topographical heights, μ is the Gravity constant times average earth density and γ is the gravity acceleration.

2.3.2 Indirect geoid determination through height anomaly

This computation is an indirect approach, which is based on computation of height anomaly and conversions to get the geoidal undulation and it follows the article "Comparison of geoid heights from the EGM2008 geopotential model and GPS/leveling data in a study area in Iran"(Nahavandchi & Mahdavipour 2012). The height anomaly is computed at the surface or above it from the geopotential coefficients of the actual model. The method is based on or above the surface, therefore it is not necessary to evaluate the geopotential at the geoid, which seldom coincides with the earths surface.

$$N(\phi, \lambda) = \zeta_E(r, \phi, \lambda) + \frac{\delta\xi}{\delta r}h + \frac{\delta\xi}{\delta\gamma}\frac{\delta\gamma}{\delta h}h + \frac{\Delta g_{Bouguer}}{\overline{\gamma}}H \tag{2.40}$$

which also can be written as

$$N(\phi, \lambda) = \zeta(r_p, \phi, \lambda) + \frac{\Delta g_{Bouguer}}{\overline{\gamma}} H \tag{2.41}$$

In Equation (2.41) r_p is the geocentric distance to the point. In Equation (2.40) and (2.42) r is the ellipsoidal radius at the point to make the computations more efficient. The two terms in the middle of Equation (2.40) is corrections to get ζ from ζ_E in point P. ζ_E is computed by the geopotential coefficients.

$$\zeta_E(r, \phi, \lambda) = \frac{GM}{r\gamma} \sum_{n=2}^{N_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\overline{C}_{nm} cos m\lambda + \overline{S}_{nm} sin m\lambda)\overline{P}_{nm}(sin\phi) \tag{2.42}$$

where ϕ and λ is the spherical coordinates, γ is the normal gravity at the Telluroid.

$\delta g_{Bouguer}$ is the Bouguer anomaly. $\overline{\gamma}$ is the average gravity between the point on the ellipsoid and Telluroid. In Equation (2.43) the gravity units are mGal. The last term in Equation (2.43) is the Bouguer plate.

$$\Delta g_{Bouguer}(\phi, \lambda) = \Delta g_{Free-air}(\phi, \lambda) - 0.1119 H(\phi, \lambda) \tag{2.43}$$

Where $\overline{\gamma}$ is set to $9.798 m/s^{-2}$ and $\Delta g_{Free-air}$ is given by:

$$\Delta g_{Free-air}(r\phi, \lambda) = \frac{GM}{r^2} \sum_{n=2}^{N_{max}} (n-1) \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\overline{C}_{nm} cos m\lambda + \overline{S}_{nm} sin m\lambda)\overline{P}_{nm}(sin\phi) \tag{2.44}$$

The correction terms are computed as followed:

$$\frac{\delta\zeta}{\delta r} h(r, \phi, \lambda) = -\frac{GM}{r^2\gamma} h \sum_{n=2}^{N_{max}} (n+1) \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\overline{C}_{nm} cos m\lambda + \overline{S}_{nm} sin m\lambda)\overline{P}_{nm}(sin\phi) \tag{2.45}$$

$$\frac{\delta\zeta}{\delta\gamma}\frac{\delta\gamma}{\delta h} h(r\phi, \lambda) = 0.3086 h \frac{GM}{r\gamma^2} \sum_{n=2}^{N_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\overline{C}_{nm} cos m\lambda + \overline{S}_{nm} sin m\lambda)\overline{P}_{nm}(sin\phi) \tag{2.46}$$

These two conversions terms are referred to as C_2, the last term in Equation (2.41) is referred to as C_1.

3 Numerical investigation

In this Chapter the geoid models computed from global gravity models, the national geoid models and the GPS/leveling derived geoid heights are presented. Followed by the results from the computations.

There is mainly three types of geoidal models, satellite-only, combined and tailored models. The satellite-only models is computed from satellite tracking of the earth's gravity field. Combined models consist of different data, terrestrial gravimetry, satellite-only models and other gravimetric data. Tailored models is improved models made of existing models obtaining higher resolution.(Featherstone 2002)

3.1   Data

3.1.1   Satellite gravity models

EGM2008, EGM96, GGM02C, GGm02S, GGM03C and GGM03S are the models used in the comparison. They are divided in two groups, GGMs and EGMs, where the EGMs are computed with the second method from geopotential coefficients. The GGMs are computed from geopotential coefficients with different correction methods.

GRACE Gravity Models

The GRACE satellites measure the Earth's gravity by circling around the Earth and was launched in March 2002(Jäggi et al. (2010), Featherstone (2002), Tapley (2004)). The main goal for GRACE is to monitor the changes in earth's gravity field by SST-ll(Visser (1999), Rummel et al. (2002)). It maps the gravity field every 30 day with a resolution varying from 400 km to 40000 km(Tapley 2004). The satellites have an accuracy of 2 to 3 millimeters with a spatial resolution of 400 kilometers(Tapley et al. 2004). SST-ll is done by two satellites which follows each other in almost the same orbit with an elevation around 450 to 480 kilometers and a distance between them varying from 100 to 400 kilometers(Rummel et al. 2002). The basis of the tracking is shown in Figure 3.1. The distance between the satellite changes due to the difference in gravity in different areas.(Hofmann-Wellenhof & Moritz 2006)
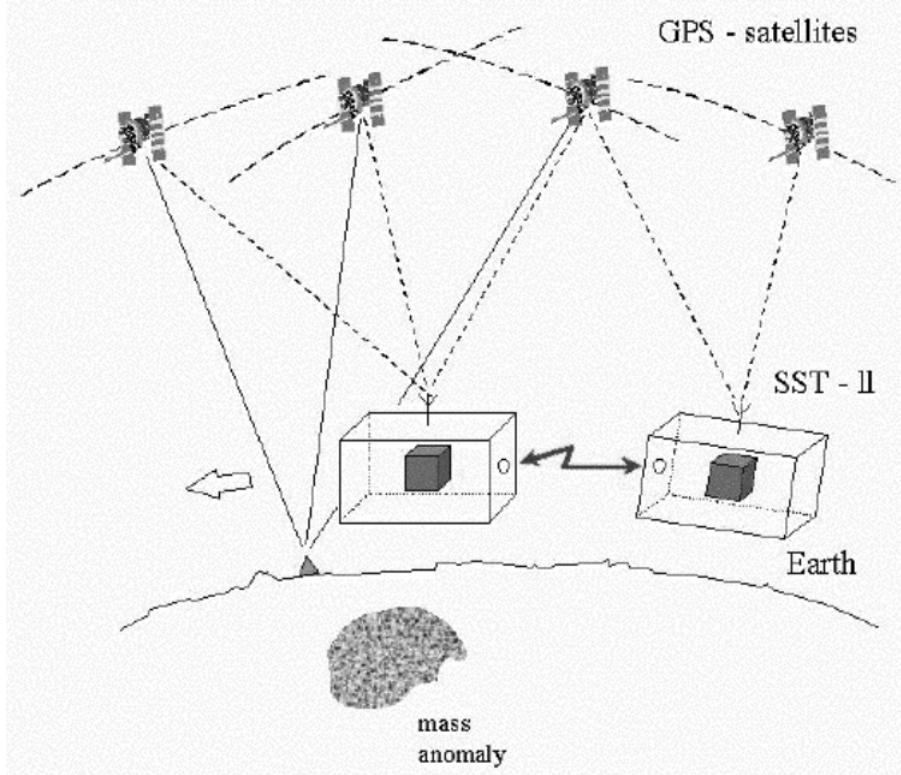
Figure 3.1: Satellite to satellite tracking method in GRACE, from Featherstone (2002)

Two types of Grace models are tested, two satellite-only models(GGM02S and GGM03S) and two combined models(GGM02C and GGM03C). The satellite-only models are based on satellite tracking data only. The theory in this section is taken from Tapley et al. (2005) and Tapley et al. (2007).

GGM02 is based on K-band range-data, altitude and accelerometer data from the GRACE satellites. The data is collected from April 2002 till December 2003, 363 of these days are used. GGM02S is made of the satellite data and has degree and order 160. It is not recommended to use GGM02S above degree 110 because of the increasing error. GGM02C is based on GGM02S and terrestrial gravity information from the TEG-4. It is complete to degree and order 200.

GGM03 are determined from GRACE K-band inter-satellite range-rate data, GPS tracking and GRACE accelerometer data. The GRACE data was collected over a period of 47 months spanning from the beginning of 2003 to the end of 2006. GGM03S is available up to degree 180, but because of the increasing error in higher degrees it should only be used up to degree 130. As for the GGM02S the GGM03S is also determined from satellite data only, while GGM03C is a combined model consisting of terrestrial gravity information and GGM03S. The terrestrial data is from the NIMAs surface anomalies, CRS mean sea surface and the Arctic Gravity Project.

Earth Gravitational Models

Here the EGM96 and EGM2008 are described, for simplicity later on if they are referred to together the EGMs are used.

EGM2008 is a static model computed from GRACE, gravimetry and satellite altimetry data (Novák 2009). The model is complete to degree and order 2159 and have additional coefficients up to degree 2190. The model is based on data from GRACE, an area-mean free-air gravity anomalies model with a 5 arc-minute resolution and a high resolution global DTM. The GRACE model that is used is ITG-GRACE03S with its belonging error covariance matrix. ITG-GRACE03S is based on 57 months of SST and complete to degree 180. The gravity anomalies models is made from measurements from satellite altimetry, terrestrial data and anomalies computed using residual terrain method(RTM) forward modeling. DTM2006.0 is used as Digital Topographical Model. The DTM and the creating of the anomalies model are described in "The development and evaluation of the Earth Gravitational Model 2008 (EGM08)".(Pavlis et al. 2012) Least-Squares adjustments was used to combine data from GRACE-only and the terrestrial data. It is not used any GPS/leveling or astronomic deflection of the vertical data in the model.(Pavlis et al. 2008). The resolution of EGM2008 is 9 km Yilmaz et al. (2010).

EGM96 is an earth gravity model of order 360 with a resolution of 30'x30'. The model is based on information from around 40 long-wavelength satellites, elevation measurements from 29 different sources, satellite altimetry for the marine areas from TOPEX, ERS-1 and GEOSAT(Kenyon et al. 2007). Surface gravity data for Europe were collected by NIMA(Lemoine et al. 1998). The model consist of a low-degree combination to degree 70, a block diagonal solution from the 71. to the 359. degree and the rest up to degree 360 is a quadrature solution(Lemoine et al. 1998).

3.1.2   National geoid of Norway

The NMA makes their own national geoid models over Norway, in this comparison NMA2013v22 and NMA2012v30 are used. The models and information about them are given from Ove Christian Dahl Omang in the NMA through personal communication.

EIGEN06C up to degree and order 250 is used as basis for NMA2012v30. Gravity data from the NKG database is used both on land and over the ocean, to supply these data new gravity measurements of Sognefjorden and Mjøsa is used. Up to degree 140 the global model is used and for degrees higher than 140 it uses local gravity data. For the computation the remove-compute-restore method is used with the RTM which is described in Dahl & Forsberg (1998).

For NMA2013v22 EGM2008 is used as global model and gravity data from NKG with data from Mjøsa and Sognefjorden added. Gaps in sea areas are filled with DTU10 data. As computational method Stokes's kernel with truncation at degree 2100-2200 is used.

Both models have a resolution of 0.04 degrees in north-south direction and 0.02 in east-west direction. The Matlab program GRID reorganizes the grid of the models, and finds the geoid height of the model at the GPS/leveling points by using the Matlab function interp2 with bicubic interpolation. The national geoid models for Norway are shown in Figure 3.2 and 3.3.
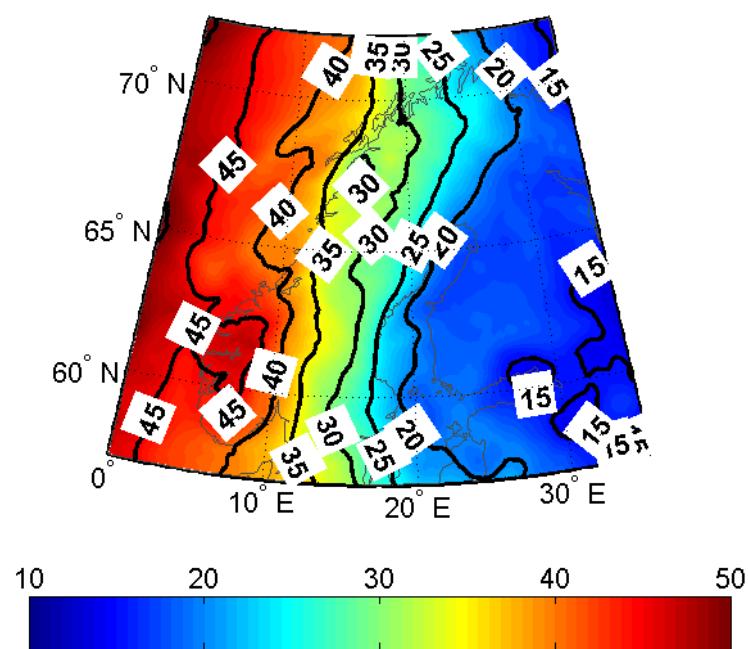

Figure 3.2: The NMA geoid model of 2012 over Norway
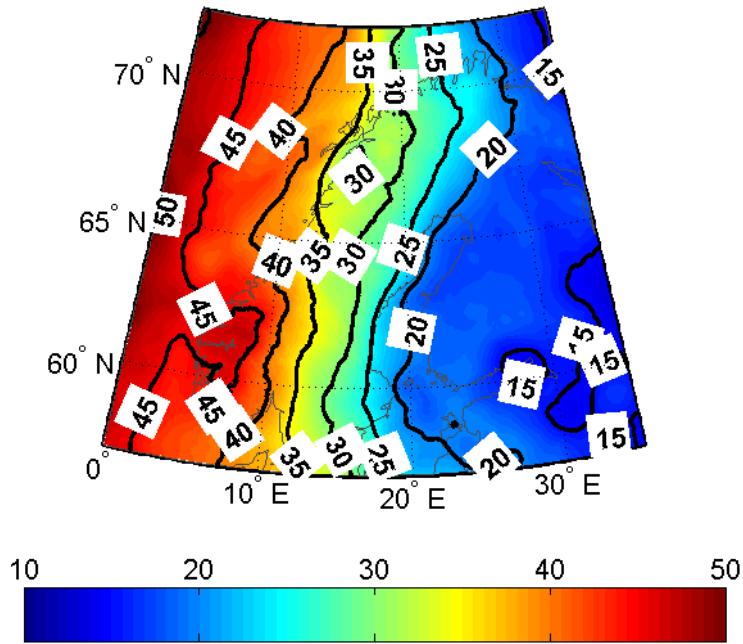
Figure 3.3: The NMA geoid model of 2013 over Norway

3.1.3 GPS/leveling points

The GPS/leveling points and information about them are given from Ove Christian Dahl Omang in the NMA through personal communication. The GPS points is a mix of vector measurements and PPP. The leveling is mainly off good quality but some of the points are of poorer quality. The accuracy lies around 15 mm in height and 6 mm in east/west direction for the GPS and the total accuracy is around 17 mm in height and 6 mm horizontally. For the computations a total of 2329 points, where no points are removed, disturbed over mainland of Norway are used. With an area of Norway at approximately 385 000 square kilometers this gives a point density of 0.006 points per square kilometer. The elevation of the points spans from sea level to 1500 meter above sea level. The distribution of the GPS/leveling points are shown in Figure 3.4.

Figure 3.4: Distribution of the GPS/leveling

3.2 Geoid computation and results

In this part the results are presented, the models are computed and compared to the national geoids of Norway. First there is a short presentation on how the error is computed, then the national geoids for Norway are presented. Following up the GRACE gravity models are presented with the error corrections. Then the EGM models and some figures with error plotted against elevation are presented. The visualizations are done with the Matlab program visualization attached.

3.2.1 Calculation method

To determine the accuracy of the model the difference in geoidal undulation is used. The difference is computed as in Equation (3.1)

$$\Delta N = N_{REF} - N_{model} \tag{3.1}$$

where

N_{REF} is the geoidal undulation from the GPS/leveling measurements
N_{model} is the geoidal undulation of the model

22

3.2.2   National geoids of Norway

The national geoids for Norway are expected to be accurate because they are tailored to fit the geoid in Norway. The results from the geoids are shown in Table 3.1, as we can see both geoids lies below the reference geoid, varying from 4 centimeters up to 46 centimeters. The standard deviation is 5.7 centimeters for NMA2012v30 and 5.6 centimeters for NMA2013v22, it has improved with 1 millimeter. As we can see from Figure 3.5 there is higher errors in the north of Norway. It also shows that there are smaller errors in the mountain areas for NMA2012v30 Figure 3.6 shows that NMA2013v22 has larger errors in the south and on the border to Finland, but good accuracy on the western coast.

Table 3.1: Accuracy of the national geoid models for Norway

Geoid model	Min[m]	Max[m]	Mean[m]	STD[m]
NMA2012	0.040	0.458	0.259	0.057
NMA2013	0.084	0.428	0.265	0.056
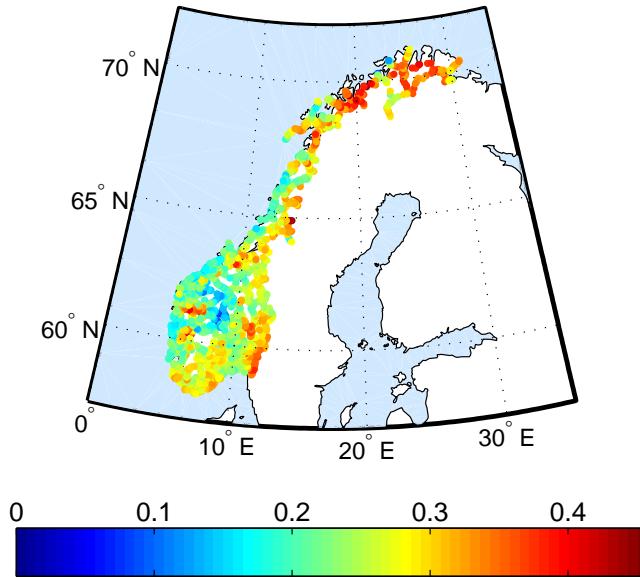
Figure 3.5: The differens in geoidal heights between NMA2012v30 and GPS/leveling in meters
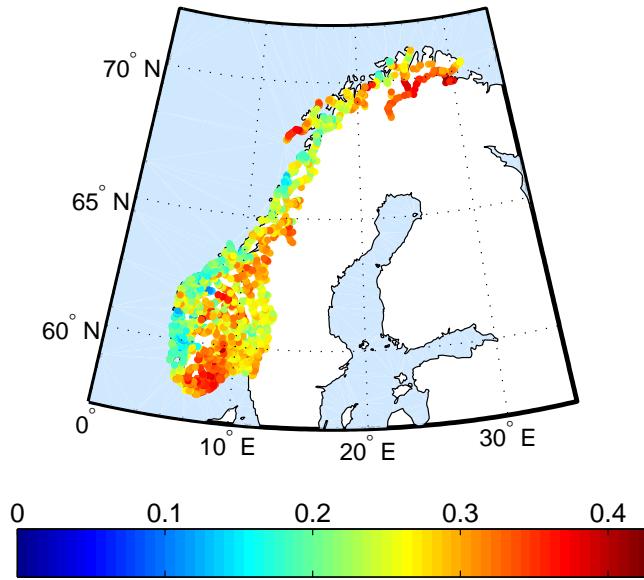
Figure 3.6: The differens in geoidal heights between NMA2013v22 and GPS/leveling in meters

3.2.3 The GRACE gravity models

The computations are done as described in Section 2.2.2 and 2.3 with geopotential coefficients obtained from `http://icgem.gfz-potsdam.de/ICGEM/modelstab.html`. The coefficient files contains both the coefficients C and S and their standard deviation. The parameters are given in the file descriptions, and are as followed:

$GM = 0.3986004415 \cdot 10^{15} m^3 s^{-2}$
$a = 0.6378136300 \cdot 10^7 m$
$\gamma = 9.81 m^3 s^{-2}$
$r = 0.63710007900 \cdot 10^7 m$

GGM02S is computed with degree 110 and GGM03S with degree 130. The computation for the geoidal undulation are done by the Matlab program GGM, the topographic corrections are computed with the program Corrections_H2 in the digital attachment. The coefficients J_{nm} and K_{nm} for the topographical corrections up to degree and order 360 are given by Hossein Nahavandchi.

The indirect computations of the geoid through height anomaly are done with the program ANOMALY.

For several of the models the errors are similar, therefore some of the visualizations are only shown in the Appendix.

The results of GGM02 are shown in Table 3.2. The difference of the computed geoidal heights for GGM02C are shown in Figure 3.7 the standard deviation is 0.502 meters which is 4 mm more than for the model with topographical corrections of H^2. For GGM02S which is shown in Figure 3.8, the standard deviation improves with 7 millimeters when the topographical corrections are added. The indirect computations through height anomalies with the C_1 and C_2 corrections gives less accuracy than the original computation.

Table 3.2: Results for GGM02

| Computation	Min	Max	Mean	STD
C N	-2.428	1.240	-0.535	0.502
S N	-3.343	2.498	-0.549	0.840
C $N + H^2$	-2.480	1.188	-0.579	0.498
S $N + H^2$	-3.389	2.446	-0.593	0.833
C $\xi_p + C_1 + C_2$	-2.482	1.650	-0.330	0.578
S $\xi_p + C_1 + C_2$	-3.308	2.609	-0.341	0.877
C $\xi_p + C_1$	-2.481	1.575	-0.335	0.573
S $\xi_p + C_1$	-3.307	2.609	-0.345	0.874
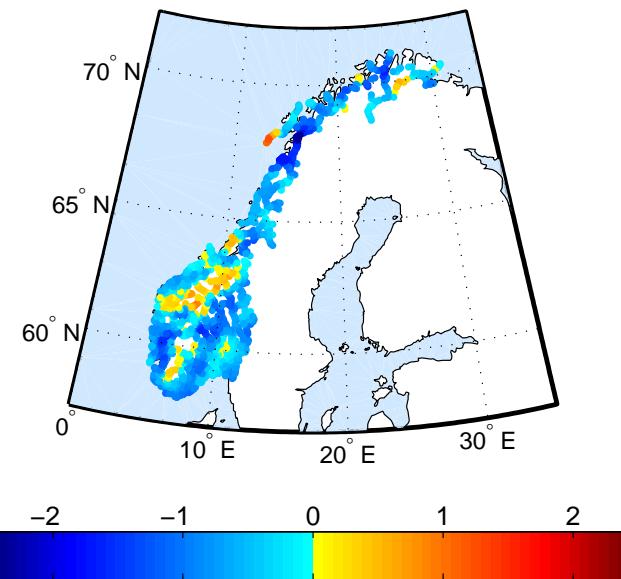
Figure 3.7: The difference in geoidal heights between GGM02C and GPS/leveling in meters
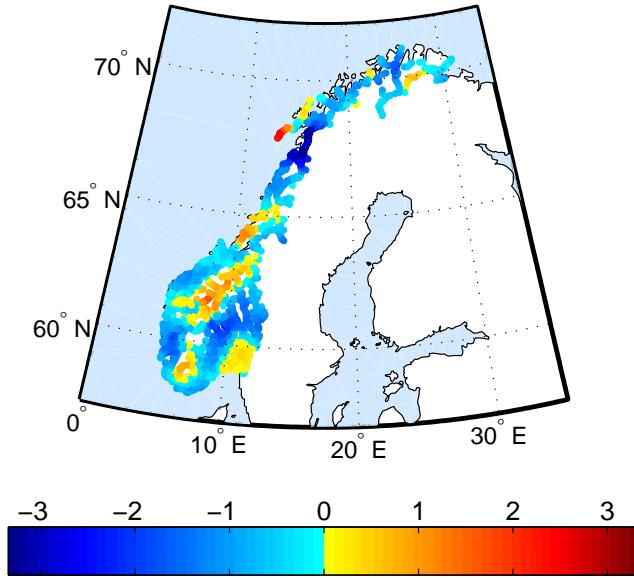
Figure 3.8: The difference in geoidal heights between GGM02S and GPS/leveling in meters

For the GGM03 models the accuracy varies between 41 and 78 centimeters, shown in Table 3.3. Figure 3.9 shows the errors for GGM03C, wich vareis between minus 2 meters to plus 90 centimeters. GGM03S which is shown in Figure 3.10 has its highest errors in a area in the middle of northern Norway. The errors for the indirect computed models of GGM03C and GGM03S are shown in Figure 3.11 and 3.12. Both these models has high errors in the mountainous area of southern Norway and in the middle of northern Norway. As for GGM02 there is higher error for the models determined from indirect geoid determination while the models with topographic correction has improvements from 3 to 7 millimeters compered to the original models.

Table 3.3: Results for GGM03

Computation	Min	Max	Mean	STD
C N	-2.040	0.827	-0.543	0.409
S N	-3.036	2.340	-0.564	0.744
C $N + H^2$	-2.091	0.775	-0.587	0.406
S $N + H^2$	-3.083	2.288	-0.608	0.737
C $\xi_p + C_1 + C_2$	-2.164	1.394	-0.337	0.506
S $\xi_p + C_1 + C_2$	-3.034	2.469	-0.355	0.784
C $\xi_p + C_1$	-2.163	1.307	-0.343	0.500
S $\xi_p + C_1$	-3.033	2.469	-0.360	0.781
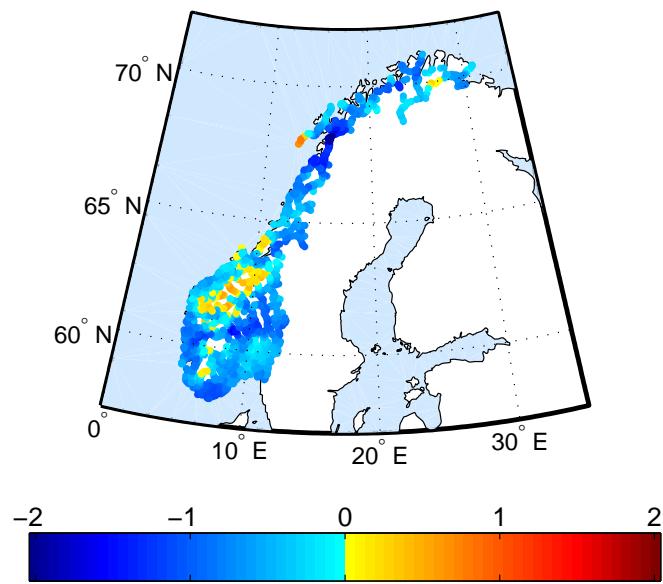
Figure 3.9: The difference in geoidal heights between GGM03C and GPS/leveling in meters
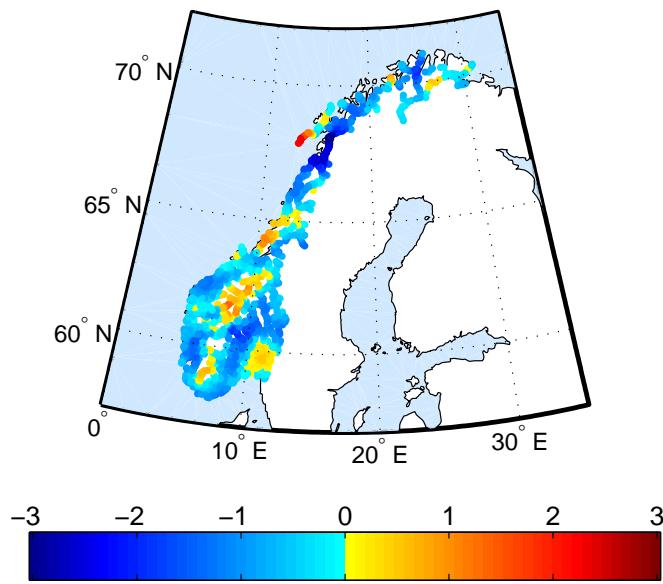

Figure 3.10: The difference in geoidal heights between GGM03S and GPS/leveling in meters
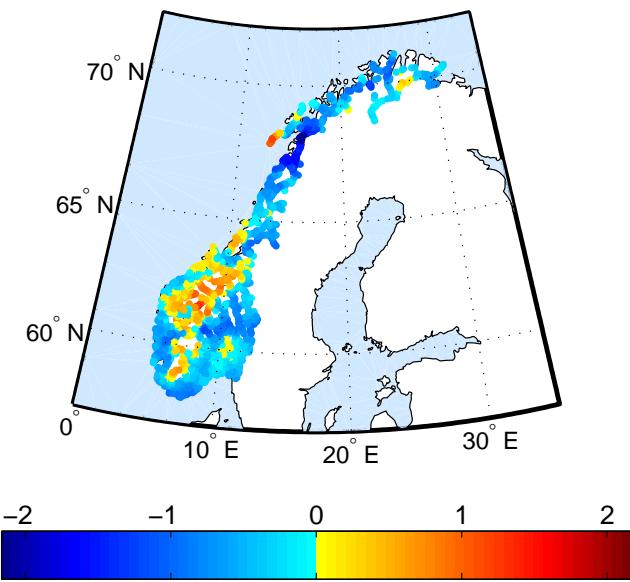
Figure 3.11: The difference in geoidal heights between GGM03C with C_1 and C_2 correction term and GPS/leveling in meters
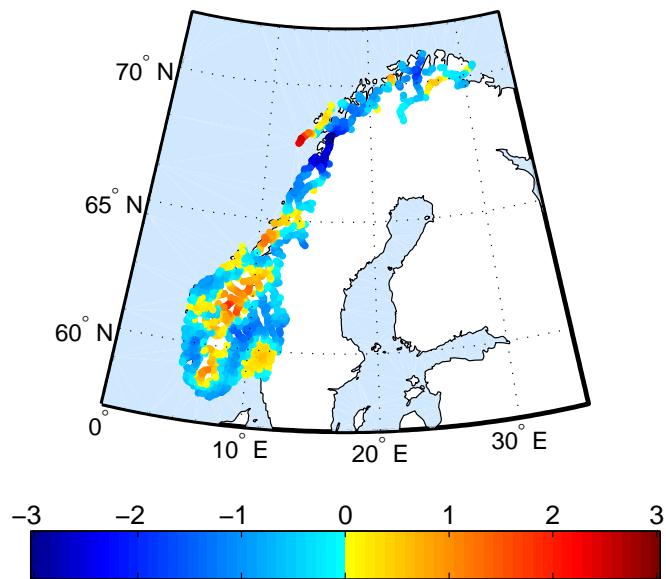

Figure 3.12: The difference in geoidal heights between GGM03S with C_1 and C_2 correction term and GPS/leveling in meters

3.2.4   The Earth Gravitational Models

These models are based on the second method, computing height anomaly first, then convert them to geoidal heights. The computation of the geoidal heights for EGM2008 and EGM96 is done with coefficients which is fully normalized, unit-less and spherical harmonic. Equation (3.2) is used to compute the geoidal undulation in the control points. EGM2008 is complete to spherical harmonic degree and order 2159, and have additional coefficients to degree 2190 and order 2159. EGM96 is complete to degree and order 360.(Pavlis et al. (2008), Lemoine et al. (1998)) For the computation the files EGM2008_to2190_TideFree.gz are used as the gravitational model and Zeta-to-N_to2160_EGM2008.gz for the correction term. The FORTRAN program hsynth_WGS84, developed by Simon A. Holmes and Nikolaos K. Pavlis is used for the computations, all the files and the program is available at the web page: `http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html`. EGM2008_to2190_TideFree.gz contains the coefficients $\overline{C}_{nm}$ and $\overline{S}_{nm}$ as well as their calibrated standard deviations. Zeta-to-N_to2160_EGM2008.gz contains the coefficients $\overline{CC}_{nm}$ and $\overline{CS}_{nm}$. (Pavlis et al. 2008). The computation of EGM96 is done with the same program and the coefficients are obtained from `http://icgem.gfz-potsdam.de/ICGEM/modelstab.html`.

$$V(r, \theta, \lambda) = \frac{GM}{r} \left[1 + \sum_{n=2}^{N_{max}} \frac{a^n}{r} \sum_{m=0}^{n} (\overline{C}_{nm} cos m\lambda + \overline{S}_{nm} sin m\lambda) \overline{P}_{nm}(cos\theta) \right] \qquad (3.2)$$

where

$GM = 3986004.415 \times 10^8 m^3 s^{-2}$
$a = 6378136.3 m$

and $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are the fully normalized, unit-less, spherical harmonic coefficients of the earth's gravitational potential.

A conversion term is added to the computation to get the ξ-to-N conventions for the WGS 84 ellipsoid the correction term in Equation (3.3) are used. Where $\overline{CC}_{nm}$ and $\overline{CS}_{nm}$ is fully-normalized spherical harmonic coefficients of ξ-to-N. (Pavlis et al. 2008)

$$C(\theta, \lambda) = \sum_{n=0}^{N_{max}} \sum_{m=0}^{n} (\overline{CC}_{nm} cos m\lambda + \overline{CS}_{nm} sin m\lambda) \overline{P}_{nm}(cos\theta) \qquad (3.3)$$

Results from the computation of the EGMs are shown in Table 3.4. The accuracy has gone from 32.5 centimeters for EGM96 to 6.9 centimeters for EGM2008. In Figure 3.13, which shows the errors for EGM96, there is an area where the computed geoid lies under the actual geoid in the area between 60° and 65° degrees north, in the rest of the country

the geoid mostly lies above the actual geoid for EGM96. EGM2008 lies above the geoid with the highest errors concentrated around the western coast shown in Figure 3.14.

Table 3.4: Results for the EGM models

Model	Min	Max	Mean	STD
EGM96	-0.825	1.507	0.010	0.325
EGM2008	-0.452	-0.037	-0.242	0.069
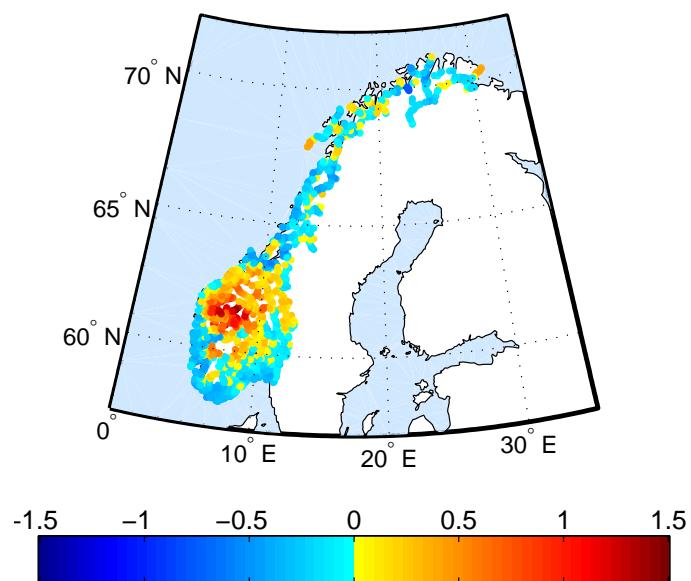
Figure 3.13: The difference in geoidal heights between EGM96 and GPS/leveling in meters
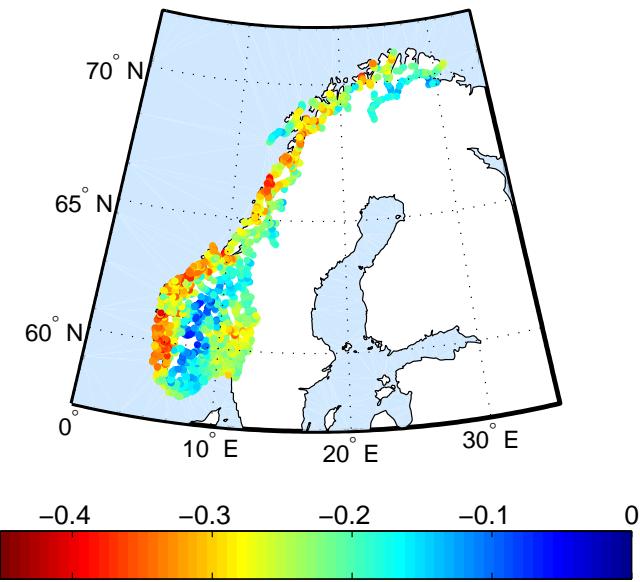
Figure 3.14: The difference in geoidal heights between EGM08 and GPS/leveling in meters

In another experiment, the EGM2008 geoid model and NMA2013v22 national geoid of Norway are compared directly. Figure 3.15 shows that EGM2008 generally lies 50 centimeters above the national geoid model for Norway from 2013. The difference in the models has a standard deviation of 3.9 centimeters.
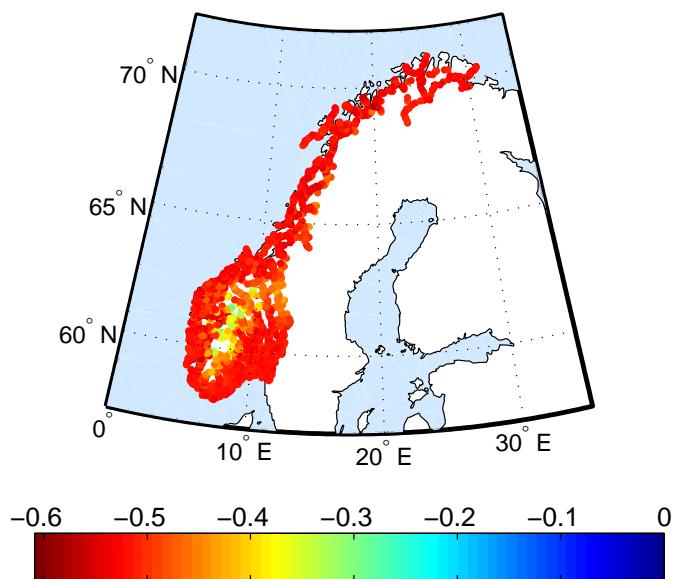
Figure 3.15: Difference in geoid heights between NMA2013v22 and EGM2008 in meters

3.2.5 Accuracy dependent on height

The Figures 3.16, 3.17, 3.18 and 3.19 are plotted to see if there are a relation between error and elevation. The figures shows the height on the x axis and standard deviation from geoidal height on the y axis. Figure 3.16 show that there is no relation between the heights and errors for the national geoid for Norway from 2013. The figure also shows that most of the GPS/leveling points has elevations under 500 meters. As shown in Figure 3.17 higher elevations has smaller errors than lower. The Figures 3.18 and 3.19 shows the error for the elevation for respectively GGM03C and GGM03S. These Figures shows a similarity for the models, which is expected because they are both based on the same observations from GRACE.
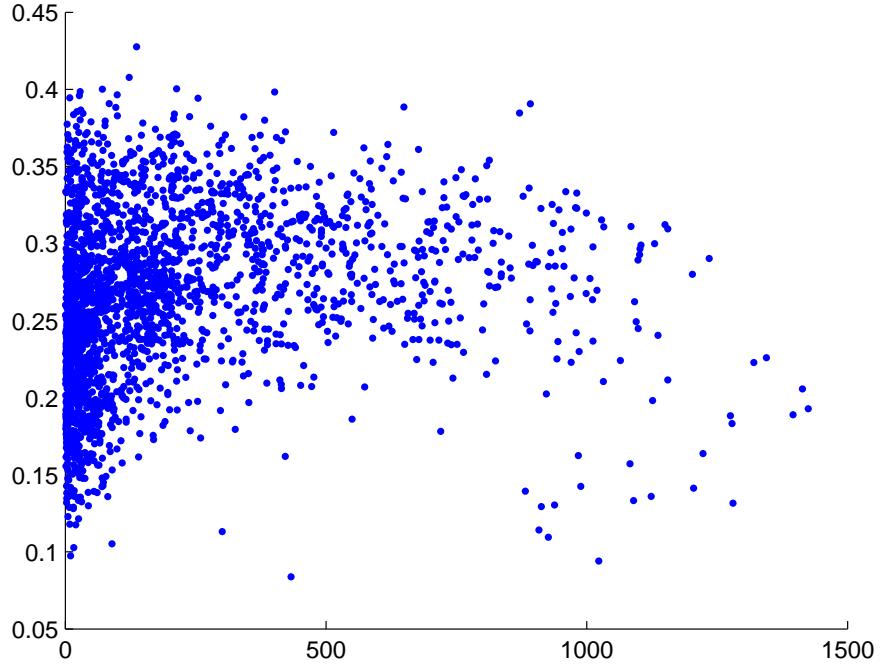
Figure 3.16: Height dependent variations of geoid difference between NMA2013v22 and GPS/leveling geoid at 2329 stations in meters
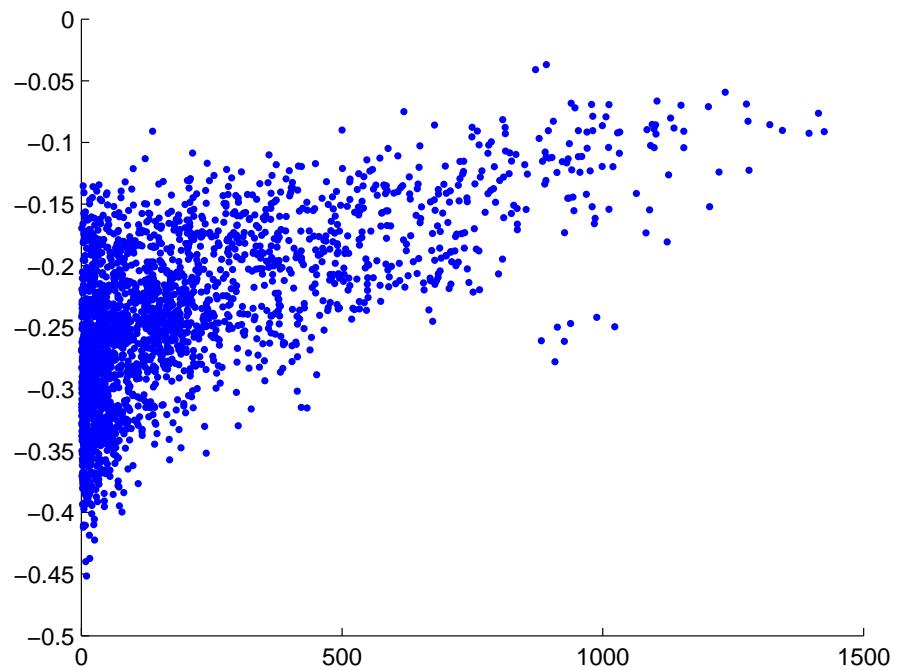
Figure 3.17: Height dependent variations of geoid difference between EGM2008 and GPS/leveling geoid at 2329 stations in meters
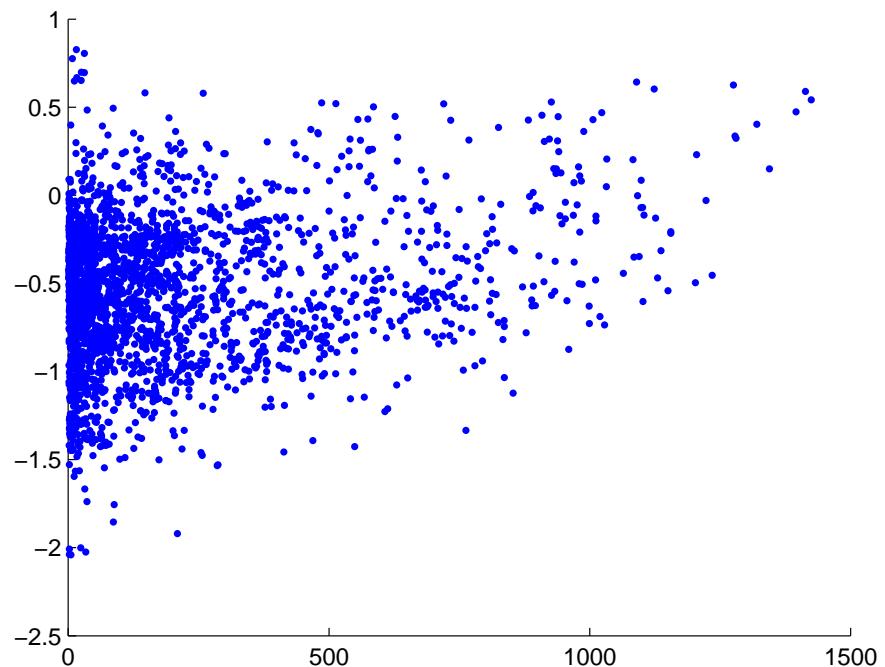
Figure 3.18: Height dependent variations of geoid difference between GGM03C and GPS/leveling geoid at 2329 stations in meters
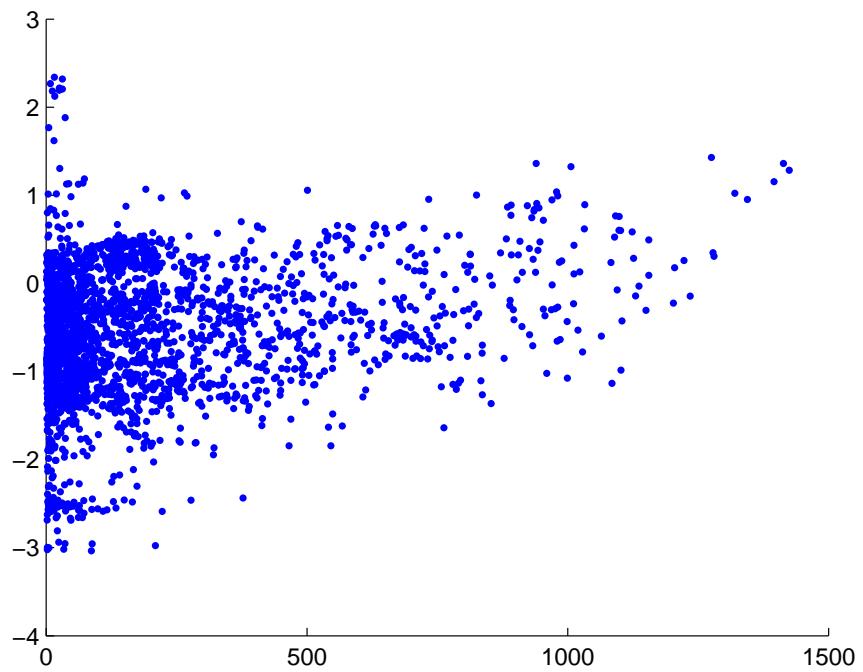
Figure 3.19: Height dependent variations of geoid difference between GGM03S and GPS/leveling geoid at 2329 stations in meters

4 Conclusion

In this part the results from Chapter 3 are discussed followed by a recommendation based on the presented results and some ideas for future work.

4.1 Discussion and conclusion

Six different global geoid models has bin computed at 2329 GPS/leveling stations distributed over mainland of Norway. The computation of different global geoid models has bin compared to national geoid models for Norway. The models accuracy was investigated by comparing their geoidal heights with geoidal heights from 2329 GPS/leveling points over whole Norway.

The GRACE gravity models different computations were used. They were computed with Stokes's formula, with and without topographic corrections from second degree of H. Indirect geoid determination through height anomaly with the correction terms C_1 and C_2. The earth gravitational models was computed by a Fortran program with the second method and proper conversion to geoidal height. Accuracy of national geoids for Norway were also computed in the GPS/leveling points.

As seen in the results the satellite only models are less accurate than the corresponding combined models that also uses terrestrial data, which is expected because of their higher degree. The GGM03 models are also more accurate than the GGM02 models, this was expected because of the difference in degree and available data to make the model. While GGM02 use approximately 12 months of data GGM03 use data from 47 months, this gives GGM03 the possibility to be computed with higher resolution, which also gives better accuracy. The accuracy of the models increases as the degree of the model increases. The error correction from the topographic masses is in the area of millimeters while the total error can be meters, Therefore it has a small effect on the total error. If the topographic correction is added to more accurate models it might have a higher influence on the accuracy. For the indirect geoid determination through height anomaly the higher error can be from a unknown error in the Matlab program. In the comparison between EGM96 and EGM2008 there seems to be no connection between the distribution and magnitude in the errors. While the error of EGM96 has a range of 2.3 meters EGM2008 only has one of 0.4 meters. This shows great improvement for EGM2008 and

als implies that there are no big errors in the GPS/leveling data.

It does not appear to be a connection between the accuracy and elevation in the height plots. In EGM2008 it seems like the accuracy improves with higher elevation, but it can also be a result of less observations at higher elevation or that EGM2008 generally lies above the geoid. The difference between the national geoid for Norway and EGM2008 implies that EGM2008 generally lies above the geoid and this is supported by the results of the computation where all the errors are negative.

As shown in Table 3.4 EGM2008 is the most accurate model with a standard deviation of 6.9 centimeters, 1.6 centimeters higher than for the national model for Norway from 2013.

4.2   Recommendation and future work

The Earth Gravitational Model 2008(EGM2008) is a good alternative to compute geoidal height. If these results are supported by other tests, EGM2008 can be used for orthometric height determination. Testing over other land areas to see if EGM2008 has the same accuracy there should be done. It would be interesting to look at the new models from GOCE which has an accuracy of 1 cm in geoid determination as a goal(Floberghagen et al. (2003), Hirt et al. (2011)). Testing the models in different topographical areas can be done to see if topography affects their accuracy. An interesting test area would be the eastern part of Norway where the topography is relatively flat, then a higher accuracy might be obtained.

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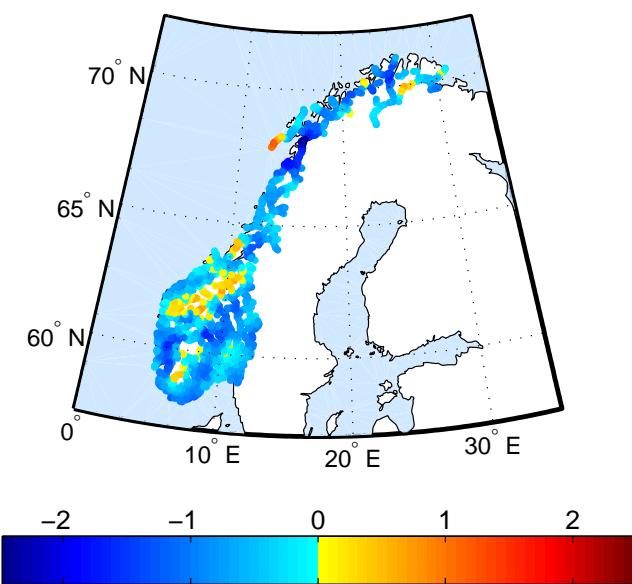
A Figures

Figure A.1: The difference in geoidal heights between GGM02C with H^2 corrections and GPS/leveling in meters
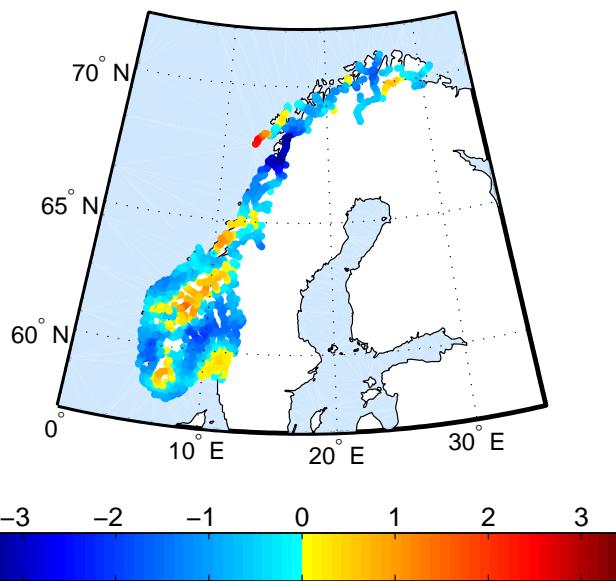
Figure A.2: The difference in geoidal heights between GGM02S with H^2 corrections and GPS/leveling in meters
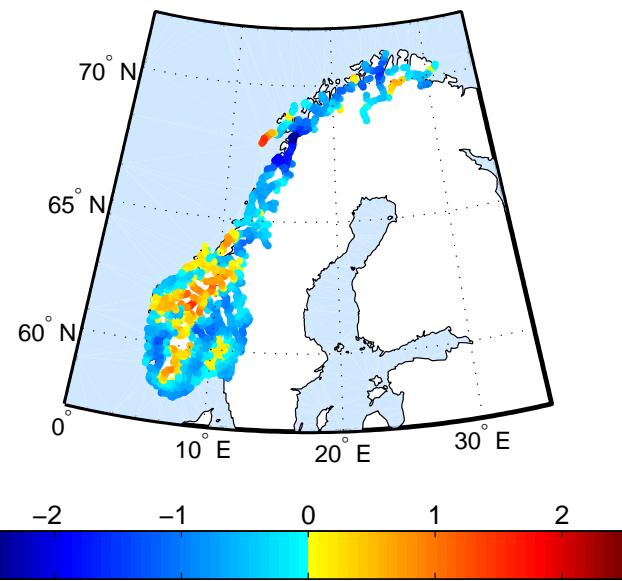

Figure A.3: The difference in geoidal heights between GGM02C with C_1 correction term and GPS/leveling in meters

b
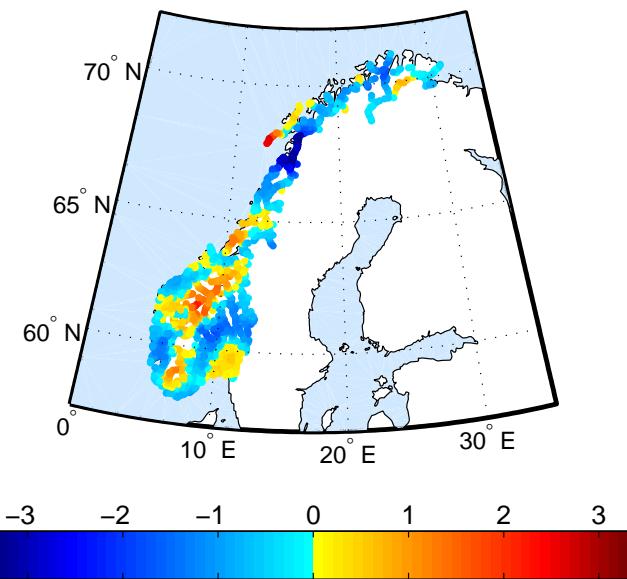
Figure A.4: The difference in geoidal heights between GGM02S with C_1 correction term and GPS/leveling in meters
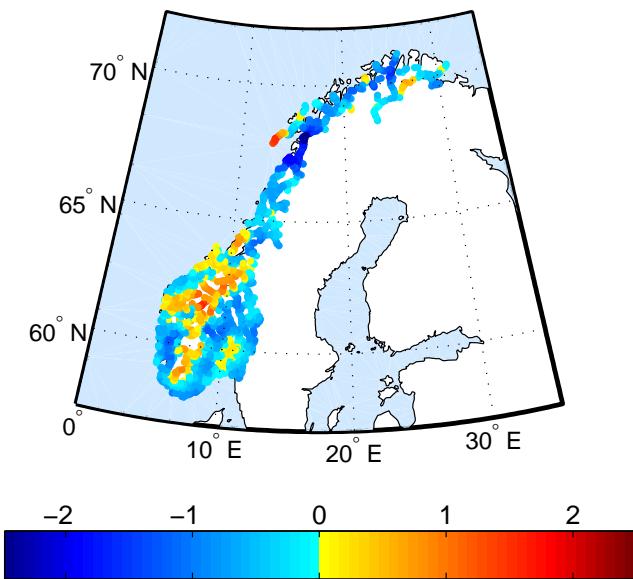
Figure A.5: The difference in geoidal heights between GGM02C with C_1 and C_2 correction term and GPS/leveling in meters
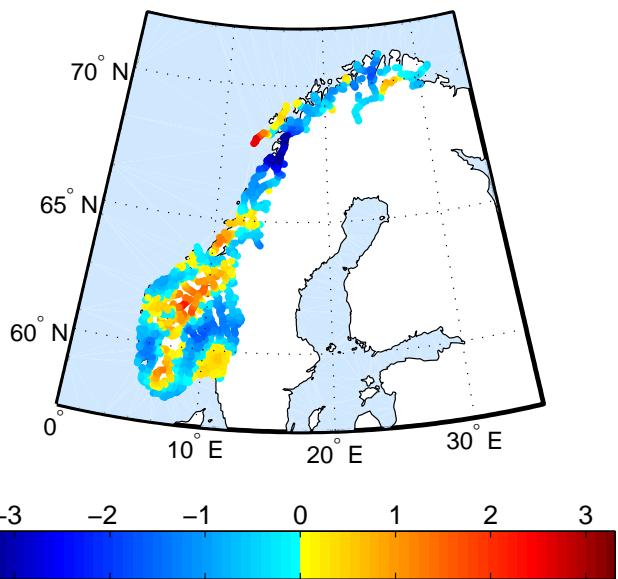
Figure A.6: The difference in geoidal heights between GGM02S with C_1 and C_2 correction term and GPS/leveling in meters
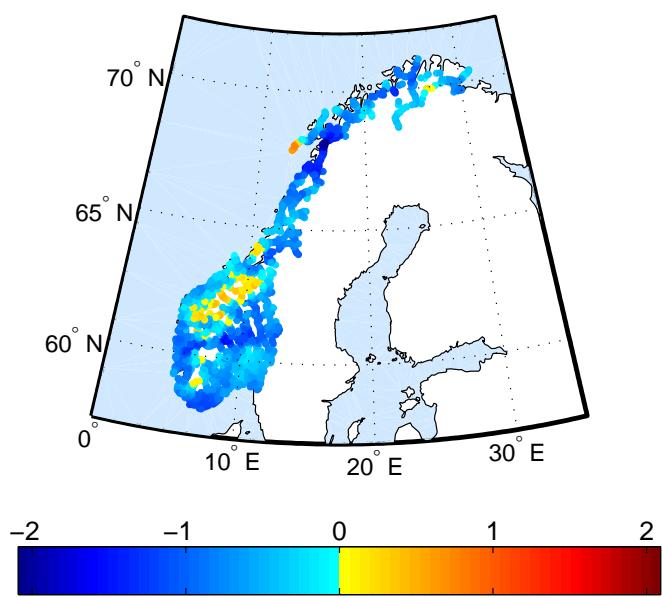

Figure A.7: The difference in geoidal heights between GGM03C with H^2 corrections and GPS/leveling in meters

d
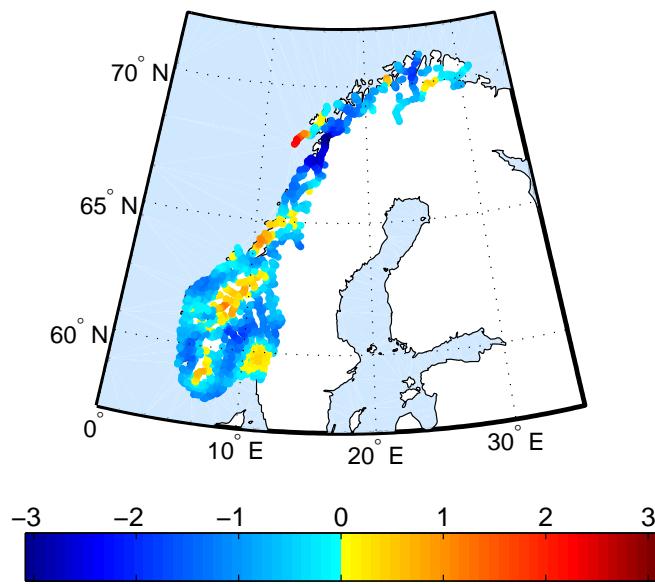
Figure A.8: The difference in geoidal heights between GGM03S with H^2 corrections and GPS/leveling in meters
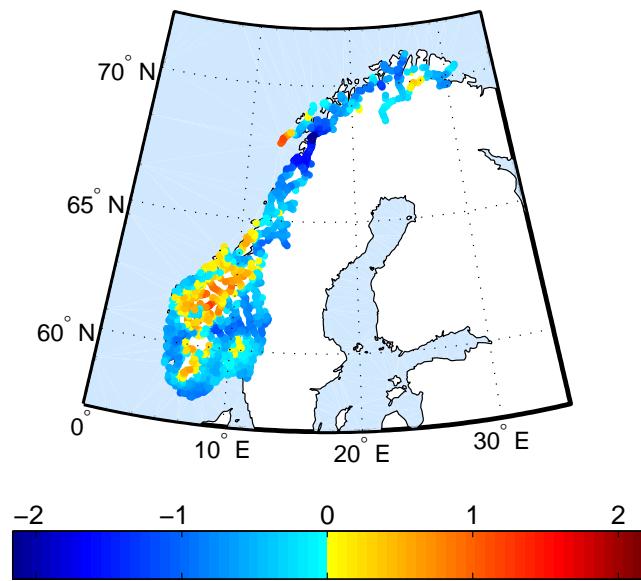

Figure A.9: The difference in geoidal heights between GGM03C with C_1 correction term and GPS/leveling in meters
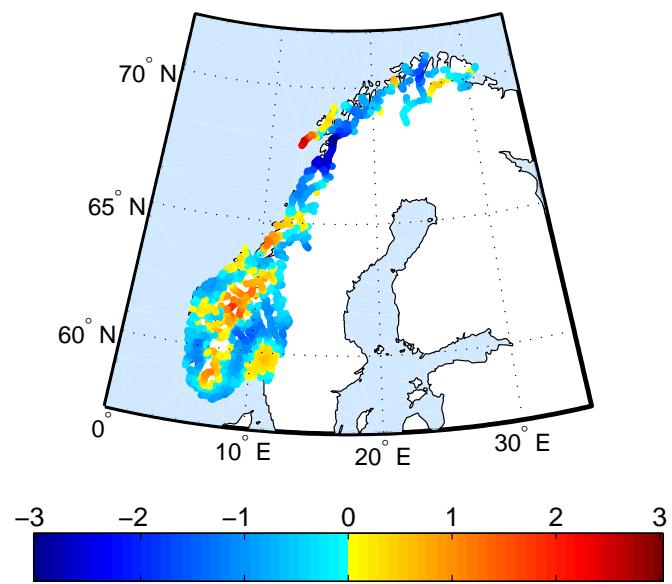
Figure A.10: The difference in geoidal heights between GGM03S with C_1 correction term and GPS/leveling in meters

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