Mathematics didactics

Masteroppgave

Richard Phiri

Students' strategies in mathematics word problem solving

A qualitative study of a group of fifth grade students and their use of strategies to solve mathematics word problems

Trondheim, august 2014

Høgskolen i Sør-Trøndelag
Avdeling for lærer- og tolkeutdanning

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Elevers strategier når de arbeider med problemløsningsoppgaver

En kvalitativ studie av en gruppe elever på femte klassetrinn og deres strategier når de arbeider med problemløsningsoppgaver

Masteroppgave, Mathematics didactics
Trondheim, august 2014

Veileder:	Ole Enge

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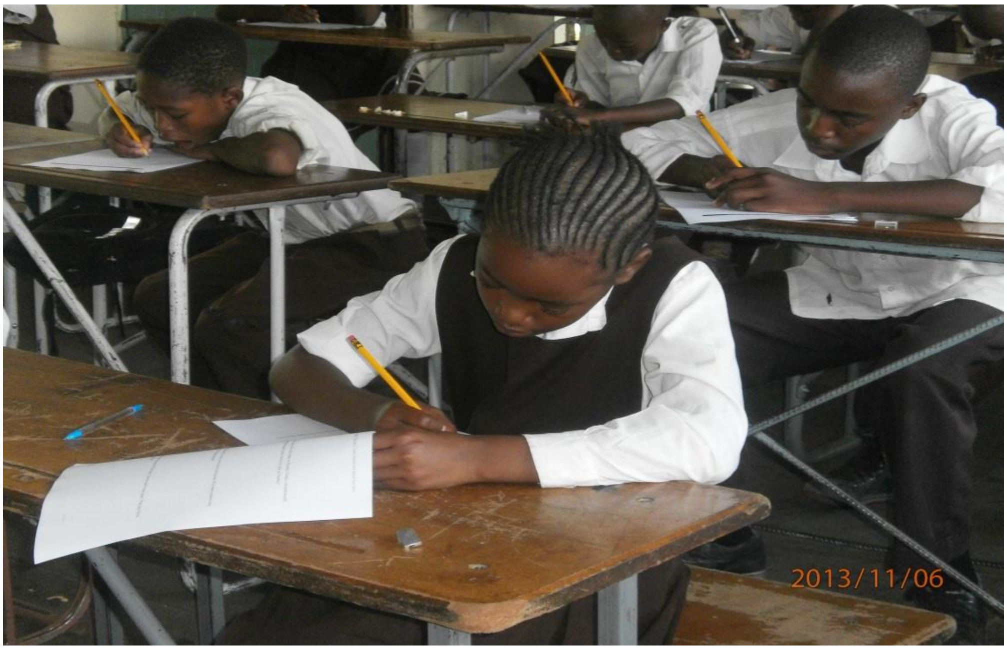

Høgskolen i Sør – Trøndelag University College
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Dedication

I wish to express warm thanks to my beloved wife Everlyn Chushi**,** my son Richard Jr, and my two daughters Chishala and Mulenga for the love and patience shown during the production of this dissertation.

I further express my thanks to my mother Elidah Mbewe, brothers and sisters for encouraging me in the production of this dissertation which they felt would help the government and other stakeholders in improving mathematics standards.

Acknowledgement

I wish to express my appreciations to my supervisor, Prof Ole who critically examined, supported and contributed towards the writing of this dissertation, my lecturers in the mathematics section and other departments at Høgskolen i Sør – Trøndelag (HiST) for generously sharing knowledge with me. I further extend my gratitude to Prof Geir as my immediate coordinator who was there for me throughout this course and not forgetting Marit the international coordinator for facilitating the provision of my scholarship.

My deepest gratitude goes to HiST University College through the Dean for offering me an opportunity to study at the institution and the Norwegian government in sponsoring the masters of didactics of mathematics at HiST University College, without which, I may not have had the opportunity to study.

I wish to appreciate the financial support of HiST University College in sponsoring my research project.

My sincere thanks also go to the Ministry of Education, Zambia for allowing me to study abroad.

Abstract

Word problems are among the components of school curriculum and are taught at all levels of education. Some tasks involving word problems have proved to be more challenging for students than others. The aim of the study was to explore the strategies that fifth grade students use to solve addition and subtraction word problems, the reason why some word problems are difficult than others and the effects of using first and second language in mathematics word problem solving.

The sample comprised of thirty nine students drawn from one school in Livingstone, Zambia. These students took part in a test of 10 word problems. Thereafter, five of them were randomly selected and interviewed. The study employed qualitative research design. Information was derived using students marked answer sheets and interview guides.

The study found that many students interviewed had difficulties to read certain mathematics text. As they read, the students committed errors such as mispronunciation and repeating of words. It was also found that subtraction word problems were more difficult than addition word problems for students to solve. Strategies used by many students in problem solving were similar though errors were noticed in some cases. The study further revealed that students use local language during small group discussions, and switch to English during a class discussion and when instructed by the teacher.

Based on these findings, the study recommended that apart from having teachers and students mathematics textbooks, there should be also parents' mathematics textbooks. Students' textbooks should include a number of word problems from real life situation which could enhance critical or advanced thinking in them. It is also recommended that students should be allowed to discover and reinvent strategies by themselves than depending on available strategies reflected in mathematics textbooks or on those given by teachers.

In relation to language, the Ministry of Education and other stakeholders should embark on a project of producing students` mathematics textbooks written in regional local language.

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Chapter One

Introduction

In the modern society, there are expectations on schools to ensure that all students have the opportunity to become mathematically literate. In this case, students are considered being literate in that they have an ability to set up problems with an appropriate operation and be able to use a variety of techniques to approach and work on problems, Siegler (1991) argues that children create their own symbolic tools to solve problems as well as using ones given to them. For example 7 to 11 years old have the ability to solve problems using informal knowledge. Informal knowledge refers to untaught methods used to solve a variety of arithmetic problem.

The questions of how children process their thinking in solving problems have been investigated over the years in the mathematics education community. For example, research has shown that children initially solve addition and subtraction word problems by directly representing the action or relationships in the problem (Carpenter & Moser, 1984; Hiebert, 1982; Franke & Carey, 1997). In this case, children`s failure to interpret questions is dependent much on the language employed. Some studies have shown that children who are capable of solving arithmetic problems numerically encounter difficulties in problem solving when the same problems are presented in words (Bernardo 1999).

Bernardo (1999) states that children`s difficulties in problem solving is dependent on the ability to understand the mathematical problem structure that is within the problem text. In his research, Bernardo showed that “children`s failure to solve the problem come from an error in just one of the range of concepts and procedures applied” (P 149). The research conducted among Philipino children showed that children faced challenges in comprehending certain types of texts in word problems (Bernardo 1999).

In the document (educating our future), the Ministry of Education (1996) embarked on development of basic numeracy and problem skills as the priority target for primary mathematics education. The aim was to ensure that “those who leave school are able to function effectively in society, while those who continue in school have an adequate basis for further education” (p. 14). Later, in the Basic School Curriculum Framework (2000a), the Ministry of Education settled for the term essential numeracy, and identified its components. That is, on completion of primary school, a child would understand the meaning of the numbers from zero to one million, thus use the range of numbers to perform the four (4) fundamental arithmetic operations of addition, subtraction, multiplication and division. Since the publication of Educating Our Future document in 1996, very little has been done practically to improve teaching and learning of numeracy problem solving in primary schools (Sampa 2004, Linehan 2005).

I wondered whether there was a connection between primary school children's unsatisfactory performance in numeracy and the dominant teaching methods used at primary level of education. Kelly (1991) argues that the prevailing teaching approaches at all levels of education in Zambia are inflexible and unimaginative, emphasising "factual knowledge and memorization" (p. 33). It is believed that teaching approaches were responsible for the difficulties many school children in Zambia are experiencing in learning and solving of mathematical problems (Kelly, 1991).

Many students at primary and junior secondary level use approaches provided to them by teachers or those reflected in mathematics textbooks to solve problems given. As a teacher and having taught students at junior secondary, I observed that many students use vertical method when given problems on addition and subtraction. In addition, students use tallying, concrete objects and finger counting to solve problems given. When using finger counting, I observed that students count from smaller to larger numbers. For example, to solve 9 plus 5, a student starts counting from 10, 11, 12, 13, 14, 15 and the last finger to be raised is considered as the answer. I also observed that students' strategies become abstract as they advance in education. Language switching was not an exceptional, as students switched between local language and English during group and class discussions.

From these considerations and my experience as a teacher, I developed interest to investigate strategies that fifth graders use in word problem solving, why some word problems are difficult than others and the effect of using local and English in word problem solving. The results of my findings could provide a unifying framework for the development of teachers' knowledge of mathematics and enhance the production as well as improving already existing mathematics resources.

1.1 Purpose of the study

Over the years, there has being a growing number of concerns involving students using rote memory skills despite having the ability to invent strategies to solve mathematical problems. In Zambia, very little is known of students inventing and using strategies in problem solving. Little research has been conducted to ascertain the strategies students use in solving various problems in mathematics and science. It is against this background that this study was conducted to find out whether students could invent strategies and use them to solve word problems, and the challenges they encountered in comprehending certain type of texts in word problems. It is imperative that student's experiences in problem solving are documented to enable stakeholders such as the ministry of education improve existing or produce news one to be used by the Curriculum Development Centre (CDC) for assessment and evaluation purpose.

1.2 Research questions

This study sought to answer the following research questions;

- What are the effects of using students' first and second language in mathematics word problems solving?
- Why are some word problems difficult for children to solve than others?
- What strategies are used by fifth graders to solve addition and subtraction word problems?

1.3 Statement of the problem

Word problems are among the components of primary school curriculum. Research shows that word problems are relatively difficulty for many children at all levels of education, and that children must learn addition and subtraction operations before solving simple word problems (Carpenter, Corbitt, Kepner, Lindquist and Reys, 1980). Solving word problems could be seen as a process of translating words into mathematics expression and then solving the problem. The study by Bebout (1990) showed that children could learn to represent word problems numerically.

1.4 Significance of the study

This study is significant in that it will provide valuable information to interested parties such as the government and agencies such as non – governmental organisation (NGO) who would like to assist by providing necessary materials such as mathematical textbooks which would enhance the use of different strategies to solve mathematics problem. This study will also provide information to scholars who may be interested in carrying further studies related to this area.

1.5 Theory

This study is grounded in (Vygotsky 1978) theory of social constructivism and framework, and also on different researchers and theorist within mathematics education (Cobb, 1994; Baroody, 1984; Groen & Parkman, 1972; Cobb, 1988; von Glasersfeld, 1987). Vygotsky (1978) as the main proponent of social constructivism argued that social constructivists emphasise on social contexts of learning and that knowledge is mutually built and constructed.

Simon (1993) argues that mathematics learning involves the individual construction of knowledge, social construction of knowledge and interaction between two or more individuals. In this case, learners have the ability to construct their own understandings of given situations rather than copying their understandings of others. For instance, research reveals that children have the ability to invent and use strategies to solve addition and subtraction word problems. It is further revealed that children have the ability to create representations of addition and subtraction word problem to be solved (Carpenter & Moser, 1983; Riley, Greeno & Haller, 1983). Jaworski (1988) argues that a child is not merely given knowledge, but actively construct it and that learning or coming to know is a process of adopting one`s view of given situation. For the child to construct meaningful knowledge, s/he must actively strive to make sense of new experiences and in so doing, must relate it to what is already known or believed about a situation.

Research reveals that children develop knowledge through active construction process, not through passive reception of information. In other words, children have the ability to build their own understanding. It is also stated that children solve word problems using concrete objects and hence use these objects to carry out the solution strategy (Briars & Larkin, 1984; Carpenter & Moser, 1984; Riley, Greeno & Haller, 1983). Vygotsky (1978) in constructivist theory argues that emphasis is mainly placed on the learner than on the teacher because it is the learner who interacts with objects and events and there by gains an understanding of the features held by such objects and events.

In short, theories are necessary because they help to understand, communicate and predict the nature of a discipline or a field of practice, it`s purpose, goals and methods. Indeed theories help to shape practice and practice in turn contribute to the development of theories.

1.6 Layout of the thesis

This thesis is divided in seven chapters and arranged as follows;

Chapter 1 introduces the study. It gives the background of the study, including the purpose of study; research questions; statement of the problem, significant of the study and theory.

Chapter 2 gives a general overview on Zambia`s profile, including the education system and research site where the study was conducted.

Chapter 3 gives a review of literature based on the problem under investigation. Its focus is centred around the following main themes; literature of language; literature on word problems and literature on mathematical strategies.

Chapter 4 discusses the research design and methodology of the study. The chapter begins with an introduction of the main aim of the study, which is followed by the research design; research procedure; study population; sample size and sampling procedure; data collection procedure and research instrument. The chapter also includes a description and justification of research instrument under the following themes; semi – structured interview; observation and document review. The chapter further discusses the validity and reliability of the study under the following themes; validity; reliability; limitation of the study and ethical considerations.

Chapter 5 presents the analysis and interpretation relating to students` work. The chapter begins with analysis, followed by interpretation of data. The data is presented qualitatively.

Chapter 6 presents the discussion of the results obtained from students` work and dialogues.

Chapter 7 presents the conclusion and recommendations of the study. It also includes an outline of suggestions for future research.

Zambia`s Profile and Research Site

This chapter presents a brief overview on Zambia's profile, education system and research site where the study was conducted.

2.1 Zambia`s profile

Zambia is one of the sub Saharan countries occupying a land of 753 612 square kilometres. According to 2010 population census, it has population of 13 million people with 39 percent of people living in urban areas and 61 percent living in rural areas. Central Statistics Office, (2010) reports that Lusaka has the largest population with 2 million people followed by copperbelt with 1, 9 million and Eastern province with 1, 7 million. The province with the least population was North – western with 7 hundred thousand followed by Western province with 8 hundred thousand and Luapula province with 9 hundred thousand.

Figure 2-1: Map of Southern Zambia (extracted from map data on 14th February, 2014)

Southern province has a population of 1, 6 million of which 49 percent were males and 51 percent were females. The population increased from 1, 2 million in 2000. Among the districts of the province, Gwembe has the highest population growth rate of 4.4 percent, followed by Siavonga of 4.3 percent and Itezi tezhi and Kalomo with 4.0 percent each. The districts with the least population growth are Choma and Monze with the rate of population growth of 1.8 percent each. Mazabuka district has the highest population (16.3 percent), followed by Kalomo with 15.8, Choma 15.2 and Monze 12.2. Gwembe and Livingstone have the smallest population share of 3.3 and 3.2 percent (Central Statistical Office, 2010).

In Zambian, the major administrative units are provinces, which are subdivided into districts. Hence, within the districts, the administrative units are the chiefdoms and constituencies, followed by the wards. The political system is openly democratic with multi – party elections for the presidency.

Zambia like many poor countries is striving hard to improve its economic well – being. In doing so, the country was boosted by reaching the completion point under the heavily indebted poor countries (HIPC) initiative which also qualified the country in the same year to the group of 8 countries initiative, in which 100 percent of all debts owed to the International Monetary Fund (IMF), the African Development Bank (ADB) and the World Bank (WB) were cancelled. The debt reduction availed more public resources for expenditure on infrastructure development and services. The annual economic growth rate improved between 2006 – 2012 averaging 6.1 percent per annum compared with an average of 4.8 percent between 2002 and 2005.

2.2 Education system

The education system in Zambia is known to consist of 7 years of primary schooling and 5 years of secondary schooling before students enter into universities, colleges or other institution of higher learning. The academic year in Zambia always runs from January to December. It has 3 months of school terms which are broken up a roughly one month holiday prior to each term. This translates to about 40 weeks of school per year.

The minimum entrance age to the first year at a public school as in grade one is 7 years old and education in public schools is free for all. Thus a child is expected to enter his/her first year of secondary school (grade 8) at 14 years old. These are government established standards. Schools, particularly private schools are very liberal in applying them. Their priority is largely on performance of each child. It's thus common to find children of varying age group throughout the schooling years (Ministry of Education, 1996).

Livingstone district in Zambia is located in the southern province. The district has a population of 133 thousand people, 26 government aided schools, 5 private and 2 mission schools, colleges and a private university.

2.3 Research site

This study was conducted at Luangwa Primary school, a government aided school situated on the eastern part of the main town centre. The school was opened in 1952 as a coronation of whites. Later, it opened its doors to children of Indian origin, coloureds and a few blacks from the elite class. After independence, the school changed its name to Luangwa primary taking its name after a local River. 25

years later in 1989, policy on basic education made the school change from primary to basic school and was incorporated in junior secondary that is grades 8 and 9. It had been a basic school until when the school reverted to primary again. Currently enrolment stands at 651 boys and 567 girls giving the total of 1218. 170 girls are orphans and 151 boys are orphaned. 29 girls and 24 boys are sponsored. Vulnerable students constitute about 30% of the school populace adding those that may not be orphans but in various vulnerable situations i.e. unemployment of parents. The school has 15 students with learning disabilities. Staffing is at 41 with only 7 male and the rest are females (Head teacher, 2013).

Literature Review

The purpose of this chapter is to scrutinise other studies in relation to the topic under discussion. The literature under this study is purposely searched and selected on the basis of relevance to the language and mathematics word problem solving.

3.1 Literature on language

Language has been described differently by different researchers. Language is regarded not just a means of communication and thinking, but it is one way in which one can define one`s adherence to group values (Barwell, Barton et al. 2007). In an education setting, teachers' and children' decide about which language to use, how and when.

In relation to the above statement, Kazima (2007) argues that children's interpretation of key vocabulary in the language of the curriculum is influenced by linguistic structure in the students' home language. A research conducted on four bilingual children's use of the home language as they solved mathematics problems showed that all children scored well in the test. Clarkson (1994) has proven that children's use of their home languages can be advantageous in schooling settings. Clarkson further argued that children's competence in their first language does have an effect on their achievement scores in mathematics.

The research by Planas and Setati (2009) showed that children in bilingual class of two languages (Catalan and Spanish) switched language to Spanish and did not go back to Catalan when explaining their solution procedures. When it was time for class discussion, however children contributed in Catalan when asked to do so by the teacher. Spanish was used when involved in discussion, and then returned to Catalan to help each other in the classification of the task and explain the main ideas to the whole class. Setati (2005) argue that children switch languages when working on solution to the problem within their small groups. When it comes to group discussion, they contribute only when they are requested by the teacher and when this happens, they use second language. Stathopoulou and Kalabasis (2007) argue that the use of second language had negative consequences on children who had difficulties in reading and understanding mathematical and other texts.

Several studies conducted previously reveal that many children encounter reading and understanding difficulties in comprehending mathematics texts leading to solution errors (Cummins, Kintsch, Reusser & Weimer, 1988 cited in Bernardo, 1999). The research by Bernardo (1999) involving Filipino –

English bilinguals showed that children were better at comprehending the problem text when it was written in a child's first language. Bernardo argues that children`s difficulties in understanding and solving word problems in mathematics might be as the result of their difficulties to understand English language among other things. It is also argued that children involved on tasks written in their first language had an improvement in their performance than before (Adetula 1990, Bernardo 1999). The study conducted by Riley and Greeno (1988 cited in Bernardo, 2002) showed that difficulties in understanding certain types of word problem texts led to errors in the solution of those types of problems.

Francis (1999 cited in French & Jacquet, 2004) suggested that much of the research supports the view that words in one language and its translation in another language share the same conceptual representation in the mind of the bilingual. Mac Gregor (1990) argues that the reader of a word problem over look searching for key information and the relation of important parts to each other. In mathematical language, words such as minuend and addend must be understood for effective word problem solving. Symbolic language as an area of comprehension needs to be mastered in mathematics by children as problem solvers (Mac Gregor, 1990).

The issue of language and education in Zambia has a long history back in colonial era and much of the federal period. From the 1927 three years after the colonial office took over the responsibility of then Northern Rhodesia and Nyasaland, the policy on education was consistent; mother tongue was used for the first two years of primary education, followed by dominant vernacular up to standard 5 and English thereafter (Linehan 2005).

The period between 1965 and 1995, saw a number of reforms to reverse this "straight – for – English" approach. In two major reviews of education policy, 1977 and again in 1991, were issues of vernacular languages and their role in ensuring quality education was made (Ministry of Education, 1977; p. 32). The policy also allowed teachers to explain concepts that might not understood through the medium of English, in one of the seven local languages, provided that majority of children in a class could understand this vernacular language.

In line with the above information, the Ministry of Education (MoE) has once more reversed its policy on education which recognises the use of familiar Zambian languages as official languages of instruction in pre – schools and early grades (1 – 4). This means that all teaching and learning in all the learning areas including mathematics at low grades are instructed in familiar Zambian languages. There is, however evidence that children learn more easily and successfully through languages that they know and understand well (Ministry of Education, 2012).

3.2 Literature on word problems

This section seeks to review articles based on word problem and mathematics strategies children use in problem solving.

According to Stanic and Kilpatrick (1989), a set of word problems have long been part of mathematics curriculum. "Primarily within the last century, a discussion of teaching word problems has shifted from advocating that children be presented with a word problem or rule of how to solve problems to developing a more general approach to problem solving" (pp.1-22). The Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) showed that the primary goal for children is that they become mathematics solvers and not waiting for hints and solutions be given to them. Hence, word problem solving is a primary goal of all mathematics teaching and an integral part of all mathematical activities.

Researchers argue that word problem solving is considered as a complex process for children at primary level (Sajadi, Amiripour & Malkhlifeh, 2013). It is a complex process in that it requires children to use efficient strategies in solving the problem. One of these strategies is the use of representation, which on the other hand children apply keywords or numbers only when faced with complex word problems. They further argue that word problem as a process, is more complex than simply extracting numbers from a story situation to solve an equation and word problem solving requires knowledge about semantics construction, knowledge of basic numerical skills and strategies (Sajadi, Amiripour et al. 2013).

For instance, Sajadi, Amiripour, and Malkhlifeh (2013) argue that the questions outlined below can be used for the purpose of identifying children's errors when working with word problem solving;

- *Please read this problem for me.*
- *What does the question wants you to solve.*
- *Explain how you got the answer.*
- *Show me how you solved the given problem.*

The study by Schoenfeld (1987) shows that knowledge of cognitive skills help children construct a thinking plan which involves strategy and skill procedures to solve word problem tasks. Lerman (2001) states that "strategies" are mathematical content knowledge which children possess, together with the ability to interpret and comprehend semantics could enable children to successfully comprehend and solve mathematics word problems.

In solving simple addition and subtraction word problems, researchers have indicated that children use a wide range of strategies which includes modelling and counting. For example, children engage vertical and empty number line methods to solve problems given (Carpenter & Moser 1984, De Corte & Verschaffel 1987, Nesher 1982). Researchers further stated that solving addition and subtraction word problems requires the involvement of at least three aspects and these are; representing the word problem situation, choosing a solution strategy and using the solution strategy to find the answer. Initially, children solve word problems using concrete objects and hence use these objects to carry out the solution strategy (Briar & Larkin 1984, Carpenter & Moser 1984, Riley, Greeno & Heller 1983).

Research has also shown that children solve problems by choosing and using arithmetic operation “addition and subtraction,” children use particular methods of adding and subtracting such as “thinking strategies, known facts, or the multidigit addition or subtraction algorithms (Carpenter & Moser 1984).

The study by Veloo (1995) reveals that children solve word problems, firstly by attempting to derive algebraic expressions using the information given in the problem. This is regarded as the most challenging step and many children have little success at this stage. In another study conducted on primary school children showed that children used different representations such as drawings to solve word problems (Lopez – Real, Veloo & Maawiah, 1992). Many researchers have reported that the process of solving word problems could easily be done through translation of information given in the problem statement into a visual diagrammatic form. For instance, Larman (1985) argue that the first step in understanding a problem is by drawing and labelling figures, diagrams or graphs. The study by researchers showed that “children perform better in mathematics word problems when diagrams are used by teachers to elicit appropriate mental images” (Yancey, Thompson & Yancey, 1989; p. 15).

Research shows that children in primary school performed better when asked to draw diagrams before solving the given problem. It is also revealed that children in primary schools draw diagrams that are concrete representations of the problem while children in high schools are more inclined to draw diagrams that are abstract in nature (Lopez & Veloo, 1993; Veloo & Lopez, 1994).

Franke and Carey (1997) argue that children perceived mathematics as a problem solving endeavour in which different strategies were considered useful. In this case, the children accepted a variety of solution strategies, with many children valuing all solutions and assuming a shared responsibility with the teacher for their mathematics learning. Children, however, begin to develop an intuitive idea of adding and subtracting as early as the age of 2 or 3 (Ginsburg & Baron, 1993). The study by Gelman and Gallistel (1978) indicate that a child aged 2 to 3 years improved an intuitive notion of things being added or taken away. By the age of 4, children begin to calculate in concrete addition situation (Ginsburg & Baron, 1993).

The study conducted by Ginsburg and Baron (1993) reveal that children usually interpreted addition as the act of combining and counting separate sets. In this case, addition involved operating on sets in order to get a new set. As children grow up, they develop efficient approaches to calculations (Groen & Resnick, 1977).

Gerofsky (1996) states that word problems are one component of the school curriculum. They require the integration of linguistic and arithmetic processing skills. In word problems, situations are described in which there is some modification, exchange, or combination of quantities. Research by Stockdale (1991) has shown that some word problems are difficult for students to solve while some are easy

Gooding (2009) argue that some word problems are difficult to solve because children are not able to interpret some texts used in a word problem, difficulties in comprehending a sentence, not able to understand specific vocabulary and lack of confidence or ability to concentrate when reading. Researchers indicate that difficulties arise more especially when children cannot imagine a context in which a word problem is prepared or children`s approach is modified by the context in which the word problem is given (Calwell & Goldin, 1979; Nunes, 1993). Many children find it harder to write or form number sentences of word problems structures than others. English (1998) stated that difficulties arise due to inability by children to choose a suitable computation strategy for the problem given or selecting an incorrect strategy. Research also shows that difficulties could occur depending on the choice of computation strategy, the context in which the word problem is set and the size of numbers involved could have a negative impact on children`s choice of computation strategies (Verschaffel, De Corte & Vierstraete 1999; Nunes, 1993; Anghileri, 2001).

Researchers have identified three types of word problems that represent addition and subtraction and these are shown in the table 1 below (Carpenter, Herbert & Moser, 1981).

Problem type	Description
Addition	
Combine:	In combine problems, there are two distinct amounts, considered in combination, for instance; *"James has 3 marbles and Joe has 5 marbles, how many marbles do they have altogether?"*
Compare:	Compare problems involve two compared quantities and the difference between them, as in this problem; *"Joe has 3 marbles; James has 5 more marbles than Joe. How many marbles does James have?"*
Change:	Join problems relate to situations in which one set is joined to another. For example; *"James has 3 marbles; Joe gives him 5 more marbles. How many marbles does James have now?"*
Subtraction	
Combine:	In combine problems, there are two distinct amounts, considered separately, as in the following example; *"Joe has 8 marbles, 5 are blue and the rest are green. How many green marbles does Joe have?"*
Compare:	In compare problems, two quantities are compared in order to find out how much greater one quantity is than another. For instance; *"James has 8 marbles, Joe has 5 marbles. How many more marbles does James have than Joe?"*
Change:	Join problems relate to situations in which some events increase or decrease the value of a quantity, for example; *"Joe has 5 marbles, how many more marbles does he need to have a total of 8 marbles altogether?"*

Table 1: Description of each type of word problem

The Change, Combine and Compare are characterisation of most addition and subtraction word problems presented in mathematics. Change problems usually begin with an initial followed by an action to cause either an increase or decrease in the quantity. In the word problem "James has 3 marbles; Joe gives him 5 more marbles. How many marbles does James have now?", "James has 3 marbles" is the initial, "Joe gives him 5 more marbles" is the action. In this case, the initial and the action produce an increase. The object (e.g. marbles) is the same among the three sets in Change situation. Combine problems involves two distinct subsets that combines to give a new set. For example; "James has 3 marbles and Joe has 5 marbles, how many marbles do they have altogether?" the 3 marbles combines with 5 marbles to give 8 marbles which is the new set. Combine problems require an understanding of part-part-whole relations (Jitendra, Griffin, Buchman & Sczesniak; 2007). Compare problems involve the comparison of two separate set (compared and referent). For each of the problem type, the position of the unknown may be any one of the three items, which can be found if the other two items are given (Jitendra, Griffin, Buchman & Sczesniak; 2007).

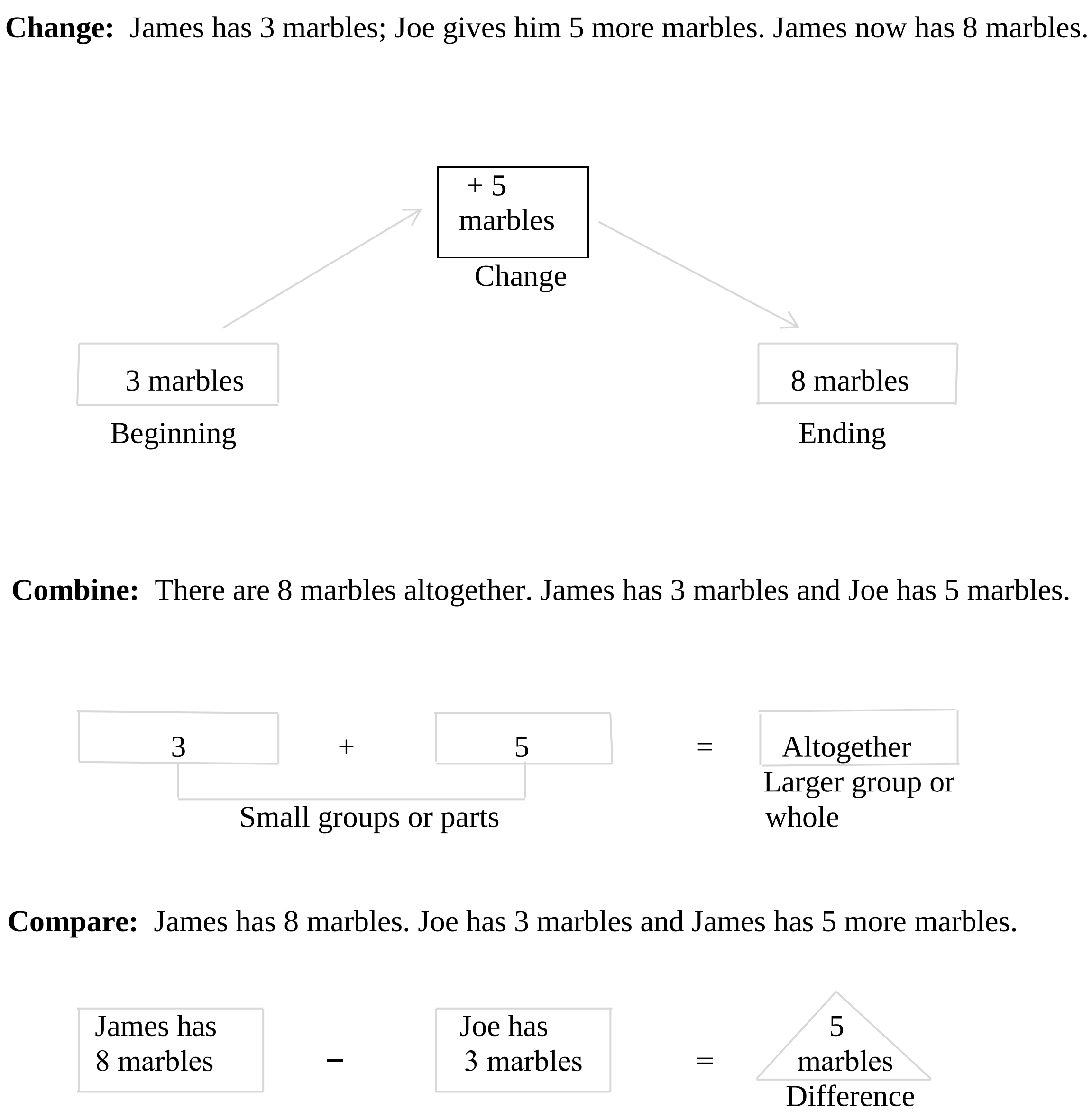

Figure 3-1: Schematic diagram for Change, Combine and Compare (Jitendra, Griffin, Buchman & Sczesniak; 2007).

Bebout (1990) investigated informal strategies for solving addition and subtraction word problems among children at primary level. The children under investigation were taught to write open number sentences symbolically presenting the structure of eight types of change and combine word problems. The sentence writing texts were administered to children before and immediately after instruction to determine children`s choice of strategy to be used in solving word problems. The result of this investigation showed that children were successful in learning to solve symbolic representation actions of change and combine problems, and this was due to instructions designed to link children's informal insights into problem structure with open number sentence forms that reflect problem structure.

In problem solving, children employ reading, language comprehension and mathematical computation skills, which happen simultaneously (Reutzel, 1983). Researchers such as Pellegrino and Goldman (1987) indicate that students use the following steps when solving word problems; reading, selecting necessary computation strategies, deciding what information to be manipulated and deciding the appropriate units for the solution. Some children find it easy to use the mentioned steps in problem solving while others have difficulties in approaching word problem tasks (Karrison & Carol, 1991). The other reason which leads to difficulties in problem solving is that children develop fear and anxiety when faced with word problems hence find it difficult to solve the problems given. Many more studies reveal that addition and subtraction of word problems provide a reasonably coherent view of how students solve addition and subtraction word problems (e.g. Bebout, 1990; Carpenter & Moser, 1984).

Furthermore, studies indicate that children solved a wide range of problems including multiplication and division situations using different strategies and hence making a remarkable success in solving word problems. The results also show that instructions encourage the use of direct modelling to solve problems (Carpenter, Ansell, Franke, Fennema & Weisbeck, 1993).

Carpenter, et al (1981) conducted another study among primary school children and their focus was based on how success the children were at solving different types of addition and subtraction word problems prior to formal instruction. The study focused on strategies children used to solve word problems and factors that lead to the selection of these strategies. The study involved 43 children from Parochial school. At the time children were tested, no formal instruction in symbolic representation of addition and subtraction was given. The children were asked to solve 10 different addition and subtraction word problems and hence were interviewed to identify the strategies used to solve each of the problems. In addition, they were asked to describe how the answer was found. When the response was unclear, the researcher continued questioning until it was clear what strategy the children used.

The result of the study showed that children used three basic counting models identified by Groen and Parkman (1972) in solving addition problems. The three strategies were counting all, counting on from the smaller number and counting on from the larger number. Counting all strategies can be carried on by the use of fingers, sticks, pencils, cubes or by counting mentally. The counting on from the smaller number may be done mentally or using cubes or fingers to keep track of the number of steps in the counting sequence. It is also revealed that children used several strategies that were not based on counting to solve addition problems.

The children were not as successful with subtraction problems as compared to addition problems. The results from this study indicated that word problems might be an appropriate context in which to introduce addition and subtraction operations. Word problems provided different interpretations that are important for children to understand. In short, the researchers suggested that by introducing operations based on verbal problems and integrating verbal problems throughout mathematics curriculum, children would develop the ability to analyse problem structure and would develop conceptions of basic operations.

3.3 Literature on mathematical strategies

3.3.1. Counting strategies used by children

This section discusses counting strategies children employ in solving addition and subtraction problems. Much of the discussion is anchored to previous research conducted on the same study.

When children begin solving addition and subtraction word problems, they seem to create representations of the problem. Most if not all children use their fingers, physical objects or counters to represent each quantity in the problem given. However, through interviews and taking to children about the problem, the researchers found that children strategies become abstract as they engage in advanced or abstract thinking. On the other hand, they tend to use advanced counting strategies such as “counting on” or “counting back” (Carpenter and Moser, 1983; Riley, Greeno & Heller, 1983).

Ma (2011) argues that “counting counters” approach is based on helping the children to easily find solutions to addition and subtraction problems by counting counters. Beside children`s own fingers, there are other types of counters which can be used such as; bottle tops, smaller pieces of sticks (approximately 15cm long), smaller stones and blocks of different sizes (ref figure 3-2 below). Also dots on a number line and tallies are often used as counters.

Figure 3-2: Different counting counters

In line with other studies carried out in this field, the process of addition strategies moves from counting all, to counting on from first, then to counting larger. For instance, to compute 3 + 4 by counting all, firstly shows two fingers and also four fingers, and then count all the fingers to get the sum. It is also mentioned that, counting from 1 is usually considered as more advanced strategy. However, a child computing 3 + 4 begins the counting at 3 and then continues for four counts instead of starting at 2 counting. Another strategy which is considered as even more advanced is counting on from larger numbers. For example, a child calculating 3 + 4 begins counting sequence at four and continues on for 3 counts. Counting all strategy can only be used when solving addition problems within 10, since we have only 10 fingers. Additionally, there are also counting strategy for subtraction counting down from, counting down to and counting up from given (Chapin & Johnson, 2006 cited in Ma, 2011).

The study conducted by Fuson and Kwon (1992) on the ability to solve word problems among the Korean children focused on addition problems with the sum of between 10 and 18 and also single digit subtraction problems with minuends between 10 and 18. In this study, six children were randomly selected and interviewed. The results showed that the children used fingers to show sums. Finger usage was important for understanding of possible developmental sequences of solution procedures of Korean children. Besides, the results also suggest that Korean first graders showed remarkable competence in solving more difficult single digit addition and subtraction combinations with sum between 10 and 18. In addition, most of the solution procedures the children used were considered advanced solutions that involved known facts or decomposition method structure around ten.

In another study, researchers identified four kinds of informal methods children seem to use to solve problems involving double – digit whole number addition and subtraction and these are; beginning with one number and moving up or down by tens and ones, decomposing tens and ones, and adding or subtracting the tens and ones separately, changing both numbers and lastly mixing the above strategies. Additionally, using the first strategy to find the value of 21 + 33 involves beginning with 21 and count up in sequences of tens and ones (i.e. 21, 41, 51) to arrive at 54. The subtraction 55 – 33 would be done in a similar way, but by counting – down from 55 (i.e. 55, 45, 35, 25, 22) until one reaches 22. Using the second method, one way to obtain the sum 21 + 33 would be to write 20 + 30 = 50 and 1 + 3 = 4, then adding up 50 + 4 gives the answer 54 (Fuson, Wearne, Hiebert, Murray, Human, Olivier, Carpenter & Fennema, 1997).

The study conducted by Ginsburg and Russell (1981) shows that children were able to solve a problem in which three objects in one set were to be added to four objects in the other set. Children did their computation accurately by means of the strategy "counting all." In this case, children normally interpret addition as the act of combining separate sets. As children grow, they seem to develop advanced approaches to computation (Groen & Resnick, 1977). Additionally, children abandon counting all, for easier strategies namely counting on which is from the larger number. As more advanced strategies are invented and used, concrete materials or supports are no longer important at this level, meaning that children are capable to calculate mentally, for instance; they can add 3 + 4 in their head. Nevertheless, this is usually accomplished through the use of clear mental imagery. This means that they can picture 3 objects and 4 in their minds and counts the sum of images, and this done from the larger numbers (Ginsburg & Baron, 1993).

Researchers argue that children's basic understanding of addition and subtraction depends much on their experience with early counting. Several studies also indicate that children solve addition problems using the strategy known as counting all. For instance, grouping two sets together and using fingers or concrete objects to count all items, they would start by counting the elements in the first set and finishing with those in the second one (Baroody & Standifer, 1993; Carpenter & Moser, 1983; Hugher, 1986; Nunes & Bryant, 1996). As children gain experience, they are seen using more efficient strategies known as counting – on. Beginning with the total of one set, usually the larger, then count on from there to attain the sum of the two sets. In the study by Carpenter and Moser (1983), despite children`s use of the above strategies over a long period of time, suddenly they begin solving addition problems using number combinations than counting. For instance; to solve 9 + 5, children would first find the more familiar sum 5 + 5 = 10, and then add 4.

Additionally, similar strategy is also used when children solve subtraction problems (Baroody, 1984; Baroody & Standifer, 1993; Carpenter & Moser, 1983). For example; James has 3 marbles; Joe gives him 5 more marbles. How many marbles does James have now? To solve this problem children depend on a number of strategies, including counting down and counting up (Baroody, 1984; Baroody & Standifer, 1993; Carpenter & Moser, 1983).

Baroody (1984) argues that separating from meanings removing a required number of elements from a larger set and then counting the remainder in order to find the answer. Additionally, counting down is also similar to separating from, in that it depends on children understanding of subtraction as taking away. However, this involves starting with the initial set, counting backward a number of steps equal to the amount to be taken away and hence announcing the last number counted as the amount remained (Baroody & Standifer, 1993).

In problem solving, children find counting backwards more of a challenge than that of counting forward as they begin to work with larger numbers in problem solving of subtraction. In this case, they engage in using the count up strategy (Baroody, 1984).

By counting up, it is the process of counting starting with a small number for instance, 3 in 7 + 3 and counting forward to reach the larger number though bearing in mind the number of steps in the forward count (Baroody & Standifer, 1993). This information indicate that children seem to use number combination than counting in solving subtraction problems though from earlier studies, it is explained that number combinations are often based on addition than subtraction (Carpenter & Moser, 1983).

3.3.2. Empty number line and place value system

The number line is regarded as an instrument which is commonly used in primary schools and often examined. As a tool, the number line supports the development of conceptual understanding and a measure of mathematical understanding. According to Fuson (1984), the number line is regarded as a measurement model, rather than a counting model, numbers on the number line are representations of the length rather than simply the points they label. To determine the position of unknown, the proximity of unknown from the known is important and is considered. On the other side, the number lines have the capacity to concretise mathematical operations. Davis and Simmt (2003) reported that the student interpreted 3 x – 4 as “three hops of lengths four” along a number line and concluded that the student`s concept of multiplication as repeated addition was combined with the concept of multiplication as movement along a number line (158). An empty number line is regarded as a new didactic model and considered as a very powerful model for the learning of addition and subtraction up to 100 (Klein et al, 1998; p. 443 cited in Diezmann et al, 2006).

The research by Gravemeijer (1994) shows that structural number line with markings on it was mainly used as a counting strategy by children. Gravemeijer argues that the structured number line was introduced with the so purpose of being used as a measurement tool and was similar to the rule with fixed distances. He stated that the structured number line could be used for counting and passive when it comes to reading of answers on the number line, as such lowering the level of strategies which the children are expected to use in problem solving.

Treffers (1991) suggests that the empty number line would be an alternative on which children draw the marks for themselves and in the process use it to solve problems given. The use of the empty number line could help the children extend their counting strategies to more abstract levels from counting by ones to multiples of tens. For instance, to solve 38 + 25, the following strategies were used to find the solution;

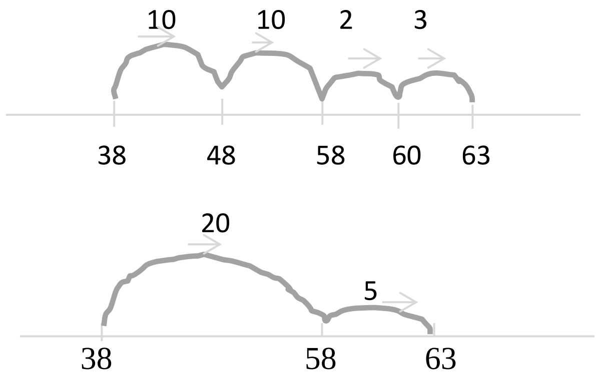

Figure 3-3: empty number line used to solve 38 + 25
(Klein & Beishuizen, 1998)

In realistic mathematics education, the use of empty number line would not only enable children find answers, but would enhance the development of more abstract strategies. Gravemeijer further stated that the empty number line does not only allow children to present their solutions, but to facilitate their procedures. Many children using the number line on addition and subtraction are seen drawing `jumps` on it (Baroody, 1987). It is, however mentioned that the development of the empty number line was to help overcome challenges children encounter connected to standard written algorithms (Gravemeijer, 1994). The research conducted by Treffers (1991) and Beishuizen (2001) shows that children`s use of the empty number line was a success as they were able to record and make sense of different solution strategies for multiple digits of addition and subtraction problems. The result of their study also shows that using the empty number line, children explained their strategies by showing them to others. This made the number line a powerful tool for enhancing communication in the classroom (Treffers, 1991; Beishuizen, 2001).

Place value system is one of the components of mathematics curriculum and taught at primary level of education. It is introduced in grade 2 and children at that level are expected to identify place values of digits in given numbers and then add by regrouping ones, tens, hundreds and thousands (Ministry of Education, 2012). Many children have difficulty to comprehend place value system despite being taught the system.

Kamii (1986) argues that children need to understand the relations that exist among powers of 10 in order for them to easily comprehend the place value system. Kamii also emphases on grouping and regrouping, which in this case children, need to understand different ways in which same quantity could be split in subsets or grouped. For instance, 17 could be seventeen 1s or as one ten and seven 1s. The emphasis is also extended to the vertical computation procedure, where carrying in addition involves formation of a new group of 10. For example, 35 and 19 in place value system involves understanding that 14 is as a result of adding 5 plus 9, which in this case is composed of one group of ten and four units. This group of ten is then “carried over” and added to the other group of 10. This situation is similar to that of borrowing in subtraction which involves breaking up of a group of 10 that already exist (Varelas & Becker, 1997). Ross (1989) states that “children need to engage in problem solving that challenge them to think about useful ways to partition and compose numbers” (p. 50).

Researchers argue that children could have conceptual understanding about the quantities and about the operations which includes grouping and regrouping but still encounter difficulties with place value system. Research shows that “children without school instruction, develop an understanding of grouping and regrouping” (Varelas and Becker, 1997). Children use regrouping procedures when solving problems on addition and subtraction. The same procedure of regrouping could also be used to solve multiplication and division problems. Research shows that children could use base ten blocks to represent numbers and to solve the problem using grouping and regrouping, but still fail to perform well in place value computation (Varelas and Becker, 1997). Therefore, children need to have an understanding of tens and ones for them to effectively solve problems on addition and subtraction.

3.4 Summary of literature review

This chapter reviewed literature on language which children use when solving problems in mathematics. The discussion also included mathematics word problems and strategies use to solve word problem.

The studies reviewed have indicated that using local language in mathematics word problems could improve their performance in mathematics than when using second language. The studies by different researchers showed that children switch language when working on solution to the task within their small groups. When it comes to group discussion, children contribute only when they are requested by the teacher and when this happens, children use second language. There is evidence that the use of local language could improve children`s performance in mathematics.

Furthermore, the studies have shown that many children use counting strategies when solving problems on addition and subtraction. Children use counters such as small stones, sticks and bottle tops. Many of the children use fingers during problem solving. The counting counter approach is based on helping children to easily find solutions to addition and subtraction by counting counters.

The studies also indicated that children use different computation procedures to solve problems. The most common used strategies are the empty number line and vertical method. The studies show that the use of empty number line could help children extend their counting strategies to more abstract levels from counting by one to multiples of tens. Children use “jumps” when using an empty number line to add and subtract. The vertical or horizontal method is used concurrently with the place value system in which children identify place values of digits in given numbers and then add by regrouping ones, tens, hundreds and thousands. The situation is similar to that of borrowing in subtraction which involves breaking up of a group of 10 that already exist.

This study was limited in that it was mainly based on research conducted outside Zambia. For example, it is clear from the report that Philipino children whose mathematics tasks were written in their local language had a gain in their performance than those whose tasks written using second language. This situation can be applicable to Zambian education system. In view of this, it is important that stakeholders in Zambia should venture on projects which aim at producing mathematics materials such as textbooks written in local language.

Chapter Four

Research Design and Methodology

4.1 Introduction

The aim of this chapter is to describe the research design and methods that were used in the study, and also give reasons why the chosen methods were appropriate to answer the research questions. In this case, the chapter pays attention on qualitative research approach, the population, the sample size, the sampling procedure, the instruments for data collection, data collection techniques and the aspects of validity and reliability of research.

4.2 Research design

The research design involves deciding what the research purpose and questions will be and what information is needed to answer a particular research question and what strategies are most appropriate for obtaining it (LeCompte & Preissle, 1993; 30). Gay (1996; 218) argues that the design of a study "is basically the overall approach used to investigate the problem of interest, i.e.; to answer the question of interest. It includes the method of data collection and related specific strategies." Muzumara (1998; 46) defines research design as; plan or procedures by which an investigator intends to answer research questions. The design is tailored to control errors of procedures and interpretations, the structure of the design delimits, the kind of observations which can be made, the persons from whom data can be collected, and the kind of analysis it is possible to make within the form of data.

There are a number of qualitative research designs and methodologies such as the case study, ethnography and ground theory study. For data collection purpose, the researcher decided to employ a qualitative case study in which 39 students were picked for testing and 5 were selected for interview.

The research design did not provide the researcher with a very rigid step by step plan; instead the researcher's choice and action during research refined the design. It is also confirmed by Leedy and Ormrod (2005; 134), who echoed that initially a qualitative researcher may select only general approach suitable for the study, e.g. case study. As a researcher learns more about what is being studied, he or she specifies the methods to be used.

Qualitative case study is an approach that facilitates exploration of a phenomenon within its context using a variety of data sources. It ensures that the issue is not explored through one lens but rather a variety of lenses which allows for multiple facets of the phenomenon to be revealed and understood.

There are two key approaches that guide case study methodology; one proposed by Stake (1995) and second by Yin (2003). Both seek to ensure that the topic of interest is well explored and that the essence of the phenomenon is revealed. Stake (1995) and Yin (2003) based their approach to case study on a constructivist paradigm. Constructivists claim that truth is relative and that it is dependent on one's perspective. This paradigm "recognizes the importance of the subject human creation of meaning, but doesn't reject outright some notion of objectivity (Miller & Crabtree, 1999, p. 10 cited in Baxter & Jack; 2008, pp. 544 – 545).

Constructivism is built upon the premise of a social construction of reality. One of the advantages of this approach is the close collaboration between the researcher and the participant, while enabling participants to tell their stories (Baxter & Jack, 2008). Through these stories, the participants are able to describe their views of reality and this enables the researcher to better understand the participants` actions (Lather, 1992; Robottom and Hart, 1993 cited in Baxter et al; 2008, pp. 544 – 545). Additionally, qualitative researchers view the world that aspects of the human environment are constructed by the individuals who participate in the environment and social reality exist only according to the meanings that individuals give them.

My interest was to find out how children solve mathematics word problems by investigating the effects of using children's second language (English) on addition and subtraction word problems, and the strategies children use to solve problems given. In order to do that, there was need to triangulate otherwise the findings could have been superficial. In this case, triangulation is the use of more than one method in order to understand the same reality. In research, no single method ever adequately solves the problem, because each method reveals different aspects of empirical reality, multiple methods of observation must be employed. This is termed as triangulation, the final mathematical principle that multiple methods should be used in every investigation.

With this in mind, the purpose of the study was to find the effect language has on word problem solving and the mathematical strategies students invent. I saw the need of confirming some of the ideas and views from children by using qualitative case study design.

4.3 Research procedure

All the research questions were answered qualitatively by the use of mathematics word problem test and interviews. Because of the nature of the research topic, the same research questions were analysed qualitatively by the use of themes.

4.4 Study population

A population is a group of elements or cases, whether individuals, objects or events that conform to particular criteria and to which we intend to generalize the results of research (McMillian & Schumacher 2001: 169). Parahoo (1997: 218) also describes a study population as a total number of units from which data can potentially be corrected. The units may be individuals, organisations, or events. The nature of the problem concerning mathematics word problem solving made it necessary to focus on the following unit; thirty nine grade 5 students at one of the primary schools in Livingstone aged between 10 to 13 years. For this age range, students are likely to invent mathematical strategies and solve word problems.

4.5 Sample size and sampling procedure

The class of 39 fifth graders (21 boys 18 girls) was sampled from a total of 5 grade five classes. The sampling procedure of a class of 39 students was facilitated by the school administration in consultation with the class teacher. The students sampled participated in a test comprising of 10 word problems.

Five students were then selected for interviews. The selection criterion of the five was based on their marked answer sheets. At the time of the interviews, the students had received instruction on how the interviews were to be conducted. Individual answer sheets were used during the interview as students gave an explanation of procedures which led to solutions obtained on 10 word problems tested. The interviewer tailored his own set of questions which were used in conducting the interview.

Table 2 below shows the details of students who took part in an interview and they names are synonymous.

Student no	Name	Gender	Age
1	S1	Female	10
2	S2	Male	12
3	S3	Male	13
4	S4	Female	12
5	S5	Female	11

Table 2: Details of participants at the time of interview

White (2005: 252) defines a sample as a “group of subjects selected from a larger population”. According to Leedy and Ormrod (2005: 133), the particular entities which qualitative researchers select comprise their sample and the process of choosing them is called sampling. McMillan and Schumacher

(2001: 319) state that "the power and logic of purposive sampling is that a few cases studied in depth yield many insights about the topic, whereas the logic of sampling depends on selecting a random sample for generalisation to a larger population."

The sample was randomly sampled because during the pilot study, it was established that students had difficulties in understanding and solving word problem questions. Additionally, the method enabled the researcher to randomly select students whom he thought had necessary information needed to answer the questions. Cohen and Manion (1994, p. 89) point out that in random sampling, "researchers handpick the cases to be included in the sample on the basis of their judgement of their typicality". In this way, they build up a sample that is satisfactory to their specific needs. It is in this random sampling that the researcher found to be important in selecting students due to the information they possessed and not by the number of students.

4.6 Data collection procedure

Data collection refers to the gathering of information to serve or prove some facts. Merriam (1998 cited in Chikuya 2007; 93) argues that case studies are normally of qualitative nature, it is logical to utilize qualitative data collection methods in a research. In this study, I used testing, interviews and observation as data collection strategies. I chose these three techniques of collecting data because they were likely to yield information about mathematical strategies used in problem solving. According to Dayman and Hollway (2002 cited in Ibrahim 2006; 64), the combination of interviews, observation and analysis as qualitative data collecting techniques are likely to yield the most needed information about the topic under investigation. Observation and interviews are the most common data collection methods in qualitative studies.

For data collection purpose, consent was sought from the head teacher of the school to carry out this research. Information needed was acquired from primary source. Primary information was gathered by means of empirical study. 39 students took part in test. The tasks were modified to suit the Zambian situation.

The 10 tasks used in the test were divided in three major categories; change, compare and combine (see table 3 below). Some tasks on; change, compare and combine are basically additive. In change 2, combine 8, 9 and 10, initial quantities gets some more added to it so that the two separate quantities are put together to form one combined quantity. In the compare 5, two quantities are compared in order to find out how much greater one quantity is than the other. The other five tasks are basically subtractive and these are change 1, 3 and 4, compare 6 and 7.

Three of these tasks were constructed to represent the semantic category of Van De Walle (2003): Change 1, 3 and 4. Change 2 and Combine 9 are based on the semantic structure of Neshar, Greeno and Riley (1982). Compare 5 and 6 are from Greeno and Kintsch (1985). Compare 7, Combine 8 and 10 are from Carpenter, Herbert & Moser, (1981).

Task	Problem
Change 1 (Van De Walle, J.A; 2003)	Everlyn has 144 spaces in her photo album. So far, she has placed 89 photos in the album. How many more photos can she put in before the album is full?
Change 2 (Neshar, P. et al; 1982)	Mary has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Mary have altogether?
Change 3 (Van De Walle, 2003)	Mulenga counted all of the teacher's pencils. Some were sharpened and some not. She counted 73 pencils in all; 46 pencils were not sharpened. How many were sharpened?
Change 4 (Van De Walle, 2003)	There were 73 students on the playground. The 46 fifth grade students came in first. How many students were still on the playground?
Compare 5 (Greeno & Kintsch, 1985)	Chola has 29 marbles. If she wins another 14 marbles, she will have the same number Sam. How many marbles does Sam have?
Compare 6 (Greeno & Kintsch 1985)	Mark has 7 mice. Ruth has 15 mice. Ruth has how many more mice than Mark?
Compare 7 (Capenter et al, 1981)	A man had 26 cars at a car sale. This was 17 more than his best friend Tim. How many did Tim have?
Combine 8 (Carpenter et al, 1981)	Seventy – two people came to the school play on Monday. Twenty – six more people attend it on Tuesday. How many people went to the play on Tuesday?
Combine 9 (Neshar et al, 1982)	There are 28 girls and 35 boys on the school playground. How many students were there on the school playground?
Combine 10 (Carpenter et al, 2010)	Richard was on page 47 of his book. Then he read 8 more pages. What page did he end up on?

Table 3: Selected tasks representing semantics categories

The students were tested as a class by the class teacher in the presence of the researcher during school hours. They wrote their names and birthdays on the answer sheet. The students were then informed that the test would last for sixty minutes and involved ten word problem solving.

They were also told that the questions would be read to them and hence need to pay attention. Additionally, the students were asked to compute solutions on the answer sheet with their names.

The teacher began testing, and as soon as the students indicated that they had finished, the answer scripts were collected by the researcher for confidentiality purpose. Having gone through and marked the scripts, five students were selected randomly for interview. Each student was given a consent letter and parents acknowledged students participation in an interview. Each interview began with the reading of questions by each interviewee, and then proceeded with explanations of procedures which led to the solutions obtained in the test.

The entire session was audiotaped with the consent of all the informants. Besides, they were told that only the researcher would hear the sounds and that their teachers and parents would not. After the session, students were given a small gift of appreciation for the time spent in this study while their friends were learning in class.

Students were assured that the information obtained would be treated confidentially and that results would be for analysis purposes only. This is a reason why in data presentation section, only pseudonyms are used.

4.7 Research instrument

Data was collected using semi – structured interview, observation and documentation. A test of 10 questions was prepared for 39 students. The major reason for using these instruments is that a test would effectively cater for large sample of respondents whereas semi – structured interview provided an in depth collection of information. Document showing the performance of students in a test was used for analysis. All the three instruments were used to answer the three research questions which sought to investigate the effect of language on word problem solving and the mathematical strategies students invented.

4.8 Description and justification of research instruments

4.8.1. Semi – structured interview

Cohen and Manion (1997 cited in Muzumara, 1998: 51), define an in – depth semi – structured individual interview as "a two person conversation initiated by the interviewer, for the specific purpose of obtaining research, relevant information as specified by research objectives of systematic description, or explanation.

White (2003: 76) has identified the following advantages of semi – structured interviews;

- *The interviewer is present to observe non – verbal behaviour and to assess the validity of the interviewee's answers.*
- *The interviewer can make sure that the interview is conducted in privacy, that there is no noise.*
- *The interviewer can ensure that the interviewee does not answer the question out of order.*

- *Spontaneous answers may be more informative than answers about which the interviewee has had time to think.*
- *The interviewer can make sure that all questions are answered.*

In research, one way of learning about things we cannot directly observe is through asking relevant questions to people. By asking people to tell us what they are experiencing, as listeners, we begin to understand the world of the other person in a deeper way. This process of understanding another person`s world view can be achieved by the use of interviews. However, interviews were used by the researcher in mathematics to collect much needed information in order to get an in depth meaning and understanding of the effect of first and second language as well as strategies used in mathematics word problem solving by the informants.

To collect data using this instrument, I recorded sounds, took pictures of informants work and wrote down the most important notes that correspond with specific research question. Questions to guide the interview were constructed by the researcher.

4.8.2. Observation

According to Leedy and Ormrod (2005: 145), observation in a qualitative study are intentionally unstructured and free – flowing, meaning that the researcher is free to shift focus from one thing to another as new and potentially significant objects. The advantage of collecting data through observation is that the researcher gathers data from various sources. However, inexperienced researcher may waste time observing things that are not important, overlooking those that are centred to the question. The researcher was aware of this disadvantage and hence concentrated on observing things that were directly related to mathematics word problem solving.

Whilst gathering data using the observation technique, the researcher adhered to the following suggestions made by Leedy and Ormrod (2005: 145 – 146);

- The researcher used various data recording strategies, for instance; audio/video recording and photographing.
- When beginning the observations, the researcher was introduced to the students whose activities were to be observed and got participants consents.
- During the observation, the researcher remained relatively quiet and friendly to students.

Observation can be participant or non – participant. In participant observation, the observer works his or her way into the group he or she is to observe so that, as a regular member, he or she is no longer regarded as an outsider against any student who needs attention (Sidhu 2003: 163). In this study, the researcher was an observer and observed the way the participants behaved during the test and the kind of mathematical strategies they used to solve word problems.

In short, observation is a technique for gathering data that are almost impossible to obtain with other methods. Researchers observe and record information relevant to the research questions (McMillan and Schumacher 2006: 348).

4.9. Validity and reliability of the study

This section of the study deals with validity and reliability of the study. The researcher has decided to discuss them in order to determine whether the study is in line with the above two criteria of study.

4.9.1. Validity

Validity of a qualitative design refers to the degree to which the interpretations have mutual meanings for the participant and the researcher. These people should agree on the description and interpretation of the events being discussed (McMillan and Schumacher, 2006: 324). Cohen and Manion (1994) argue that the most important quality of any research in the validity or extent to which an instrument measures what it supposed to measure. In a qualitative study, a researcher is also an instrument. Hence the definition of validity cited above is applicable to this study.

4.9.2. Reliability

Many qualitative researchers regard reliability as the elimination of casual errors that can influence results. Reliability can be classified into two parts, internal and external reliability. Internal reliability is achieved during the study through triangulation, cross examination, careful selection, by reaching consensus regarding the findings with the participants and using audio/video recordings to store information and computers for processing of data, while external reliability refers to the verification of the findings of the research, when the same research is conducted by an independent researcher under the same circumstances and using the same participants (White 2005: 201).

According to Muzumara (1998: 49), reliability refers to the consistency between independent measurements of the same phenomenon. However, the same methods used by different researchers at different times under similar conditions should yield the same results.

In regard to my study, the reliability of mathematical strategies and problem solving were based on the responses from the five participants, 2 boys and 3 girls from one primary school. The differences in the responses were very minimal.

4.10. Limitation of the study

There are three main challenges faced during the research. Firstly, 3 weeks were spent for data collection instead of 4 weeks as earlier planned. This was due to local and national examinations which took place within the school, and hence all students in the school were busy with these examinations. However, to successfully collect much needed data within 3 weeks, I spent more than 1 hour interviewing each student in a day.

Secondly, there was insufficient room to be used for the study as most of the rooms were used for examinations. At first, I was allowed to use a grade 5 class just for a day, and then moved to the library. The library was later found not suitable for interviews because many students were studying and this situation prompted the administration to allocate me to the mathematics department where the interviews were successfully conducted.

The last limitation of the present study was late coming by students selected for interviews. Late coming, however was due to the distance some student had to cover before reaching the school. Furthermore, it was the start of the rainy season and most of them had to wait until rain fall reduces before starting off for school. In line with the above challenge, time for interviews was changed from 12.30 hours to 13.30 hours to accommodate students from far places.

The research was concluded in the first week of December as students went on break as in line with the school calendar.

A great effort was made to ensure that enough data was collected before the school went on break.

4.11. Ethical considerations

My study involved fifth graders aged 9 – 11. Therefore, I anticipated my research not to go without ethical issues in relation to access, confidentiality and privacy, informed consent and distress.

In order to conduct a research, I involved and co – operated the “gatekeepers” (Muzumara, 1998), argues that access to students might be gained through gatekeepers that is, those who control access. In this case, access to students was gained through the institution where i conducted my study. I visited the institution with an introduction letter from Høgskolen I Sør Trøndelag were permission was granted (informed consent) to gain an access to students and the institution. I also had consent from parents/guardians of the students who were my informant.

I explained to my participants the purpose of the study, what was involved and their right to withdraw from the research when they felt like not continuing to allow them make their own decision either to participate or not.

Analysis and Interpretation

The first part of this chapter qualitatively discusses 39 students` performance in each problem solved in the test and the second part is based on five students interviewed.

Table 4 below shows problem type and 39 students` performance in each problem.

Problem type	Correct answers	Wrong answers	No attempt
Problem 1 (subtraction)	7	29	3
Problem 2 (addition)	17	19	3
Problem 3 (subtraction)	8	26	5
Problem 4 (subtraction)	12	24	3
Problem 5 (addition)	19	14	6
Problem 6 (subtraction)	8	23	8
Problem 7 (subtraction)	6	29	4
Problem 8 (addition)	17	12	10
Problem 9 (addition)	23	13	3
Problem 10 (addition)	25	10	4

Table 4: students` performance in each problem

The information in table 4 shows that students` performance on problems which required addition was much better than subtraction word problems. Many students performed better on problems 2, 5, 8, 9 and 10. The performance on the other five word problems was low because each of the problems required the use of subtraction. For example, on problem one, 29 students had wrong solutions and only 7 of them had the correct solution. Three of the students did not solve the problem. A similar situation is also reflected on problems three, four, six and seven. The results in table 4 shows that students` performance on problems which require subtraction is generally low.

5.1 Analysis of five students interviewed

Qualitative analysis was employed to understand students' use of mathematics strategies during word problem solving. Firstly, the recorded interviews were coded and transcribed. Secondly, the students` performance in the interview was interpreted in order to determine commonalities of their mathematical strategies and procedures.

Students names used in this study are pseudonyms and these are; S1, S2, S3, S4 and S5. Students 1, 4 and 5 were better speakers than the other two students.

5.2 Responses to question 1 (Subtraction)

5.2.1. Case of student 1 (S1), over – average learner

S1 is an over – average learner due to performance in class and past performance records. In this study, S1 used two methods in solving the word problem and was able to explain how the problem was solved using the vertical method. The student had a problem in explaining how the number line was used to solve the problem.

The dialogue below is based on the interview with student 1.

Q1. *Everlyn has 144 spaces in her photo album. So far, she has placed 89 photos in the album. How many more photos can she put in before the album is full?*

1. Me: I want you to begin reading question 1 on your question paper.

2. S1: "Am not good at reading, but I will try to read the question."

3. Me: Write the words space and album.

4. S1: "spice and albom"

5. Me: Good, what did the first part of the question mean to you? You can explain in Chinyanja!

6. S1: [she remained quiet for some time until question in utterance 5 was asked]

7. Me: Look at the two words "photo album", what do the two words mean?

8. S1: "a photo album ilikwati buku yoyikamo ma photos" [it's like a book were photo are put].

9. Me: Ok, what does "144 spaces" in the 1st phrase of the question mean?

10. S1: "144 spaces mean that there are 144 ma places yama snaps" [144 places for snaps].

11. Me: How did you find the remaining spaces?

12. S1: "by subtracting."

13. Me: Why did you have to subtract?

14. S1: "the question required me to find the remaining spaces from 144"

15. Me: Explain how you found the remaining spaces.

16. S1: "Firstly I wrote 144 minus 89 equals to question mark. Then I borrowed 1 ten and then added it to 4 take away 9 equals to 5. So 100 plus 30 minus 80 equals 50, then 50 plus 5 equals 55."

17. Me: Why did you use this approach?

18. S1: "that's how my teachers taught me in my previous grades; I also copy from maths textbooks."

19. Me: Good, which other approach did you use to solve the problem?

20. S1: "number line "

21. Me Ok, explain how you used a number line?

22. S1: "I drew a number line with markings 0, 89, and 144. Firstly i moved from 0 to 89 and I wrote 55 as the distance. Then i moved 144 to 89 and wrote the distance as 89. So the answer is 55."

23. Me: Are you sure, that's how a number line is used in problem solving?

24. S1: "I was just trying to use the number"

25. Me: Good.

S1 read question 1 despite indicating of not been good at reading (ref to utterance 2). The student in this study encountered reading difficulties in that some word were wrongly pronounced as well as skipping of certain words. The word *space* was pronounced as "spice" and *album* as "albom" and was written as pronounced (ref to utterance 4). These errors often results from school or home environment were reading materials are inadequate or not readily available for the students.

The student`s responses to some questions asked during the interview were in local language, a good example is where a S1 responded that "a photo album ilikwati buku yoyikamo ma photos" [a photo album is like a book were photos are put] (ref utterance 8). The use of local language in the discussion resulted in the student becoming more active in responding to the questions. The use of English in the beginning of the discussion had an impact as S1 was taking long to respond to questions. I argue that local language usage in mathematics could be an added advantage to student's performance and understanding of mathematics word problems. In solving the word problem, S1 used two different methods.

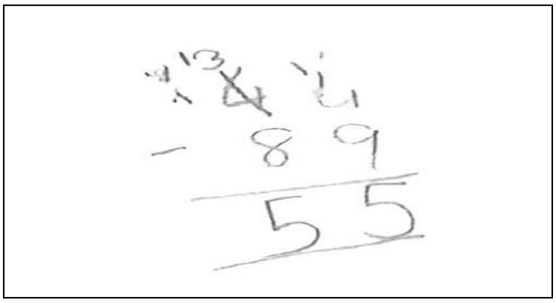

Figure 5-1: Borrowing and regrouping

S1: "Firstly I wrote 144 minus 89 equals to question mark. Then I borrowed 1 ten and then added it to 4 take away 9 equals to 5. So 100 plus 30 minus 80 equals 50, then 50 plus 5 equals 55."

The first step S1 took was to subtract the ones, then the tens, moving from right to left. The solution procedure above shows that the student used borrowing and regrouping technique and subtracted digits in ones and tens columns and then dropped 1 on hundreds` answer space. The student pictured 144 and 89 written vertically and applied the traditional algorithm. The second method used in solving the problem was a number line.

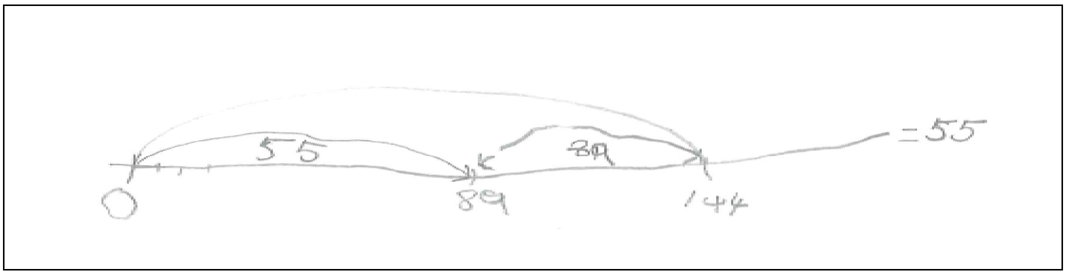

Figure 5-2: student solution using a number line

S1: "I drew a number line with markings 0, 89, and 144. Firstly I moved from 0 to 89 and wrote 55 as the distance. Then i moved 144 to 89 and wrote the distance as 89. So the answer is 55."

S1 drew the number line which is associated with the positive number ray. The distance between 0 and 89 was indicated as 55 and 89 between 89 and 144.This study revealed that the student was not sure about the distance between 0 and 89, and also between 89 and 144. My findings of the study show that the student used a number line without fully understanding how it works and has only remembered what to do.

5.2.2. Case of student 2 (S2), average learner

The student used vertical method and the number line to solve the problem in the test. S2 explained the procedure in the first method though had difficulties in explaining how the number line was used to solve the problem. The figures (5-3 and 5-4) below are based on two methods the student used to solve the problem given.

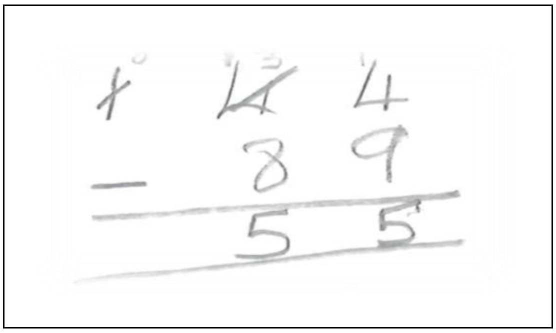

Figure 5-3: Borrowing and regrouping on ones and tens

S2: "4 minus 9 it can`t, I borrowed 1 from 4 leaving 3. 14 minus 9 equals to 5. Then 3 minus 8 it can`t, I borrow 1 from hundred`s column, so 13 minus 8 equals to 5. So then my answer is 55."

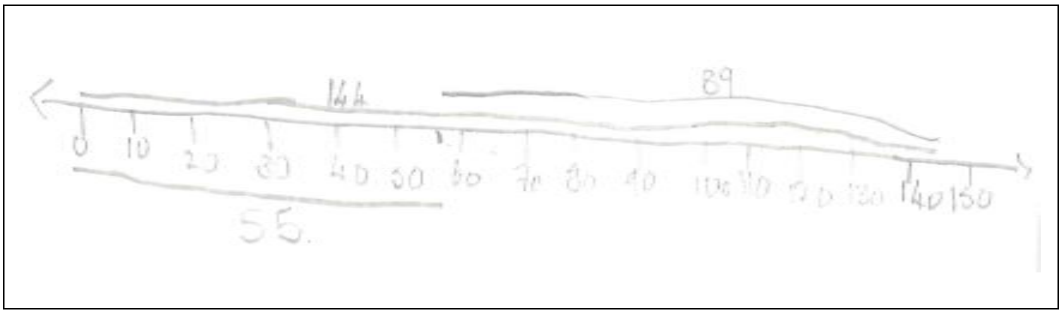

Figure 5-4: Model of a number line used in problem solving

S2: "I drew a straight line from 0 to 150. Each interval is 10 units. A line measuring 144 is drawn moving to the right direction of 0. Then the second line measuring 89 is drawn moving left of 144. The remaining 55 is the answer and is shown by the line below."

The number line shows the multiples of 10 from 0 to 150. I observed that the student estimated the positions of 55 and 144. The arrows pointing to the direction of movements are not indicated on figure 5-4. The number line shows two lines originating from zero. The first line runs from 0 to 144 and the second line is from 0 to 55. This finding reveals that the student used the number line without fully understanding how to use it.

5.2.3. Case of student 3 (S3), below – average learner

Student 3 also used two methods to solve question 1 in the test. The procedure on the first method was correct though had incorrect solution because the student subtracted smaller numbers from larger ones. The second method shows that the two sets of numbers are split in hundreds, tens and units and arranged vertically. Thereafter, student had to subtract the digits according to their positions and hence obtained the correct solution.

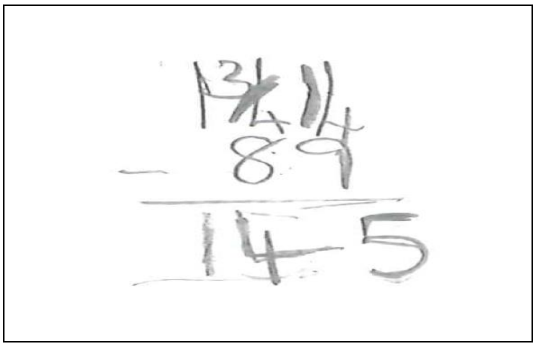

Figure 5-5: student`s solution to the problem

S3: “9 minus 4 equals to 5. Remove 4 units from 9 units. 8 minus 4 equals to 4. Remove 4 from 8 tens. Drop 1 down in the answer space. 145 is the answer.”

S3 started with the unit column where 4 was subtracted from 9. Then went on to subtract 4 from 8 and lastly dropped 1 under hundreds column in the answer space. In short, the student did not regroup the minuend, but subtracted the smaller minuend from the larger subtrahend in both ones and tens columns. The finding of this study shows that the student understands subtraction despite getting a wrong answer.

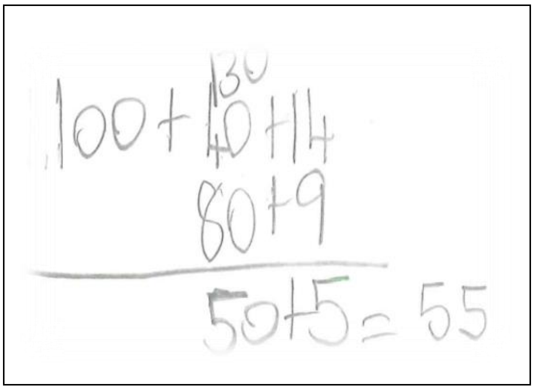

Figure 5-6: Borrowing and regrouping on ones and tens

S3: “First I borrowed 1 ten from 40, and added 10 plus 4 equals 14. Then 14 minus 9 equals to 5. I also borrowed 100 from column for 100 and added it to the reminder which is 30, and then got 130. So I removed 80 from 130 and the answer I got was 50. So then, 50 plus 5 equals to 55.”

In method two (figure 5-6), the student solved the problem by splitting two sets of numbers into hundreds, tens and ones and then subtracted the digits in columns. In this study, I observed that the student carried out an appropriate calculation on two methods despite giving an incorrect solution on the first method. My findings reveal that the student has the ability to solve problems which require subtraction.

5.2.4. Case of student 4, average learner

S4 solved question 1 using two methods. The first method was where numbers were not split while the second method was where numbers were split in hundreds, tens and units (ref to figures 5-7 and 5-8).

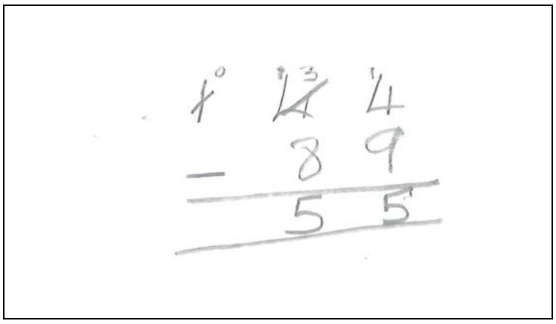

Figure 5-7: Borrowing and regrouping on ones and tens

S4: "144 minus 89, so 4 minus 9 it can`t, borrow 1 from 4 and leave 3 and then I did 14 minus 9 equals 5. Again, I did 3 minus 8 it can`t, borrow 1 from left, then 13 minus 8 equals 5. So 55 is the answer."

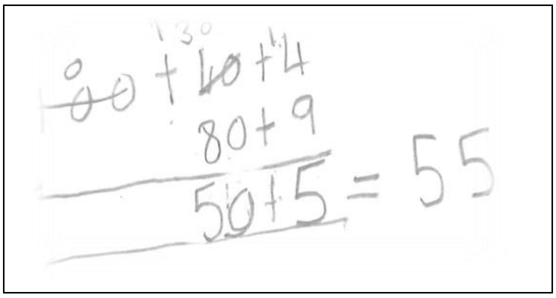

Figure 5-8: Borrowing and regrouping on ones and tens

S4: "144 has 1 hundred, 4 tens and 4 units. 89 has 8 tens and 9 units. 1 hundred plus 4 tens plus 4units can be written as 100 plus 40 plus 4. 8 tens plus 9 units can also be written as 80 plus 9. 4 minus 9 it can`t, borrow 1 ten. 10 plus 4 minus 9 equals 5. 30 minus 80 it can`t, then borrow 100 add to 30. So 130 minus 80 equals to 50. Then 50 plus 5 equals 55."

The student`s first step was to separate hundreds, tens and units from 144, tens and units also from 89. The second step was to subtract the ones and then the tens, moving from right to left. This study reveals that the student has the ability to work with problems which require subtraction. My findings indicate that the student used appropriate computation strategies leading to correct solution. I also observed that the student used fingers besides other counting tools when explaining how the problem was solved in the test. Finger counting is commonly used by many students in schools to solve the problem given.

5.2.5. Case of student 5 (S5), over – average learner

S5 is considered by her teacher as an over – average student because of her performance academically. The student`s class teacher indicated that "the student`s performance in middle and end of term test has been good". The findings of this study shows that the student solved the problem using two methods as can be seen from figures 5-9 and 5-10 below.

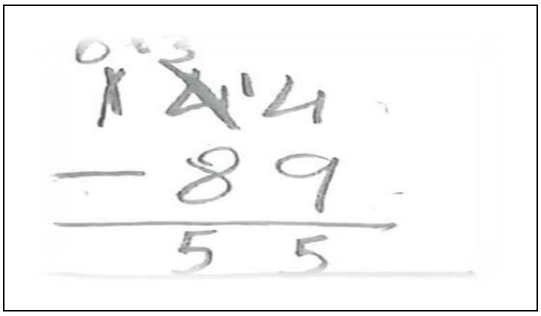

Figure 5-9: Borrowing and regrouping on ones and tens

S5: "I started by subtracting 4 minus 9, it can`t, borrow 1 from tens column and so 14 minus 9 equal to 5. Then, 3 minus 8 it can`t, so I borrow 1 from the column of hundred and then 13 minus 8 equals to 5. So 55 was my answer."

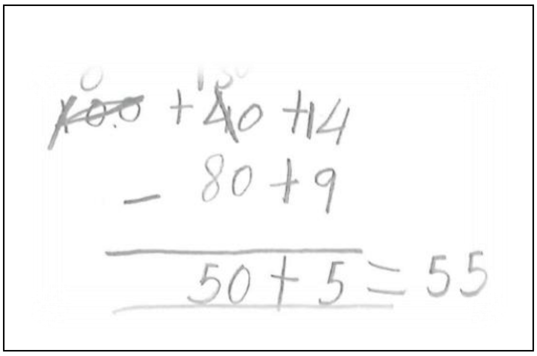

Figure 5-10: Borrowing and regrouping on ones and tens

S5: "144 minus 89 equals question mark. Borrow 1ten plus 4, minus 9 equals 5. Borrow 1hundred plus 3tens minus 8tens equals 50. Then I added 50 plus 5. So the answer is 55."

The student used borrowing and regrouping strategy in her computation. Firstly, the student borrowed 1ten plus 4units equals 14, minus 9units equals 5. Secondly, the student borrowed 1hundred plus 3tens equals 130, minus 8tens equals 50. Then, 50 plus 5 equals 55. I argue that the student has the ability of writing number sentences from a word problem and use appropriate strategies to solve the problem.

In conclusion, students interviewed in this study encountered challenges when it came to reading of mathematical texts. As they read through the texts, the following errors were observed and these are;

reading of texts without observing punctuation marks, mispronunciation and skipping of certain words. I also observed that students switched between local and second languages during the interviews. I argue that the use of local language in teaching and learning could enhance students' participation in class. This study has also revealed that many students used more than one method to solve the problem. I observed that many students used vertical method more than any other methods in addition and subtraction problem solving. My findings reveal that many students have the ability to solve addition and subtraction problems using different methods. The findings of this study show that many students used appropriate computation procedures though a student had an incorrect solution to the problem.

My findings also revealed that many students used finger counting and countered from smaller numbers in problem solving. I argue that finger counting as a tool does help students solve addition and subtraction problems more especially when working with smaller numbers. This study has also revealed that many students had difficulties in explaining how the number line was used to solve the problem. The difficult encountered was as a result of not been able to understand how it works despite using it. The findings of this study show that students used a number line without fully understanding how it works and only remembered what to do.

5.3 Responses to question 2 (Addition)

5.3.1. Case of student 2, an average learner

The student employed two methods in solving question 2. The first method is the use of drawings while the second one is where the student used a number line. The student explained the procedures leading to the attainment of the solution, though had difficulties in explaining how the number line was used in the test. The dialogue below is based on student 2 interviewed.

Q2. *Mary has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Mary have altogether?*

26. S2: Reads question 2

27. Me: How did you understand the question?

28. S2: "there are 3 packages and each has 6 pieces of gums."

29. Me: What were you required to find?

30. S2: "the total number of pieces of gums in all the three packages."

31. Me: How did you find the total number of gums?

32. S2: "I drew circles in rows and columns, 3 rows and 6 columns. Total count of circles gave 18 as the answer." [Pointing to her drawing]

33. Me: Ok, which other way was used to find the answer?

34. S2: "added 6 plus 6 plus 6 which is equals to 18.

35. Me: Good, why did you add 6, 3 times?

36. S2: "because there are 6 pieces of gums in each package and there are 3 packages."

37. Me: Alright, which other method did you use to calculate the answer?

38. S2: "I used a number line" [pointing at the drawing]

39. Me: How did you use the number line to solve the problem?

40. S2: "I firstly drew a straight line and marked from 0 to 40 at interval of 5 units. First, second and third lines drawn above the markers are 6 units each in length. I counted all the intervals from 0 to 18; the answer I got was 18."

41. Me: Alright.

S2 read the question in which errors such as repeating and mispronouncing of words were observed. The student also read through the texts without considering punctuation marks. I observed that the student used two different strategies in order to answer the question.

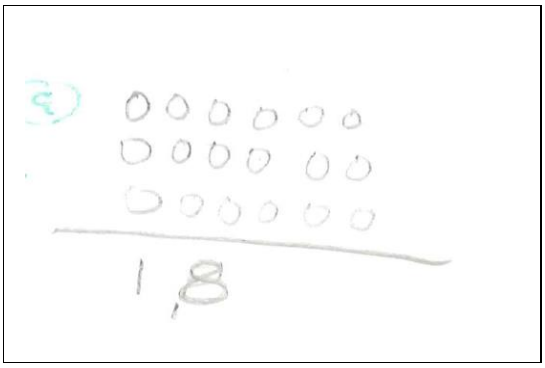

Figure 5-11: student`s solution to problem

S2: “I drew circles in rows and columns, 3 rows and 6 columns. Total count of circles gave 18 as the answer.”

Firstly, the student drew circles in rows of 3 and columns of 6. Thereafter, S2 counted the circles which gave a total of 18. The answer was written below the circles drawn. The result shows that the student demonstrated the ability to solve problems using pictorial. Besides counting of circles, the student might have used repeated addition of 3 plus 3 plus 3 plus 3 plus 3 plus 3 or 6 plus 6 plus 6 as can be seen from the diagram above. This is also reflected in response to the question “add 6 plus 6 plus 6 which is equals to 18.” Each row of 6 circles represents a package of gum and the circles in each row indicate pieces of gums.

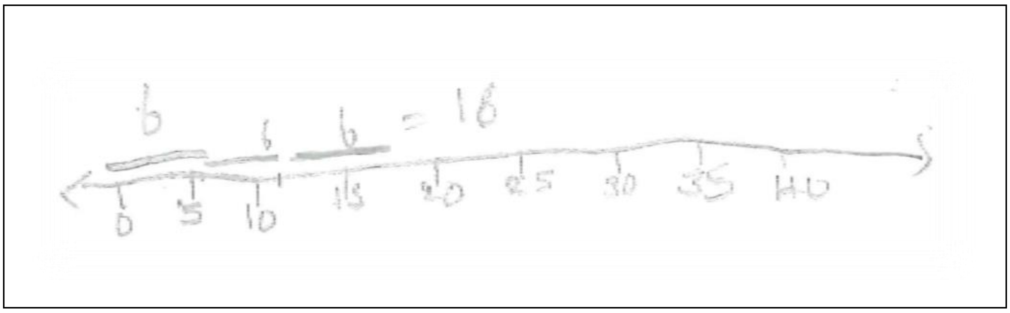

Figure 5-12: Student`s solution to question 2

S2: “I firstly drew a straight line and marked from 0 to 40 at interval of 5 units. First, second and third lines drawn above the markers are 6 units each in length. I counted all the intervals from 0 to 18; the answer I got was 18.”

The number line drawn by S2 has similar errors to the one the student used to solve question 1 (ref figure 5-4). In this case, multiples of 5 were used in drawing the number line. It was also noticed that the answer was written immediately after the last line drawn above the markers. I argue that the student has the ability of using the empty number line in problem solving despite errors observed on figure 5-12.

5.3.2. Case of student 1, over – average learner

The student found the solution to the problem in two different ways; firstly, the student used repeated addition where the additive of 3 and 6 were involved. This study revealed that the student understands that repeated addition could yield same results as that of multiplication. The number line was used as a counting tool and hence counted the intervals which gave a total of 18. The student was also able to explain the procedures on how the solution to the problem was found in a test.

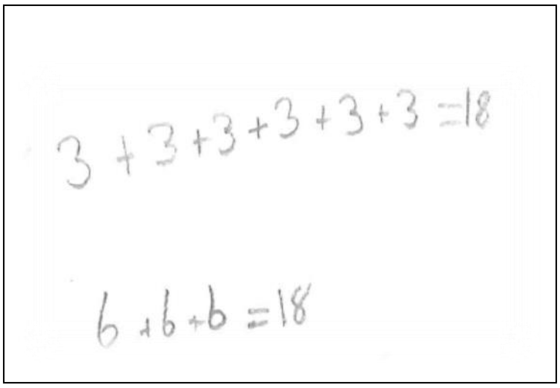

Figure 5-13: Repeated addition of 3 and 6

S1: "Firstly, I wrote 3 times 6. Then, I added 3 plus 3 plus 3 plus 3 plus 3 plus 3 and got 18. I also tried to add 6 plus 6 plus 6 which as gave me 18. So I realized that addition simpler than times."

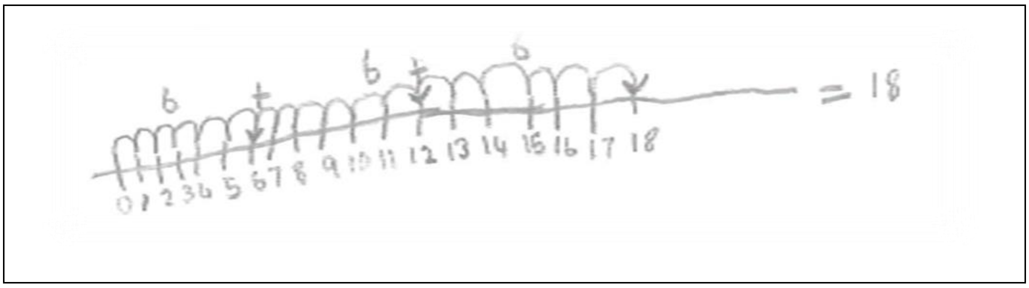

Figure 5-14: Student`s solution using a number line

S1: "I marked and drew a number line from 0 to 18. I counted the first 6 intervals from 0 to 6. Counted next intervals from 6 to 12 and the last 6 intervals from 12 to 18 and the answer I found was 18."

S1`s first step was to draw a number line with markings from 0 to 18 and then counted the intervals. The student started by counting the first 6 intervals from 0 to 6, the next intervals from 6 to 12 and last sets of intervals from 12 to 18 and then counted the intervals from 0 to 18. The last interval to be counted was considered as the answer. My findings in this study reveal that the student used the number line as a counting tool to solve the problem.

5.3.3. Case of student 3, below – average learner

The student used circles and number line to solve the problem. S3 presented circles in a similar way as presented by student 2 and was able to find solution to the problem. The number line drawn was used as a counting tool and counted the intervals. I also observed that the student used repeated addition and multiplication to find solution to the problem.

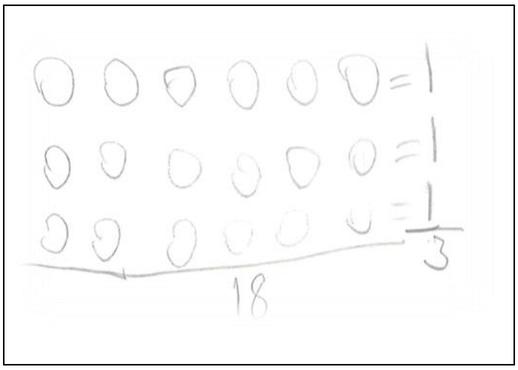

Figure 5-15: student`s solution to the problem

S3: "I drew circles in rows and columns. Each set of 6 circles equals to 1 row. Each column consists of 3 sets of circles. 3 represent the total number of packages of gums. 18 represent the total number of pieces of gums."

S3 started by drawing the circles in rows and columns. The student indicated that each set of sixes is 1, meaning that there 3 sets of sixes. Then S3 counted all the circles which gave a total of 18 circles. I argue that the student has the ability to solve problems using of drawings.

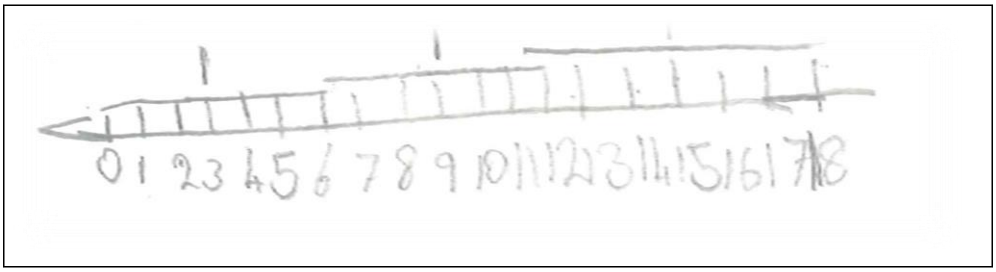

Figure 5-16: student`s solution to problem

S3: "Firstly I drew the number line with markings from 0 to 18. Each count of 6 intervals is one set. I wrote one above each set of 6 intervals. The total number of sets is 3. Then I counted all the intervals and found 18 as the answer."

S3 started measuring from 0, and then counted the intervals and grouped them in sixes. Each set of 6 intervals was labelled as 1 and written above each set. Then the student added together the 3 sets of intervals which gave 18 as an answer. My findings reveal that the student used the number line as a counter in problem solving.

5.3.4. Case of student 4, average learner

S4 in this study used pencils as counting tools that helped her find solution to the problem.

Pencils were grouped in 3 sets, each consisting of 6 pencils. 3 sets of pencils represented packages of gums and 6 pencils in each set as pieces of gums. The quotation below is an extract of S4`s explanation “6 set of pencils represent 1 package, so, 18 pencils represent 3 packages of gums” (ref to appendix 3, question 2). Other findings reveal that counting counters as an approach is mainly to help young children easily find solutions to mathematical problems by counting counters.

Figure 5-17: Pencils as counters

I also observed that the student used finger counting. To solve 3 x 6, each finger represents 6 and therefore the first finger to be raised is 6, followed by 12 and 18 as the answer. It was my general observation that many students used finger counting besides other counting strategies to solve the problem.

5.3.5. Case of student 5, over – average learner

The student solved the question in three different ways in the test. Firstly, S5 used tallying and counted 3 sets sixes. Secondly, repeated addition and multiplication were used and found the same results in all three methods. Hence, figures 5-18 and 5-19 below shows how the student solved the problem.

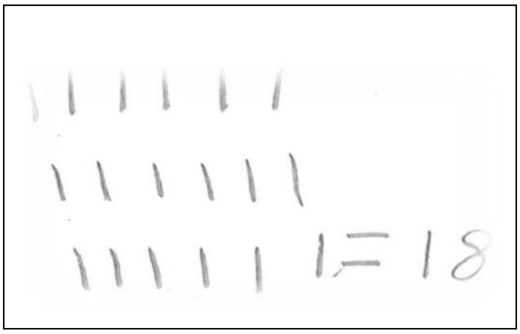

Figure 5-18: solution to the problem

S5: “Firstly i had to write down 3 sets of 6 marking each. Then I counted all the markings and got a total of 18 markings.”

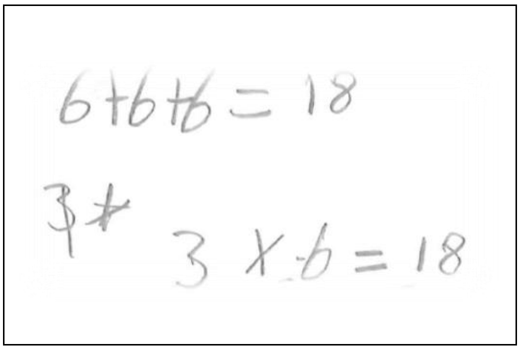

Figure 5-19: student`s solution to the problem

S5: "I started by 6 plus 6 plus 6 equal to 18. I did 3 times 6 equal to 18. So then the answer I got was 18"

In figure 5-18, the student used tallying to solve the problem. Firstly, the tallies were arranged in rows and columns. Each row had six tallies and each column three tallies. The student counted the tallies or multiplied the row by the column in order to find the answer. Figure 5-19 shows two methods and these are repeated addition and multiplication. I argue that multiplication could be written as repeated addition and still get the same results.

Mulligan (1992) argues that young children can solve a variety of multiplication problems prior to instruction. Furthermore, it is indicated that children`s informal use of additive procedures is both efficient and meaningful in solving multiplication problems.
In this study, I observed that some students presented their solution using drawings and gave the same sum as the product of multiplication. Rows and columns were used on their drawings. For instance, the circles drawn were placed in three rows and six columns and the total count of circles resulted into 18 as the final solution. My findings of this study show that students demonstrated the ability to handle problem by the use of drawings.

Other findings show that when students begin solving addition and subtraction word problems, they seem to create representations of the problem (Carpenter and Moser, 1983; Riley et al., 1983). I also observed that many students used the number line and repeated addition to solve the problem.

I argue that repeated addition is commonly used by many students at primary level when it comes to multiplication problem solving. It is also a general observation that many students use number line as counting tools. Word problem solving using a number line has its own challenges as many students count markers than intervals and begin counting at one resulting to incorrect solution. The number line is also essential as students could use it as counting tool beside finger counting and other forms of counters.

5.4 Responses question 5 (Addition)

5.4.1. Case of student 4, average learner

Student 4 solved the problem in two different ways. Firstly, S4 calculated without splitting the numbers in ones and tens. Secondly, the number was split in ones and tens, and then added from ones. The method used was also used by many students in this study. The dialogue below is based on student 4.

Q5: *Chola has 29 marbles. If she wins another 14 marbles, she will have the same number as Sam. How many marbles does Sam have?*

40. S4: Reads question 5

41. Me: How many marbles has Chola and how many did she win?

42. S4: "has 29 and won 14 mobles."

43. Me: Which strategy did you use to find the solution for question 5?

44. S4: "add 29 and 14"

45. Me: Good, explain how you found the answer?

46. S4: "9 plus 4 equals 13. Carry the 1 and add it to the 2 plus 1. Then 1 plus 2 plus 1 equals 4 So the answer is 43."

47. Me: Ok, 43 marbles, which other method did you use to find the answer?

48. S4: "place value"

49. Me: How do you find the answer?

50. S4: "I did 9 plus 4 equals 13 and 20 plus 10 equals 30. Then I added 30 plus 10 plus 3 equals 43."

51. Me: What did you notice about the methods used?

52. S4: "Mmm! There are different"

53. Me: What made you say that the methods are different?

54. S4: "because in the first method, it is 29 plus 14 and in the second method, it is (20 plus 9) plus (10 plus 4)"

55. Me: Are you sure that 29 and (20 plus 9) are different?

56. S4: yes

57. Me: why?

58. S4: "because in the first number, 2 and 9 are connected and in the other one, 20 and 9 are separate".

59. Me: Alright.

S4 read question though certain words were mispronounced (ref to utterance 42). Addition was used in solving the problem. I observed that the student solved the problem in two ways as reflected in figures 5-20 and 5-21.

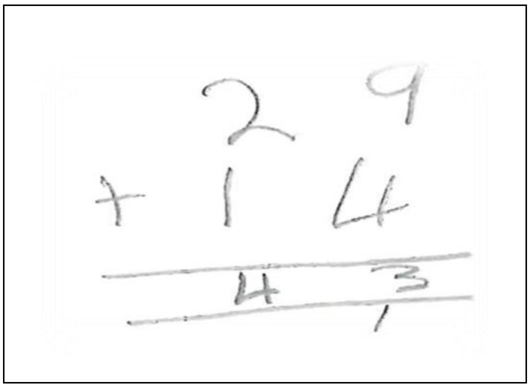

Figure 5-20: Carrying and regrouping in tens

S4: "9 plus 4 equals 13. Carry the 1 and add it to the 2 plus 1. Then 1 plus 2 plus 1 equals 4 so the answer is 43."

S4`s first method (ref figure 5-20) was used by other students interviewed in this section. When solving the problem, the student pictured 29 and 14 written vertically and applied the traditional algorithm. On the other side, the student does not sense that 1 plus 2 plus 1 plus 4 really is 10 plus 20 plus 10 equals 40. I argue that the student knows more than she actually does.

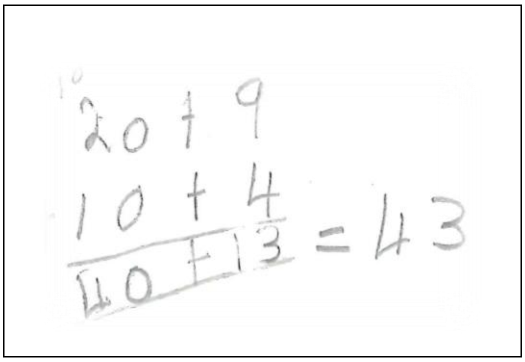

Figure 5-21: Carrying and grouping in tens

S4: "I did 9 plus 4 equals 13 and 20 plus 10 equals 30. Then I added 30 plus 10 plus 3 equals 43." I observed that the student added the ones and then the tens, moving from right to left. The method (ref figure 5-20) makes regrouping easier, because of the zero in the 30. The methods used in this case are similar to those used previously by the student.

The student commented that two methods used are different because in the first method, all the numbers are treated as one whole while in the second method; the numbers are separated from each other, for instance, 29 and (20 + 9). The following response is for student 4; “because in the first number, 2 and 9 are connected and in the other one, 20 and 9 are separate”.

My findings reveal that the student used fingers to model the numbers to be added, for example the student mentioned 29 then started on 30 counting (30, 31, 32, 33……43), hence the last finger to be counted was the solution. Other findings revealed that students use fingers to show sums. It is also argued that finger counting enhances an understanding of possible developmental sequences of solution procedures (Fuson et al., 1992).

5.4.2. Case of student 1, over – average learner

The student used two different methods and these are vertical calculation and the number line. When using the vertical method, I observed that the numbers were split in ones and tens, hence added from ones. Secondly, S1 used a number line to perform addition. I argue that the student used the number line as a counting tool, and counted the intervals to get 43.

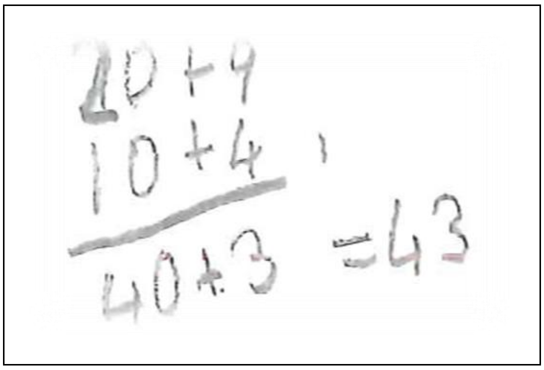

Figure 5-22: Carrying and regrouping in tens

S1: “9 plus 4 equals 13. Then I split 13 into two numbers (10 plus 3). I added 10 to 20 plus 10, and then added 3 to get 43.”

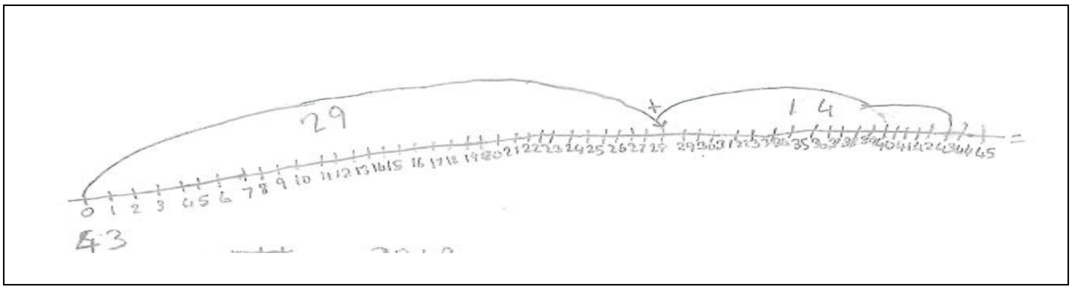

Figure 5-23: Student`s solution to the problem

S1: “I drew a number line of intervals ranging between 0 and 45. I counted 29 intervals and then 14 intervals. Then I added 29 and 14 intervals and got the answer is 43.”

S1`s first step was to draw a number line of intervals ranging between 0 and 45 and then marked 29 intervals followed by 14. Secondly, the student combined the intervals marked which gave a total of 43 intervals. My findings reveal that the student has the ability to solve problems using different methods.

5.4.3. Case of student 2, an average learner

S2 solved the problem using vertical calculation. The student added from right to left (ref figure 5-23). Many students in this study used similar method to solve the problem.

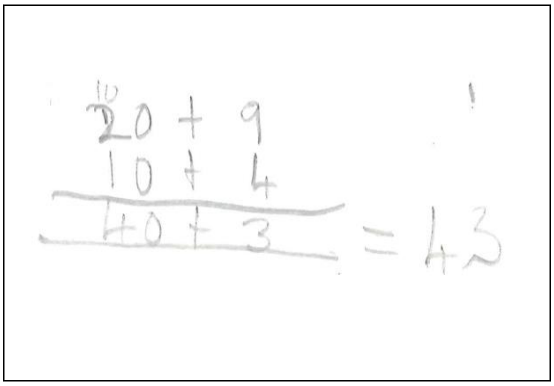

Figure 5-24: Carrying and regrouping in tens

S2: "Firstly, I did 20 plus 10 equals 30. Then I had 9 and 4, I add 9 plus 4 equals 13. I split 13 into two numbers (10 and 3). Finally I add 30 plus 10 plus 3 equals 43."

The student solved the question 5 using vertical method; the two sets of numbers (29 and 14) were split in hundreds and tens. The student explained the procedure which led to solution in figure 5-23. Murmuring and finger counting was used in S2`s explanation. I observed that the student was limited to one method as shown in figure 5-24 and was able to use it to solve the problem. I argue that the student has the ability to invent different strategies which could to solve problems.

5.4.4. Case of student 3, below – average learner

The student took much time in explaining the procedure on how the problem was solved. The student`s procedural explanation to problem solving was accompanied by the use of finger counting resulting into correct solutions (ref figure 5-25 and 5-26). I observed that the student used two different methods though similar to the methods other students used in this study.

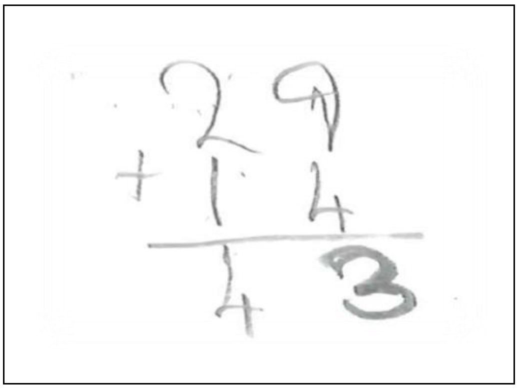

Figure 5-25: student`s solution on vertical method

S3: "29 plus 14, 9 plus 4 is 13, put 3 and carry 1. 2 plus 1 is 3, and then put together 3 and 1 is 4, so the answer is 43."

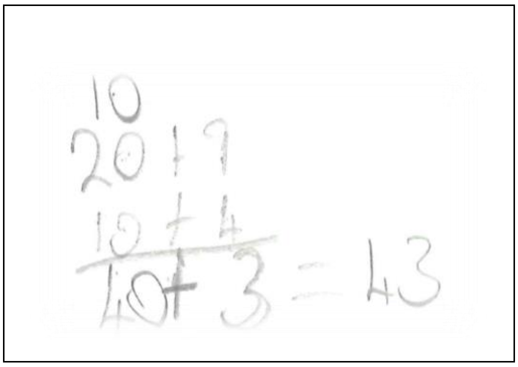

Figure 5-26: Carrying and regrouping in tens

S3: "First I took 9 from 29 and 4 from the 14. I add 4 plus 9 equals 13, add 20 plus 10 equals 30, then I added 10 plus 20 plus 10 plus 3 equals 43."

My findings reveal that the student made an effort to solve the problem in two different ways; firstly by not splitting the two sets of numbers into ones and tens, secondly, by splitting numbers in ones and tens. This study revealed that the student gave an explanation of the process which gave results in figure 5-25 and 5-26. I argue that the methods used in this case were also used by other students in this study.

5.4.5. Case of student 5, over – average learner

The student solved the problem using two different methods. I observed that the two sets of numbers (29 and 14) in the first method were not split in ones and tens (ref figure 5-26), but in the second method (ref figure 5-27), the numbers (29 and 14) were split in ones and tens and hence calculated from the ones.

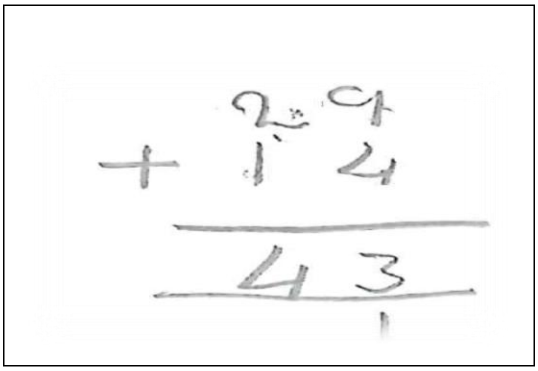

Figure 5-27: Carrying and regrouping in tens

S5: "I started by adding 9 plus 4, I got 13. I wrote 3 and carried 1, and then I added the 1 carried to 2 plus 1 and got 4. So I wrote 4 on the answer space. The answer I got was 43."

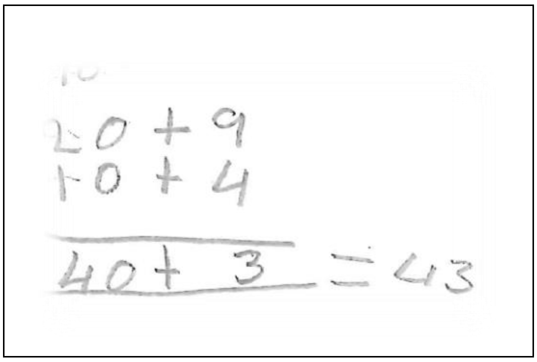

Figure 5-28: Carrying and regrouping in tens

S5: "I took the 9 from 29 and the 4 from 14. And I added; 9 plus 4 equals 13. Then I added 20 plus 10 equals 30. Finally, I had to add 30 plus 13 equals 43."

The findings of this study reveal that the student used vertical calculation to solve the problem. Vertical calculation was in two ways, firstly, the problem was solved without splitting the numbers (ref to figure 5-27) and secondly, the numbers were split in ones and tens (ref to figure 5-28). The student was able to explain the procedure of how solution to the problem was found. I argue that the student understands the rule of addition and how it works.

In conclusion, the students in this study used one or more methods to solve the word problem. I observed that the methods used involved either breaking the numbers (29 and 14) apart into tens and ones components or combining numbers to form multiples of ten. I also observed that many students used finger counting as their explained the procedures leading to solutions obtained in a test. Some students used pencils as counters to aid them in solving word problem. From the students' explanations and solutions, I argue that students used almost similar methods in problem solving.

The study revealed that the student got a little confused on the mathematical operation to be used in word problem solving. At first, the student stated that subtraction was appropriate for problem solving, but later changed to addition (ref utterance 200 and 202, appendix 4). I argue that the change from subtraction to addition was initiated by the word "win". The student referred the word "win" to lose of marbles, but later on realized that to "win" meant gain, meaning that addition was appropriate in this situation.

My findings reveal that students used local language in giving explanations to certain questions asked which the student could not explain in English, a good example was in the student`s response to question 5 "ku win ni ku yikapamozi [to win is to add]". Other findings revealed that word in one language and its translation in another language share the same conceptual representation in the mind of the learner (French et al, 2004).

5.5 Responses to question 10 (Addition)

5.5.1. Case of student 5, over – average learner

S5 solved the problem in two different ways; firstly, the student used the book and counted the pages starting on page 48 and ended on page 55. Secondly, the student used vertical calculation and found the same answer as in the first method. The dialogue below is based on student 5.

***Q10**. Richard was on page 47 of his book. Then he read 8 more pages. What page did he end up on?*

60. S5: Reads question **10**

61. Me: How did you find the answer?

62. S5: "nina count ma pages mu buku 2 ya ma maths" [I counted pages in maths book 2].

63. Me: Explain how you counted the pages using the mathematics textbook?

64. S5: "48, 49, 50, 51, 52, 53, 54, 55, the answer is 55." [Flapped the pages in a mathematics textbook, pupils book 2]

65. Me: Ok, which other method did you use to find the answer?

66. S5: "adding"

67. Me: Ok, how did you add?

68. S5: "47 plus 8 equals to 55."

69. Me: Good, how did you get 55?

70. S5: "7 plus 8 equals 15, leave 5 and carry 1, then 4 plus 1 carried is 5, then the answer is 55".

71. Me: What do the numbers 4 and 7 represent in 47?

72. S5: "4 stand for tens and 7 for units"

73. Me: How many tens and units are in 55?

74. S5: "5 tens and 5 units, oh I have remembered another method I used"

75. Me: Which other method did you use and how was it used?

76. S5: "First I added 7 plus 8 equals 15 put 5 and carry 1ten. 40 plus 0 equals 40. Then 40 plus 10 equals 50. So 50 plus 5 equals 55."

77. Me: Where did you get the 40 from?

78. S5: "4 tens is the same as 40"

79. Me: Alright.

The student read the question well, and answered to certain questions asked in local language. A good example was student`s response to question 10 "nina count ma pages mu buku 2 ya ma maths" [I counted pages in maths book 2] (ref utterance 62). For counting purpose, I observed that the student used a grade 2 mathematics textbook instead of fingers. The student counted the pages in a textbook by flapping them. The student began counting from page 48; 48, 49, 50, 51, 52, 53, 54 and 55 meaning that 55 was the final solution. The counting strategy used was known as counting forward. Other findings revealed that students find counting backward more challenging than counting forward as they begin counting from larger numbers in problem solving of both addition and subtraction Baroody (1984) and Ginsburg (1981). I argue that students choose their counting strategies according to the size and familiarity of the numbers involved in the problem to be solved.

I also observed that the student used two other methods in problem solving as could be seen in figure 5-29 and 5-30. In figure 5-29, the student added 47 and 8 which gave 55. In figure 5-29, the student had to split 47 into tens and ones, hence found the correct solution to the problem.

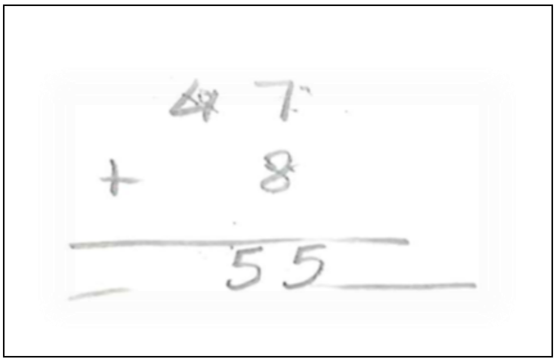

Figure 5-29: Carrying and regrouping in tens

S5: "7 plus 8 equals 15, leave 5 and carry 1, then 4 plus 1 carried is 5, then the answer is 55".

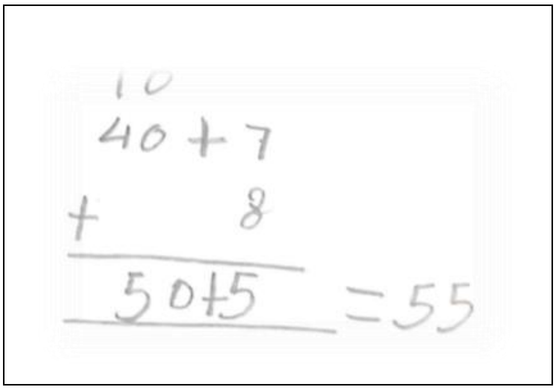

Figure 5-30: Carrying and regrouping in the tens column

S5: "First I added 7 plus 8 equals 15 put 5 and carry 1ten. 40 plus 0 equals 40. Then 40 plus 10 equals 50. So 50 plus 5 equals 55."

My findings in this study reveal that the two methods used to solve the problem are similar in that 47 plus 8 is actually 40 plus 7 plus 8 though the student did not realize that the methods in figure 5-29 and 5-30 are actually the similar. I argue that the student has the ability to invent strategies and use them to solve problems given.

5.5.2. Case of student 1, over – average learner

Student 1 used two methods to solve the problem. The student used vertical calculation in which the number was split in ones and tens and added starting from ones. S1 also used a number line as the second method. In all the methods used, the student found the same results (ref figure 5-31 and 5-32).

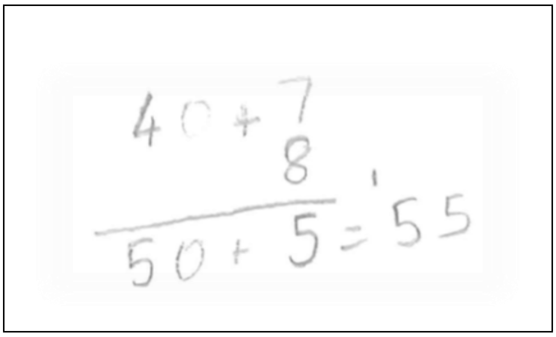

Figure 5-31: Carrying and regrouping in the tens column

S1: “47 plus 8, first I added 7 plus 8 equals 15. I wrote 5 and then carried 1. The one I carried stand for tens, it was then added to 40 to give 50. Then 50 plus 5 equals 55.”

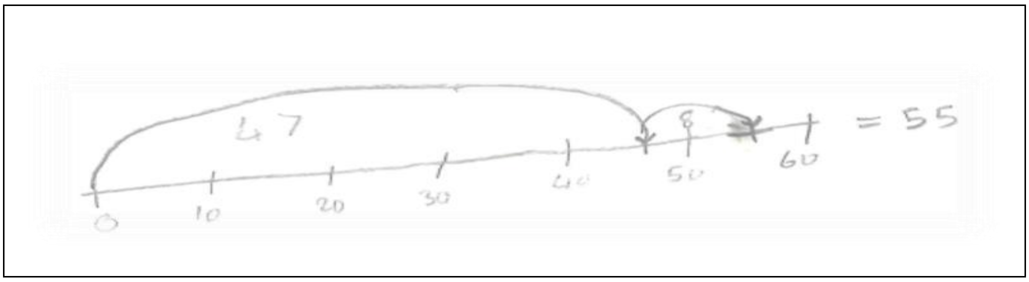

Figure 5-32: student`s solution to problems

S1: “I drew a straight line and marked 0, 10, 20, 30, 40, 50, and 60. Then I counted up to 40 and added a 7 to get 47. I also counted 8 more to get 55. I guessed the positions of 47 and 55.”

I observed that the student used estimations for the positions of 47 and 55 on the number (ref figure 5-32). My findings in this study indicate that the student used use the number line without fully understanding how it works and only remembered how to use it. I also argue that finger counting is helpful in that it helps keep track of the sequence.

5.5.3. Case of student 2, average learner

The student demonstrated the ability of solving the problem in two different ways though similar to other students in this study. I observed that the student used book 5 and counted pages starting on page 48 and ended on page 55. Secondly, the student used vertical calculation in which the numbers were split in ones and tens as could be seen below (ref figure 5-33).

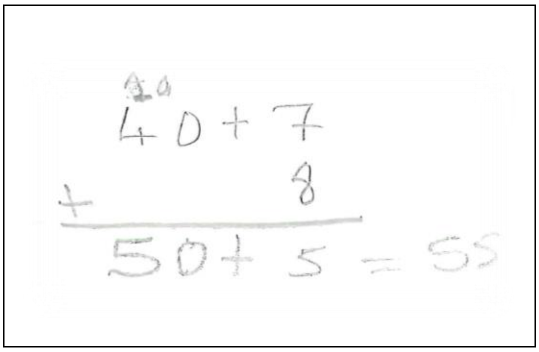

Figure 5-33: Carrying and regrouping in the tens column

S2: "I had 40 from 47. I had also the 10 from 15 because I had to split 15 into 10 and 5. Then I added 40 plus 10 equals 50, so 50 plus 5 equals 55."

The findings of this study reveal that the student solved the problem in two ways and these are; counting of pages in a mathematics textbook and vertical calculation. The student counted pages beginning on page 48, 49, 50, 51, 52, 53, 54 and 55, hence considered 55 as the final solution. The student also used vertical method in problem solving (ref figure 5-33). In the process of computation, the student calculated from right to left. Additionally, I observed that the student used finger counting and mental computation during the interview.

5.5.4. Case of student 3, below – average learner

In this study, I observed that the student used two different methods to solve the problem and had the same results. Figure 5-34 and 5-35 below shows how the student worked out the problem in the test.

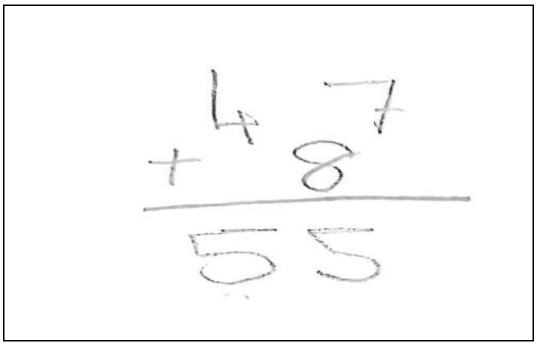

Figure 5-34: Carrying and regrouping in tens

S3: "I just added 47 plus 8 equals 55. I used my fingers to count 8 starting on 48, and got 55 as my final answer"

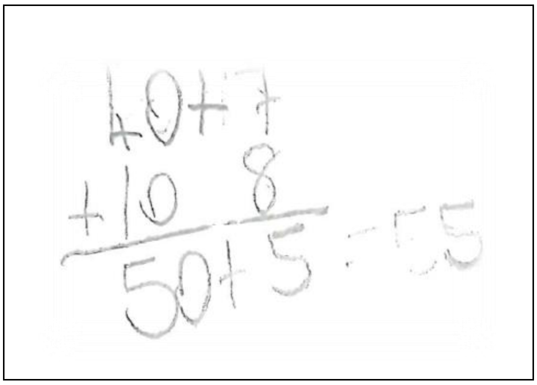

Figure 5-35: Carrying and regrouping in the tens column

S3: "First I added 7 plus 8 equals 15. Then I split 15 in ones and tens, which means I have 10 and 5. Then I added 40 equals 10 equals 50, so 50 plus 5 equals 55."

The findings of this study reveal that the student has the ability to solve addition and subtraction problems vertically, though does not know how it works, but only remembered how to use it. I observed that in the first method (figure 5-34), the digit 8 was wrongly placed and hence placed 8 under 47 though had the correct solution. I argue that the student remembered how to solve addition problems horizontally. In the second method, the student solved the problem by splitting 47 into ones and tens and then added from ones. In the end, the student added 50 and 5 which gave 55 as the solution to the problem (ref figure 5-35). I also argue that the student has the ability to solve problems using different methods.

5.5.5. Case of student 4, average learner

The student used two methods to solve the given problem. The methods used in this case are similar to that of other students in this study.

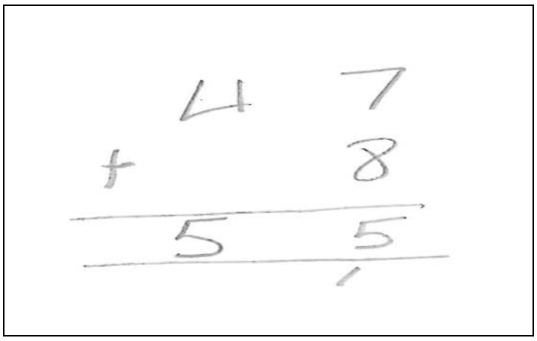

Figure 5-36: Carrying and regrouping in tens

S4: “47 plus 8, 7 plus 8 equals to 15. Leave 5 and carry 1, 4 plus 1 equals to 5, so, the answer is 55.”

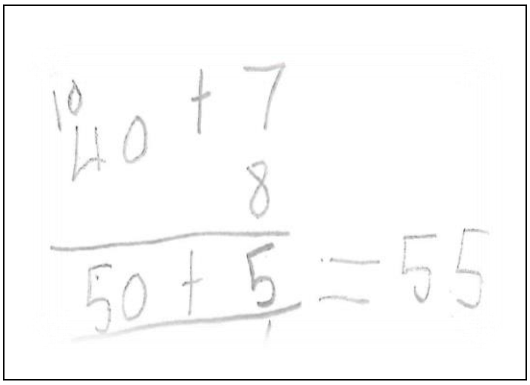

Figure 5-37: Carrying and regrouping in the tens column

S4: “First I added 7 plus 8 equals 15. Then I split 15 in ones and tens, which means I have 10 and 5. Then I added 40 equals 10 equals 50, so 50 plus 5 equals 55.”

The findings of this study reveal that the student solved the problem in a similar way as that of other students. I observed that the student followed the rule of vertical method and calculated from right to left. My findings reveal that the student used finger and murmured counting. The student`s computation strategies has the same results.

The findings of this study revealed that reading of mathematics text was a problem for many students. For instance, students read the text without considering punctuation marks. Pronunciation errors were also observed in some students. Some students pronounced the word space as “spice” and album as “albom” while the word marbles pronounced as “mebles”. I observed that students switched language between local and second language as they responded to some questions during the interview. I argue that language switching is used when students discuss problems in small groups and switch to English during whole class discussion. My findings in this study reveal that students` computation strategies used are appropriate in this situation. I observed that many students calculated vertically more often than other students. I argue that the students have the ability to solve word problems using different strategies. Other findings reveal that the topic on addition and subtraction of word problems is embedded in the Zambian mathematics curriculum and taught at all levels of education (Ministry of Education, 2003).

Mixed method of calculation means different ways of calculations, for instance, use of vertical calculations, the number line, pictorial representations and finger/sticks counting. I observed that students interviewed used mostly vertical calculation, number line and counted using fingers/tallying. Pictorial representation was less used to solve the problem. I argue that many students used vertical calculations, with less or no incidences of mental calculations. The mathematics syllabus indicates that fifth graders are expected to carryout abstract calculation when solving word problems. Students begin learning calculations from grade 1 (Ministry of Education, 2003). I also argue that many students know how to solve addition and subtraction problems vertically, and prefer using fingers for counting. Mental calculation is mainly applicable to single digit numbers, but for multiple digit numbers, they switch to vertical calculations.

Discussion of the Results

Word problems are among the components of school curriculum. Most curricular programs show that word problems are relatively difficulty for many students at all levels of education. Dwelling on the analysis of my empirical data, i argue that many students were successful in solving addition than subtraction word problems. Students come in contact with addition before getting admitted into school. Similar finding have been reported that subtraction word problems are more difficulty than addition word problems for students to solve (Bebout, 1993; Carpenter et al., 1993; Fuson et al., 1997). Furthermore, I argue that students` failure to solve some mathematics word problems is due to the language employed in the question and this argument is in line to that of Bernardo, (1999).

The findings also revealed that students solve word problems by representing the action in the problem. For instance, some children in this study drew circles to represent pieces of gums in a package before solving the problem (ref figure 5-11 & 5-15). My findings coincide with other researchers who indicated that students initial solve word problems by directly representing the action or relationships in the problem (Carpenter & Moser, 1984; Hiebert, 1982; Franke & Carey, 1997). In the same vein, Lopez and Veloo (1993, 1994) argue that students at primary school perform better when asked to draw diagrams before solving the problem.

The results from the test in this study suggested that most students did better on tasks which required addition than those of subtraction (ref table 4). Therefore I argue that some students add the values on the tasks which require subtraction and hence obtaining incorrect solutions. This argument is similar to that of Barwell et al, (2011) in which they have argued that students combine numbers in a problem without understanding the question and give unrealistic solution.

In this study, students were asked to read the questions and explain how they solved each problem. The findings revealed that many students encounter reading difficulties, for instance some words in the texts were pronounced wrongly (ref utterances 4); hence I argue that reading difficulties cut across students at all level of education and the society in general. Similar finding have been reported in which many students encounter reading and understanding difficulties in comprehending mathematical texts leading to solution errors (Cummins, Kintsch, Reusser & Weimer, 1988 cited in Bernardo, 1999).

In this study, I also observed that many students used mixed methods to solve word problems given. Similarly, Franke and Carey (1997) argued that students perceive mathematics as a problem solving in which different strategies are considered useful and hence use them to solve problems given. I argue that many students used vertical method and empty number line to solve the problems given. On vertical computation procedure, many students solved addition word problems by carrying and regrouping which also involved the formation of new group of tens. This procedure is similar to that of borrowing in subtraction and this argument resonates with findings by Varelas and Becker (1997).

Errors were also observed in some procedures, for example, a student subtracted 4 from 9 and wrote 5. Then, subtracted 4 from 8 in tens column and brought 1 down in hundreds column, and wrote 145 as the answer (ref figure 5-5). In this situation, I argue that many students tend to subtract smaller numbers from larger numbers regardless of their positions. This argument is similar to that of other researchers who argued that students' common errors when solving subtraction problems are either subtracting smaller from larger numbers or mistakes with borrowing (Dickson et al., 1984 cited in Resnick, 1982). I also observed that a student placed digit 8 under 47 instead of it being under 7 on unit column and still had the correct answer (5-33). Similar arguments have been presented by Dickson et *al* (1984) who argue that addition seems to present students with the least of the four operations though the common errors relate to the positioning of numbers in vertical presentation of addition and the process of carrying (Dickson et al., 1984 cited in Resnick, 1982).

Many students also used different counting strategies in problem solving. I observed that finger counting was used by many students who counted from smaller numbers. For instance, one student used pencils as counters to solve a given problem. The student grouped pencils in three sets and each set had six pencils. Then the student counted by putting together the pencils from the three sets and wrote 18 as the solution to the problem. Similarly, other researchers have reported that counting strategies could be carried on by the use of fingers, sticks, pencils, cubes or by counting mentally. For instance, grouping two sets together and using fingers or concrete objects to count all items, children start by counting the elements in the first set and finishing with those in the second one (Baroody and Standifer, 1993; Carpenter and Moser, 1983; Hugher, 1996; Nunes and Bryant, 1996, Groen and Parkman, 1972). Ma, (2011) argued that "counting counters" approach is mainly there to help students easily find solutions to addition and subtraction problems.

My findings also revealed that many students use counting strategies similar to those given to them by teachers, though there were few instances in which students strategies became abstract, for example, many students counted on from a smaller number by either using sticks, pencils or fingers. Similar findings have been reported by other researchers who argue that students use advanced counting strategies such as "counting on" or "counting back" to solve problems given (Carpenter and Moser, 1983; Riley et al., 1983). Therefore, I argue that as students grow and advance in education, they become more advanced in thinking and hence use strategies which are more abstract. In this case, many students move away from a situation where they count using concrete objects such as sticks, small stones, blocks and fingers. Similarly, Groen and Resnick (1977) argue that students develop advanced approaches to computation as they grow.

Language switching was observed during the interview as student switched between local (Chinyanja) and second language (English). Many students in this study used local language in expressing their ideas about a problem. I argue that students use first language when discussing in small groups or explaining certain concepts to their friends. English is mainly used in a whole class discussion and when instructed by the teacher. Similarly, Setati (2009) and Kazima (2009) argue that students switch languages when working on solution to task within their small groups and when it comes to class discussion, students contribute only when they are requested by the teacher and when this happens, they use second language. Clarkson (1994) also argued that children`s use of local language as they solved mathematics problems performed well in a test, an indication that home language is advantageous in a school setting.

In line with the above arguments, the Ministry of Education (MoE) in Zambia reversed its education policy requiring the pre – school and early grades (1 – 4) to be taught in all subjects in regional languages and English be taught as a subject (MoE, 2012). Based on the analysis of my findings, I argue that the use of local language in teaching and learning would improve students' performance in mathematics and other subjects which are taught in English. Bernardo (1999) also argued that students whose tasks are written in local language perform better than those students with tasks written in English. Similarly, Adetula (1990) and Bernardo (1999) argued that students involved on tasks written in local language improved their performance than before. Furthermore, Clarkson (1994) argued that students` use of local language could be advantageous in schooling setting. He further argues that students` competence in their local language does affect their achievement scores in mathematics positively.

Conclusion and Recommendation

7.1 Conclusion

From the finding of this research, it is clear that students were successful in solving addition and subtraction word problems. Students were more successful in solving addition word problems than subtraction word problems. This finding concede with the result from previous research that showed subtraction word problems as being more problematic than addition word problem for students to solve (e.g. Bebout, 1993; Carpenter et al., 1993; Fuson et al., 1997). This difficulty became more pronounced when borrowing was required.

In solving mathematics word problems, different strategies such as vertical, empty number line and counters were used. The results however, showed that the number line was wrongly used by all students despite using it to solve problem. The other research was where students count markers rather than the intervals. It is also clear that students used different counters such as pencils, small pieces of sticks and fingers to help them find solutions to problems given. Counting counters are commonly used by primary school going students though finger counting is applicable to most students at all levels of education.

It was a common observation that students switch languages during discussions within small groups and class discussion. First language is used when students help each other to understand certain mathematics concepts (Bernardo, 1999; Setati, 2009). Similar research conducted showed that students performed well during the test which was written in their local language (Kazima, 2009; Bernardo, 1999; Setati, 2009; Clarkson, 1994). My research is consistent with the results from previous research that shows home language as being advantageous than second language in a school setting.

7.2 Recommendation

Based on the findings of this research, the following are the recommendations;

- The government through the Ministry of Education and other stakeholders to embark on producing mathematics handbooks which parents could use to help children with problem solving at home.
- The Ministry of Education through the CDC should include mathematics word problems based on real life which would enhance critical or advanced thinking in children.

- Children at all levels of education should be given opportunity to re-invent strategies on their own or under the guidance of teachers than depending on those strategies given in mathematics textbooks.
- The Ministry of Education should also embark on publication of mathematics textbooks in local languages for all levels of education, as this would enhance easy understanding of mathematical concepts by children.
- The Ministry of Education should encourage workshops where teachers and parents meet to discuss ways of how best to improve mathematics performance among children.
- Refresher courses are necessities for teachers, as this would enable them acquire latest information based on mathematics education. This on the other hand would also improve their teaching skills.
- The mathematics textbooks at primary level should include activities or games which could inculcate in children the concepts of addition, subtraction, multiplication, division and counting.

8. Suggestions for future research

1. This study only looked at one school in Livingstone and was limited to one class of 39 students. A similar study should be conducted in other schools in Zambia.
2. The main focus of this study was on relationship between problem solving and language, the effects of using students` first language and second language in mathematics word problem solving and strategies used to solve addition and subtraction word problems. Therefore, more studies are recommended to cover strategies used by primary school children in solving addition and subtraction word problems and also to ascertain factors leading to subtraction word problems being difficult for students.
3. This study focused on mathematical text written in English. A similar study could be carried out to look at the effects of mathematical text written in students' first language on their performance.

In summary, it is my hope that the Zambian government through the MoE and other stakeholders would embark on production of mathematics textbooks for lower primary written in local language.

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10. Appendices

Appendix 1: Consent form for parents/guardian

My name is Richard Phiri a student at Høgskolen i Sør – Trøndelag (HiST) where I am studying didactics of mathematics. I came here to learn from your children about their experiences in mathematics problem solving. This study will take approximately one month.

I am willing to allow my child (name) ………………………………... to participate in the mathematics teaching to be conducted in the school. I agree that his/her utterances in the whole class during mathematics be recorded. Recordings will be done by the use of a camera.

I am aware that data will be transcribed and analysed, and that part of the collected data will be used in reports from the study. The data will be anonymous by using pseudonyms in transcribing and reporting from the study.

The study is scheduled to end on 28th November, 2013 and data collected will be discarded within a stipulated period of time.

My child is free to withdraw from the study at any time.

____

Signature of parent/guardian

____

Date

Appendix 2: Mathematics word problem tasks for primary

Instructions:

i. Give each student a question paper.
ii. Read the questions provided to the whole class.
iii. Students should show all their calculations on the spaces provide after each question

Please provide the following background information

Name:______________________________

Gender:_________

Age:___________

1. Everlyn has 144 spaces in her photo album. So far, she has placed 89 photos in the album. How many more photos can she put in before the album is full?

2. Mary has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Mary have altogether?

3. Mulenga counted all of the teacher's pencils. Some were sharpened and some not. She counted 73 pencils in all; 46 pencils were not sharpened. How many were sharpened?

4. There were 73 students on the playground. The 46 fifth grade students came in first. How many students were still on the playground?

5. Chola has 29 marbles. If she wins another 14 marbles, she will have the same number Sam. How many marbles does Sam have?

6. Mark has 7 mice. Ruth has 15 mice. Ruth has how many more mice than Mark?

7. A man had 26 cars at a car sale. This was 17 more than his best friend Tim. How many did Tim have?

8. Seventy – two people came to the school play on Monday. Twenty – six more people attend it on Tuesday. How many people went to the play on Tuesday?

9. There are 28 girls and 35 boys on the school playground. How many students were there on the school playground?

10. Richard was on page 47 of his book. Then he read 8 more pages. What page did he end up on?

Thank you for participating

Appendix 3: Interview guide

1. What does the first part of the question mean to you?

2. What do the two words mean?

3. What does 144 spaces in the 1^{st} phase of the question mean?

4. How did you find the remaining spaces?

5. Why did you have to subtract?

6. Why did you use the approach chosen?

7. Explain how you used a number line?

8. How did you understand the question?

9. What were you required to find?

10. How did you find the total number of gums?

11. Which other way was used to find the answer?

12. Apart from the methods mentioned, which other method did you use to find the answer?

13. How did you use the number line to solve the problem?

14. How many marbles has Chola and how many did she win?

15. What strategy did you use to find solution for question 5?

16. Explain how you found the answer?

17. What did you notice about the two methods used?

18. What made you say that the two methods are different?

19. Why?

20. How did you count the pages using the mathematics textbook?

21. How did you add?

22. How did you get 55?

23. What do the numbers 4 and 7 represent in 47?

24. How many tens and units are in 55?

25. Where did you get 40 from?

Appendix 4: Dialogues of five students interviewed

The interviews conducted to 5 students was interesting, as the students participated fully and hence much of the data needed to the writing of my thesis was successfully collected.
The dialogue below is based on 5 students interviewed and their names are pseudonyms.

Q 1

1. Me: I want you to begin reading question 1 on your question paper.

2. S1: "Am not good at reading, but I will try to read the question."

3. Me: Good, what does the first part of the question mean? You can explain in Nyanja if possible!

4. S1: [she remained quiet for some time until question in utterance 5 was asked]

5. Me: Look at the two words "photo album", what do you understand by the two words?

6. S1: "a photo album ilikwati buku yoyikamo ma photos" [it's like a book were photo are put].

7. Me: Ok, what does "144 spaces" in the 1st phrase of the question mean?

8. S1: "144 spaces mean that there are 144 ma places yama snaps" [144 places for snaps].

9. Me: If 89 photos are placed in a photo album, how you would find the remaining spaces?

10. S1: "Mmm….subtract".

11. Me: Why did you subtract?

12. S1: "to find the answer"

13. Me: How did you subtract then?

14. S1: "Firstly I wrote 144 minus 89 equals to question mark. Then I borrowed 1 ten and then added it to 4 take away 9 equals to 5. So 100 plus 30 minus 80 equals 50, then 50 plus equals 55."

15. Me: Why did you use the method the first method?

16. S1: "that's how my teachers taught me; I also copy from maths textbooks."

17. Me: That's good, which other method did you use apart from what you have just explained?

18. S1: "number line "

19. Me: why did you use a number line?

20. S1: "I saw my brother in grade 7 using it pamene eza chita homework" [when he was doing homework]

21. Me Ok, explain how you used it?

22. S1: "I drew a number line with markings 0, 89, and 144. Firstly i moved from 0 to 89 and wrote 55 as the distance. Then i moved 144 to 89 and wrote the distance as 89. So the answer is 55."

23. Me: Are you sure, that's how a number line is used in problem solving?

24. S1: "I just tried to use it"

25. Me: Alright.

Q 2

26. S1: Reads question 2

27. Me: Ok, what does the question mean to you?

28. S1: "Mary has 3 packages of gum and each package has 6 pieces of gums".

29. Me: Are you sure?

30. S1: yeah!

31. Me: If so, then how did you find the answer?

32. S1: "Mmm! 3 times 6 equals 18"

33. Me: Good, which other method did you use to find the answer?

34. S1: "add 3 plus 3 plus 3 plus 3 plus 3 plus 3 giving the answer 18"

35. Me: Why did you add the 3s?

36. S1: "because there are 3 packages of gums"

37. Me: 3 packages of gums, but why add 3s?

38. S1: "half a package has 3 gums"

39. Me: Alright, which other way did you use to find the answer besides the one used above?

40. S1: "add 6 plus 6 plus 6 giving the answer 18"

41. Me: Good, explain how you used a number line?

42. S1: "I marked and drew a number line from 0 to 18. I counted the first 6 intervals from 0 to 6. Counted next intervals from 6 to 12 and the last 6 intervals from 12 to 18 and the answer I found was 18." [Pointing to the number line drawn in a test]

43. Me: Alright.

Q 3

44. S1: Reads question 3

45. Me: What do you understand by this question?

46. S1: "there are 73 pencils altogether. 46 pencils are not sharpened; ati nima pencils a ngati amena ana shengedwa".

47. Me: Ok, then, how many pencils are sharpened?

48. S1: "Mmm…minus 46 from 73"

49. Me: What answer did you get?

50. S1: 27 pencils

51. Me: Good, which other method did you use to solve for sharpened pencils?

52. S1: "I used empty number line and place value".

53. Me: Explain using your diagram how you found the answer.

54. S1: "this is how I added using the number line" [explaining while pointing at a number line]

55. Me: Good.

Q 5

56. S1: Reads question 5

57. Me: What did you do to find the answer?

58. S1: "add"

59. Me: Why did you add?

60. S1: "because Chola WINS some marbles, that's why I need to add 29 and 14".

61. Me: Ok, when 29 and 14 are added together, which answer did you get?

62. S1: 43

63. Me: Ok, explain how you got 43?

64. S1: "9 plus 4 equals 13. Then I split 13 into two numbers (10 plus 3). I added 10 to 20 plus 10, and then added 3 to get 43."

65. Me: Alright, which other method did you use to find the answer?

66. S1: "empty number line"

67. Me: How did you use the number line to solve the problem?

68. S1: "let me explain using my answer sheet" [pointing at her answer sheet]

69. Me: Explain using your number line how you got 43.

70. S1: "I drew a number line of intervals ranging between 0 and 45. I counted 29 Interval and then 14 intervals. Then I added 29 and 14 intervals and got the answer is 43."

71. Me: Good.

Q 6

72. S1: Reads question 6

73. Me: What does the question mean to you?

74. S1: "Mark ana tumbeba tuli 7, Ruth ana tuli 15, I need to find the number when added to 7 gives 15".

75. Me: Good, how did you find the answer?

76. S1: " ni kuchita subtract"

77. Me: What answer did you get?

78. S1: 8 mice

79. Me: Ok, which other methods did use to find the answer?

80. S1: "I used the number line and place value".

The student used the two methods (empty number line and place value) and found the answer as 8 mice.

Q 7

81. S1: Reads question 7

82. Me: So, what does the question mean?

83. S1: "in a car sale, a man has 26, and it is 17 more than his friend Tim"

84. Me: Ok, how did you find the number of cars Tim has?

85. S1: "ni kuchita subtract (it is to subtract)".

86. Me: Ok, why did you need to subtract?

87. S1: "ku peza ma galimoto ya Tim (it's to find the number of cars Tim has)."

88. Me: Good, how did you subtract?

At this stage, the student used different methods such as empty number line, place value and tally to come up with 9 as answer.

89. Me: How did you use the number line to come up with 9 as an answer?

90. S1: "Mmm…move 26 steps to the right of 0, then 17 steps left of 26 and then the answer is 9".

91. Me: Good. Why moving 26 steps to the right not to the left of 0?

92. S1: "Mmm. I think 26 is total number of cars, I think it is greater than 0, so go to the right. The teacher said a number without minus is greater than 0."

93. Me: Ok, so why did you move 17 steps left of 26?

94. S1: "Mmm! ba sir, because it was to remove!"

95. Me: Good, why using place value to find the answer?

96. S1: "sir, place value was easy to use, our teacher taught us how to use it"

97. Me: Ok, why did you use tallies to find the answer?

98. S1: "Mmm! You ask too many questions!"

99. Me: I know that! But why did use tallies to find the answer?

100. S1: "tallying is always easy to use and you find the answer easily."

101. Me: Good.

Q 8

102. S1: Reads question 8

103. Me: What does the question mean?

104. S1: "(pause) 72 people were present at school play on Monday and 26 more joined the 72 people on Tuesday"

105. Me: Ok, if 26 more people joined the 72 on Tuesday, then, how many people were there on Tuesday?

106. S1: "I added 26 and 72".

107. Me: Are you sure that you needed add?

108. S1: yes

The student using different methods such as empty number line, place value, tally and grouping (using pencils) came up with 9 as an answer.

Q 9

109. S1: Reads question 9

110. Me: What were you required to find in this question?

111. S1: "the total number of students on the playground".

112. Me: How did you find the total number of students on the playground?

113. S1: "I added 28 and 35".

114. Me: Ok, which other methods did you use to find the total number of students on the playground?

115. S1: "number line, place value and tallies".

116. Me: How did you use them to find the answer?

117. S1: "let me use my workings I wrote in the test" [pointing at her illustrations while explaining]

118. Me: Good.

Q 10

119. S1: Reads question 10

120. Me: Ok, which method did you use to find the answer?

121. S1: "I added 47 to 8 and found the answer 55"

122. Me: How did you add then?

123. S1: "47 plus 8, first I added 7 plus 8 equals 15. I wrote 5 and then carried 1. The one I carried stand for ten, it was then added to 40 to give 50. Then 50 plus 5 equals 55."

124. Me: Good, which other method did you use to find the answer?

125. S1: " number line"

126. Me: How did you use the number line to solve the problem?

127. S1: "I drew a straight line and marked 0, 10, 20, 30, 40, 50, and 60. Then I counted up to 40 and added a 7 to get 47. I also counted 8 more to get 55. I guessed the positions of 47 and 55."

128. Me: How did you know if that is 47th position?

129. S1: "I just guessed"

130. Me: Why guessing?

131. S1: "mmm! the paper was too small to write numbers up to 47"

132. Me: Ok, what is your answer then?

133. S1: 55

134. Me: alright.

Q 1

135. S2: Reads question 1

136. Me: What does the question mean to you?

137. S2: "ati Everlyn has 144 spaces in a photo album that she puts photos".

138. Me: Good, what else can you say?

139. S2: "she has put 89 photos in her photo album."

140. Me: Ok, then what were you required to find?

141. S2: "Mmm! the number of photos which can put so that the album is full."

142. Me: Good, how did you find the number of photos which will make the album full?

143. S2: "Mmm! Can I explain using my answer sheet?"

144. Me: Yes! You can!

145. S2: "144 minus 89 equals to 55."

146. Me: How did you get 55?

147. S2: "I removed 89 from 144."

148. Me: Explain using necessary steps how got the answer?

149. S2: "4 minus 9 it can`t, I borrowed 1 from 4 leaving 3. 14 minus 9 equals to 5. Then 3 minus 8 it can`t, I borrow 1 from hundred`s column, so 13 minus 8 equals to 5. So then my answer is 55."

150. Me: Ok, which other method did you use apart from the one used in figure 3

151. S2: "number line"

152. Me: How did you use a number line?

153. S2: "I drew a straight line from 0 to 150. Each interval is 10 units. A line measuring 144 is drawn moving to the right direction of 0. Then the second line measuring 89 is drawn moving left of 144. The remaining 55 is the answer and is shown by the line below."

154. Me: Explain how you arrived at 55?

155. S2: "I counted steps from 144 to 55"

156. Me: looking at your number (figure 4) only lines are drawn, where are the steps you moved?

157. S2: "Mmm, but that's how I learnt it"

158. Me: Ok.

Q 2

159. S2: Reads question 2

160. Me: What does the question mean?

161. S2: "there are 3 packages and each has 6 pieces of gums."

162. Me: What were you required to find?

163. S2: "the total number of pieces of gums in all the three packages."

164. Me: How did you find the total number of gums?

165. S2: "I drew circles in rows and columns, 3 rows and 6 columns. Total count of circles gave 18 as the answer."

166. Me: Ok, which other way was used to find the answer?

167. S2: "I added 6 plus 6 plus 6 which is equals to 18."

168. Me: Good, why did you add 6, 3 times?

169. S2: "because there are 6 pieces of gums in each package and there are 3 packages."

170. Me: Alright, which other method did you use to calculate the answer?

171. S2: "I used a number line."

172. Me: Explain how you used a number line to find the answer?

173. S2: "I firstly drew a straight line and marked from 0 to 40 at interval of 5 units. First, second and third lines drawn above the markers are 6 units each in length. I counted all the intervals from 0 to 18; the answer I got was 18."

174. Me: Alright.

Q 3

175. S2: Reads question 3

176. Me: Ok, what does the question require you to do?

177. S2: "to find the number of sharpened pencils."

178. Me: Good, so, how did you find the number of sharpened pencils?

179. S2: "the total number of pencils is 73; the number of unsharpened pencils is 46, then 73 minus 46." The student used vertical method to solve the problem in question.

180. Me: Which other method did you use to answer the question?

181. S2: "count from 46 to 73, the total number of counts is 27, 27 plus 46 gives 73."

182. Me: Ok, apart from what you have done! Which other way did you use to find the answer?

183. S2: "I used a number line and place value."

184. Me: Good, how did you find the answer using the empty number line?

185. S2: "I moved from 0 to 73 and from 73, 46 steps backwards and end up on 27."

186. Me: Good, are you able to use place value to find the answer?

187. S2: Yes

Q 4

188. S2: Reads question 4

189. Me: Ok, what were you asked to find?

190. S2: "the number of students who remained on the playground."

191. Me: Good, how did you find the number of students who remained on the playground?

192. S1: "I subtracted 46 from 73; the answer I found is 27."

193. Me: Ok, what did you notice between question 3 and 4?

194. S2: "Mmm! they are almost similar, have the same answer and have the same methods."

195. Me: Good.

Q 5

196. S2: Reads question 5

197. Me: So, you are saying Chola has 29 marbles and wins 14 more, have you ever seen marbles before? What do you understand by the word WIN in relation to the question?

198. S2: "it's like you are playing the game, then you do better and collect some marbles to yourself."

199. Me: How did you find the number of marbles Sam has?

200. S2: "subtract"

201. Me: Ok, are you sure that you need to subtract?

202. S2: "Mmm! no, it's adding 29 and 14."

203. Me: Alright, how did you add?

204. S2: "Firstly, I did 20 plus 10 equals 30. Then I had 9 and 4, I add 9 plus 4 equals 13. I split 13 into two numbers (10 and 3). Finally I add 30 plus 10 plus 3 equals 43."

205. Me: Good, which other method did you use to find the answer?

206. S2: "I can get the answer by just counting"

207. Me: Ok.

Q 6

208. S2: Reads question 6

209. Me: Ok, how did you find the answer?

210. S2: "subtract"

211. Me: Why not adding?

212. S2: "because we must find more mice Ruth has than Mark."

213. Me: Ok, what did you do to find answer?

214. S2: "15 minus 7, 7 from 5 it can't, borrow 1 ten from tens and add it to 5 giving us 15. Then 7 from 15 is 8"

215. Me: Good, which other method did you use to find the answer?

216. S2: "Mmm! using the place value, I think!"

217. Me: Good.

Q 7

218. S2: Reads question 7

219. Me: What were you required to find?

220. S2: "the total number of cars Tim has."

221. Me: Ok, then what did you do to find the answer?

222. S2: "subtract"

223. Me: Good, how did you subtract?

224. S2: "26 minus 17 equals to 9 cars for Tim."

225. Me: Which other method did you use to find the answer?

226. S2: "number line and place value."

227. Me: Good.

Q 8

228. S2: Reads question 8

229. Me: What were you required to find?

230. S2: "the total number of people who came to a school play on Tuesday."

231. Me: Ok, how did you find the total number?

232. S2: "add 72 and 26."

233. Me: Good, when you added 72 and 26, what answer did you get?

234. S2: "2 plus 6 equals 8 and 7 plus 2 equals 9, then the answer I got was 98."

235. Me: Ok, which other method did you use to get 98?

236. S2: "place value"

The student was able to use the place value and hence found the answer as 98.

Q9

237. S2: Reads question 9

238. Me: What were you asked to find?

239. S2: "total number of students who were on the playground."

240. Me: Good, how did you find the answer?

241. S2: "add 28 and 35."

242. Me: When you add 28 and 35, what answer do you get?

243. S2: "8 plus 5 equals to 13 put 3 and carry 1. 2 plus 3 equals 5 plus 1 carried from 13 gives 6, and then the answer is 63."

244. Me: Good, which other methods did used and how do you use them?

245. S2: "number line and place value."

The student used both the number line and place value to find the answer 63.

Q10

246. S2: Reads question 10

247. Me: How did you find the last page he read?

248. S2: "by counting 8 more pages starting on 48, 49, 50, 51, 52, 53, 54, and 55." [Flapped the pages in a mathematics textbook, pupils book 5]

249. Me: Ok, which page did he end on?

250. S2: "page 55"

251. Me: Apart from counting the pages, which other method did you use in finding the answer?

252. S2: "Mmm! I used the place value."

253. Me: Illustrate how you used the place value.

254. S2: "I had 40 from 47 and I had the 10 from 15 because I had to split 15 into 10 and 5. Then I added 40 plus 10 equals 50, so 50 plus 5 equals 55."

255. Me: good.

Q1

256. S3**:** Reads question 1

257. Me: What does the question mean to you?

258. S3: **[**he remained quiet for some time**]**

259. Me: What did you understand by the term photo album?

260. S3: "album yo fakamo ma photos or ma snaps." [an album where photos or snaps are put]

261. Me: Good, yo fakamo ma snaps [where snaps are put], so what are you required to find?

262. S3: "niyenela kupeza ma space yamene ana salapo mu photo album" [I need to find the remaining spaces in the photo album]

263. Me: So udzachita chyani [what are you going to do]?

264. S3: "minus"

265. Me: How did you subtract then?

266. S3: "i can explain here" [point at his marked answer sheet]

267. Me: Ok, how did you find 145?

268. S3: "nachita [I did]; 9 minus 4 equals to 5. Remove 4 units from 9 units. 8 minus 4 equals to 4. Remove 4 from 8 tens. Drop 1 down in the answer space. 145 is the answer."

269. Me: What did you do with 1?

270. S3: "i dropped it."

271. Me: What was your answer in this case?

272. S3: 145

273. Me: Are you sure that 145 was the answer?

274. S3: [he remained quiet for some time and then answered "yes"]

275. Me: Ok, write 144 and 89 according to place value?

276. S3: "100 plus 40 plus 4 and 80 plus 9."

277. Me: Then, can you subtract the second set of place value from the first one.

278. S3: "First I borrowed 1 ten from 40, and added 10 plus 4 equals 14. Then 14 minus 9 equals to 5. I also borrowed 100 from column for 100 and added it to the reminder which is 30, and then got 130. So I removed 80 from 130 and the answer I got was 50. So then, 50 plus 5 equals to 55."

279. Me: Why do you think 55 is the answer?

280. S3: [he remained quiet for some time and then continued "when i add 50 plus 5 it gives me 55"]

281. Me: Good

The student solved the problem using the place value resulting into 55 as the answer; hence he realized that the answer he got at first as 145 was wrong.

Q2

282. S3: Reads question 2

283. Me: What does the question require you to find?

284. S3: "to add"

285. Me: Are you sure, you add?

286. S3: yes

287. Me: How did you get the answer then?

288. S3: "3 plus 6"

289. Me: Mmm! what answer did you get?

290. S3: 9

291. Me: Ok, read the question once more, what are you supposed to do?

292. S3: "is to add"

293. Me: How did you added?

294. S3: "Firstly, I wrote 3 times 6. Then, I added 3 plus 3 plus 3 plus 3 plus 3 plus 3 and got 18. I also tried to add 6 plus 6 plus 6 which as gave me 18. So I realized that addition simpler than times."

295. Me: What answer did you get after adding?

296. S3: 18

297. Me: Good, which other method did you use to find the answer?

298. S3: "3 times 6"

299. Me: Ok, what did you get when 3 was multiplied by 6?

300. S3: 18

301. Me: Good, any other method did you use to get 18?

302. S3: "a number line and drawings, like the one I have done."

303. Me: Explain how you found the answer using circles.

304. S3: "I drew circles in rows and columns. Each set of 6 circles equals to 1 row. Each column consists of 3 sets of circles. 3 represent the total number of packages of gums. 18 represent the total number of pieces of gums."

305. Me: Good, explain also on how you used a number line.

306. S3: "Firstly I drew the number line with markings from 0 to 18. Each count of 6 intervals is one set. I wrote one above each set of 6 intervals. The total number of sets is 3. Then I counted all the intervals and found 18 as the answer."

307. Me: alright.

Q3

308. S3: Reads question 3

309. Me: What were you asked to find in this question?

310. S3: "ati tipeze ma pencil yoshengewa (sharpened pencils)."

311. Me: Ok, how did you find the number of sharpened pencils?

312. S3: "minus 46 from 73"

313. Me: How did you subtract?

314. S3: "6 minus 3 equals 3, 7 minus 4 equals 3 and the answer is 33."

315. Me: Ok, think of another way of getting the answer.

316. S3: "place value"

317. Me: How did you use the place value to get the answer?

318. S3: "70 plus 3 minus 40 plus 6, so, 3 tichoseko (take away) 6, it can't, borrow 1 ten from 70 and add it to 3, I get 13 then I remove 6, the answer remaining is 7. 60 tichoseko (take away) 40, the answer is 20. So, 20 plus 7 gives me 27."

319. Me: Good, was your answer the same with the one you got previously?

320. S3: "no, I just made a mistake."

321. Me: Ok.

Q4

322. S3: Reads question 4

323. Me: What is the meaning of question 4?

324. S3: "ati 73 bana basikulu ana yenda ku ground yamasobelo. Bana bali 46 ana bwelela ku sikulu, ni bangati amene ana sala ku ground?" [There were 73 students on the playground. The 46 fifth grade students came in first. How many students were still on the playground?]

325. Me: Ok, how did you find the number of those who remained at the playground?

326. S3: "subtract"

327. Me: Good, how did you subtract?

328. S3: "iyi question ni same na question 3 [this question is the same as question 3], i use same methods and get same answer."

329. Me: what was the answer?

Q5

330. S3: Reads question 5

331. Me: Can you spell the word marble without looking at your question paper.

332. S3: "merble"

333. Me: Check for word and compare it to what you have spelled.

334. Me: What did you do to find the answer?

335. S3: "add"

336. Me: Why did you add?

337. S3: "ku win ni ku adding (to win is to add)"

338. Me: Ok, how did you find answer?

339. S3: "29 plus 14, 9 plus 4 is 13, put 3 and carry 1. 2 plus 1 is 3, and then put together 3 and 1 is 4, so the answer is 43

340. Me: Alright, which other method did you use in finding the answer?

341. S3: "place value, I can explain using my answer sheet" [pointing at the answer sheet]

342. Me: Explain how you found the answer.

343. S3: "First I took 9 from 29 and 4 from the 14. I add 4 plus 9 equals 13, add 20 plus 10 equals 30, then I added 10 plus 20 plus 10 plus 3 equals 43."

344. Me: Are the two methods used the same?

345. S3: Yes

346. Me: Why are you saying they are the same?

347. S3: “addition is done the same”

348. Me: Alright.

Q 6

349. S3: Reads question 6

350. Me: What does the question require you to do?

351. S3: “subtract”

352. Me: Ok, how did you subtract?

353. S3: “15 minus 7 equals to 8”

354. Me: Why 8?

355. S3: “5 minus 7 it can’t, borrow 1 ten and add to 5 giving me 15. Then 15 take away 7 equals to 8.”

356. Me: Good, which other method did you use to find the answer?

357. S3: “place value, 10 plus 5 and 7. 5 take away 7 it can’t, borrow 10 plus 5 equals 15 take away 7 equals 8.”

358. Me: Good.

Q 7

359. S3: Reads question 7

360. Me: What did you do to get the answer?

361. S3: “added”

362. Me: Are you sure, you needed to add?

363. S3: “no, I subtracted”

364. Me: Why did you subtract and not adding?

365. S3: “26 is 17 more, so Tim ana (has) small cars.”

366. Me: Good, what answer did you get when you subtracted?

367. S3: 11

368. Me: How did you subtract?

369. S3: “7 minus 6 equals 1, and 2 minus 1 equals 1, then the answer is 11.”

370. Me: Are you sure?

371. S3: “Mmm! no it is to borrow.”

372. Me: How did you work it out then?

373. S3: “2 take away 1, 16 take away 7 equals 9, 1 take away 1 equals 0, the answer is 9.”

374. Me: Good, which other method/s did you to find the answer?

375. S3: “place value.”

376. Me: Ok

Q 8

377. S3: Reads question 8

378. Me: What were you required to do to get the answer?

379. S3: “add”

380. Me: Why adding?

381. S3: “kuti tipeze number ya bantu onse amene ana bwela ku ma sobelo yapa sikulu pa Tuesday.” [to find the total number of people who attended the school play on Tuesday]

382. Me: Ok, how did you find the answer?

383. S3: “72 plus 26 equals to 98”

384. Me: Which other method did you use to find the answer?

385. S3: “place value”

386. Me: Good.

Q 9

387. S3: Reads question 9

388. Me: Good, how did you find the answer?

389. S3: “subtract”

390. Me: Why subtracting?

391. S3: [he remained quiet for some time and then said “I think it is adding”]

392. Me: Ok, if you think it is addition, how do you add then?

393. S3: “8 plus 5 equals 13 put 3 carry 1. 2 plus 3 is 5 plus 1 equals to 6, and then the answer is 63.”

394. Me: Good, which other method did you use to find the answer?

395. S3: “place value”

396. Me: Ok, how did you use the place value?

397. S3: “20 plus 8 plus 30 plus 5 equals 50 plus 13 equals 63.”

398. Me: Good.

Q 10

399. S3: Reads question 10

400. Me: Good, how did you understand question 10?

401. S3: “ati Richard anali pa page 47 mu buku. Abelenga ma page ali 8, ana pelela pa page chyani?” [Richard was on page 47 in a book. He read 8 pages, on which page did he end on?]

402. Me: Ok, what did you do to find the answer?

403. S3: "add"

404. Me: How did you add?

405. S3: "I just added 47 plus 8 equals 55. I used my fingers to count 8 starting on 48, and got 55 as my final answer"

406. Me: Good, which other method are you going to use to find the answer?

407. S3: "place value"

408. Me: Ok, how did you find the answer using the place value?

409. S3: "40 plus 7 plus 8 equals to 40 plus 15 equals to 40 plus 10 plus 5 which gives me 50 plus 5 equals to 55."

410. Me: Alright, are the two strategies the same?

411. S3: "yeah"

412. Me: Why are two strategies the same?

413. S3: "mmm! In the first method, there is 40 and 7 plus 8, though it is not shown, but in the second strategy I did show them"

414. Me: Good.

Q1

415. Me: Reads question 1

416. S4: Read question 1[reading was a major challenge for this student]

417. Me: What is a photo album?

418. S4: "buku yoyikamo ma snaps." [book were snaps are put]

419. Me: Ok, how many spaces are there in a photo album?

420. S4: "Mmm! 144 spaces."

421. Me: How many spaces were used in a photo album?

422. S4: 89

423. Me: Ok, how did you find the remaining spaces in a photo album?

424. S4: [she remained quiet for some time until question in utterance 424 was asked]

425. Me: How did you find the remaining spaces, if 89 spaces were used up?

426. S4: "add"

427. Me: Ok, are you sure that you needed to add?

428. S4: "no, subtraction."

429. Me: Good, explain how you found the remaining spaces.

430. S4: "144 minus 89, so 4 minus 9 it can`t, borrow 1 from 4 and leave 3 and then I did 14 minus 9 equals 5. Again, I did 3 minus 8 it can`t, borrow 1 from left, then 13 minus 8 equals 5. So 55 is the answer."

431. Me: Ok, which other method did you use to find the answer?

432. S4: “place value”

433. Me: Ok, explain how you found the answer using the place value.

282. S4: “144 has 1 hundred, 4 tens and 4 units. 89 has 8 tens and 9 units. 1 hundred plus 4 tens plus 4units can be written as 100 plus 40 plus 4. 8 tens plus 9 units can also be written as 80 plus 9. 4 minus 9 it can`t, borrow 1 ten. 10 plus 4 minus 9 equals 5. 30 minus 80 it can`t, then borrow 100 add to 30. So 130 minus 80 equals to 50. Then 50 plus 5 equals 55.”

283. Me: Good.

Q 2

284. S4: Reads question 2

285. Me: How did you use pencils to find solution to the problem?

286. S4: “6 set of pencils represent 1 package, so, 18 pencils represent 3 packages of gums.”

287. Me: Good, which other way did you to find the answer?

288. S4: “3 plus 6”

289. Me: Ok, what was the answer?

290. S4: 9

291. Me: Are you sure that 9 was the answer?

292. S4: “Mmm! no, it was to times 3 and 6.”

293. Me: What answer did you get?

294. S4: 18

295. Me: Good

Q 3

296. S4: Reads question 3

297. Me: Ok, how did you find the number of pencils not sharpened?

298. S4: “add”

299. Me: When you added, what answer was your answer?

300. S4: 119

301. Me: How did you get 119 as the answer?

302. S4: “3 plus 6 equals 9 and 7 plus 4 equals 11, so the answer is 119.”

303. Me: Read the question once more and tell me how you understood it?

304. S4: “I needed to find the number of pencils sharpened.”

305. Me: Ok, how did you find the number of sharpened pencils?

306. S4: “take away”

307. Me: How did you take away?

308. S4: "73 minus 46, which gave me 27." [pointing at the marked answer sheet]

309. Me: Good.

Q 5

310. S4: Reads question 5

311. Me: How many marbles has Chola and how many did she win?

312. S4: "has 29 and won 14 mobles."

313. Me: Which strategy did you use to find solution for question 5?

314. S4: "add 29 and 14"

315. Me: Good, explain how you found the answer?

316. S4: "9 plus 4 equals 13. Carry the 1 and add it to the 2 plus 1. Then 1 plus 2 plus 1 equals 4 So the answer is 43."

317. Me: Ok, 43 marbles, which other method did you use to find the answer?

318. S4: "place value"

319. Me: How do you find the answer?

320. S4: "I did 9 plus 4 equals 13 and 20 plus 10 equals 30. Then I added 30 plus 10 plus 3

equals 43."

321. Me: What did you notice about the methods used?

322. S4: "Mmm! There are different"

323. Me: What made you say that the methods are different?

324. S4: "because in the first method, it is 29 plus 14 and in the second method, it is (20 plus 9) plus (10 plus 4)"

325. Me: Are you sure that 29 and (20 plus 9) are different?

326. S4: yes

327. Me: why?

328. S4: "because in the first number, 2 and 9 are connected and in the other one, 20 and 9 are separate".

329. Me: Alright.

Q6

330. S4: Reads question 6

331. Me: How did you find the answer?

332. S4: "times"

333. Me. Ok, can you multiply!

334. S4: "15 times 7 equals to 42"

335. Me: Explain how you got 42.

336. S4: "5 times 7 equals 32, put 2 and carry 3, 3 plus 1 equals 4, the answer is 42."

337. Me: Explain how you understood the question?

338. S4: "oh, I should find the difference between 15 and 7."

339. Me: Right! How did you find the difference?

340. S4: "take away"

341. Me: When you "take away", what is the answer?

342. S4: 9

343. Me: Good

Q 7

344. S4: Reads question 7

345. Me: Ok, a man has 26 cars and is 17 more than Tim, how do you find the total number of cars for Tim?

346. S4: "add"

347. Me: Why did you add?

348. S4: "to find the number of cars for Tim."

349. Me: How did you add?

350. S4: "26 plus 17 equals to 43"

351. Me: If 43 is the answer, how do you work it out?

352. S4: "6 plus 7 plus 13, put 3 and carry 1, then 2 plus 1 plus 1 plus 4, so the answer is 43."

353. Me: Are you sure that 43 is the answer?

354. S4: "Mmm! ni kuchosa [take away]"

355. Me: What was your answer in this case?

356. S4: 9

357. Me: Good, which other method did you use to find the answer?

358. S4: "place value"

359. Me: How did you use the place value to find the answer?

360. S4: "20 plus 6 minus 10 plus 7, 6 minus 7 it can't, borrow 1 tenth and add 10 plus 6 equals to 16. Then 16 minus 7 equals to 9 cars for Tim."

361. Me: Good.

Q 8

362. S4: Reads question 8

363. Me: What does the question require you to find?

364. S4: "the total number of people who watched the play on Tuesday."

365. Me: Good, how did you find the total number of people who watched the play on Tuesday?

366. S4: "added"

367. Me: Ok, how did you add?

368. S4: "72 plus 26 equals to 98."

369. Me: Good, which other methods did you use to find the answer?

370. S4: "70 plus 2 plus 20 plus 6 equals to 70 plus 20 plus 2 plus 6 equals to 90 plus 8, and the answer is 98."

371. Me: Good.

Q 9

372. S4: Reads question 9

373. Me: What does the question require you to do?

374. S4: "bakazi bali 28 na bamuna 35 banapezeka ku place yamasobela, bali bangati pamozi ku place yamasobelo?" [28 girls and 35 boys were at the playground. how many are there at the playground?]

375. Me: Ok, what was the total number of students at the playground?

376. S4: "ni ku [it is] adding 28 plus 35."

377. Me: Good, how did you add to find the answer?

378. S4: "8 plus 5 is 13, leave 3 and carry 1. 2 plus 3 plus 1 equals to 6, then 63 is the answer."

379. Me: Good, which other method did you use to find the answer?

380. S4: "place value"

381. Me: Ok, how did you use the place value to find the answer?

382. S4: "20 plus 8 plus 30 plus 5 equals to 20 plus 30 plus 8 plus 5, gives 50 plus 5 equals to 55."

383. Me: Good.

Q 10

384. S4: Reads question **10**

385. Me: How did you find the answer to question 10?

386. S4: "add"

387. Me: Ok, how did you add?

388. S4: "47 plus 8, 7 plus 8 equals to 15. Leave 5 and carry 1, 4 plus 1 equals to 5, so, the answer is 55."

389. Me: Good, which other methods did you use to find the answer?

390. S4: "First I added 7 plus 8 equals 15. Then I split 15 in ones and tens, which means I have 10 and 5. Then I added 40 equals 10 equals 50, so 50 plus 5 equals 55."

391. Me: What does number 40 represent in your procedure?

392. S4: "it is tens"

393. Me: How many tens are there in 40?

394. S4: 4

395. Me: Ok, how many tens and ones are there in 15?

396. S4: "just one ten and five ones"

397. Me: What did you do with the one ten from 15?

398. S4: "add it 40, and then get 50"

399. Me: Alright, what was your final answer?

400. S4: "50 plus 5 equals 55"

401. Me: Good.

Q 1

402. S5: Reads question **1**

403. Me: Ok, what does the first question mean to you?

404. S5: "it means that Everlyn has 144 spaces in her photo album."

405. Me: What is a photo album, you can explain in Chinyanja?

406. S5: "ni buku yo fakilamo ma photo" [it's a book were photos are put]

407. Me: Good, how many spaces are there in total and how many spaces have been used?

408. S5: "144 spaces and 89 spaces are used."

409. Me: What were you required to find?

410. S5: "bafuna nipaze ma spaces yasalako" [they me to find the remaining spaces]

411. Me: Alright, how did you find the remaining spaces?

412. S5: "I can use my answer sheet to explain."

413. Me: What did you do then?

414. S5: "subtracting"

415. Me: Ok, how did you subtract?

416. S5: "I started by subtracting 4 minus 9, it can`t, borrow 1 from tens column. So 14 minus 9 equal to 5. Then, 3 minus 8 it can`t, so borrow 1 from the column of hundred and then 13 minus 8 equals to 5. So 55 was my answer."

417. Me: Ok, which other method did you use to find the answer?

418. S5: "yes, I counted 89 spaces from 144 until I got 55."

419. Me: Good, apart from counting, which other way did you use to find the answer?

420. S5: "place value"

421. Me: How did you use the place value"

422. S5: "144 minus 89 equals question mark. Borrow 1ten plus 4, minus 9 equals 5. Borrow 1hundred plus 3tens minus 8tens equals 50. Then I added 50 plus 5. So the answer is 55."

423. Me: Alright

Q 2

424. S5: Reads question 2

425. Me: What does the question require you to do?

426. S5: [she remained quiet for some time and then continued "to find the total number of gums"]

427. Me: Good, how did you find the total number of gums?

428. S5: "Firstly i had to write down 3 sets of 6 marking each. Then I counted all the markings and got a total of 18 markings."

429. Me: Ok, which other method did you use apart from counting?

430. S5: "Mmm! add 6 and 6 and 6."

431. Me: what else did you do to get the answer?

432. S5: "3 times 6"

433. Me: Good.

Q 3

434. S5: Reads question 3

435. Me: What does the question mean?

436. S5: "I understood it by, Mulenga counted 73 pencils, some were sharpened and some were not sharpened."

437. Me: How many pencils were not sharpened?

438. S5: "Mmm! 46"

439. Me: What were you required to find if 46 pencils were not sharpened?

440. S5: "I was to find pencils which were sharpened."

441. Me: How did you find the total number of pencils which are sharpened?

442. S5: [she remained quiet for some time and then responded "subtract"]

443. Me: Good, how did you subtract?

444. S5: "73 minus 46, the answer equals to 27."

445. Me: Which other method did you use to find the answer?

446. S5: "place value."

447. Me: Good.

Q 4

448. S5: Reads question 4

449. Me: How did you understand this question? Explain as much as you can.

450. S5: "I understand it by 73 students were on the playground and 46 students came back in school."

451. Me: How did you find the total number of students who remained on the ground?

452. S5: "subtract."

453. Me: Good, how did you subtract?

454. S5: "73 minus 46, 27 is the answer, which is the same as the one in question 3."

455. Me: Good

Q 5

456. S5: Reads question 5

457. Me: What is the meaning of the question?

458. S5: "It means that Chola has 29 marbles and win 14 marbles."

459. Me: So, how did you find the answer?

460. S5: "I added"

461. Me: How did you add in this case?

462. S5: "I started by adding 9 plus 4, I got 13. I wrote 3 and carried 1, and then I added the 1 carried to 2 plus 1 and got 4. So I wrote 4 on the answer space. The answer I got was 43."

463. Me: Which other method did you use in finding the answer?

464. S5: "place value"

465. Me: Explain how you found the answer?

466. S5: "I took the 9 from 29 and the 4 from 14. And I added; 9 plus 4 equals 13. Then I added 20 plus 10 equals 30. Finally, I had to add 30 plus 13 equals 43."

467. Me: What else did you do to find the answer?

468. S5: "I counted the 29 sticks and then 14 sticks to get 43 sticks"

469. Me: Good.

Q 6

470. S5: Reads question 6

471. Me: How did you understand by this question?

472. S5: "Mark has 7 mice and Ruth has 15 mice."

473. Me: How many more mice does Mark need to have the same number as Ruth?

474. S5: 8

475. Me: Ok, how did you get 8 as the answer?

476. S5: "Mmm! 5 minus 7 it can't, I borrowed 1 to make it 15, then 15 minus 7 equals to 8."

477. Me: Good

Q 7

478. S5: Reads question 7

479. Me: Ok, what did you do to find the answer?

480. S5: "subtract"

481. Me: Good, how did you subtract?

482. S5: "6 minus 7 it can't i borrow 1 from 2 to give 16 and I am going to remove 7 and the answer equals to 9. 1 minus 1 equals to 0, and then the final answer is 9."

483. Me: Good, which other method did you use to find the answer?

484. S5: "place value"

485. Me: Good.

Q 8

486. S5: Reads question 8

487. Me: What were you required to do to find the answer?

488. S5: [she remained quiet for some time until an example was given in utterance 634]

489. Me: If 2 students came to school play on Monday and Tuesday 4 more student came to school play, how many are there all together?

490. S5: "oh…I know! It is adding altogether"

491. Me: Ok, how did you find the answer?

492. S5: "72 plus 26 equals to 98"

493. Me: Good, which other method did you use to find the answer?

494. S5: "place value"

495. Me: Good.

Q 9

496. S5: Reads question 9

497. Me: How did you find the answer?

498. S5: "add"

499. Me: Ok, how did you add then?

500. S5: "8 plus 5 equals to 13, put 3 and carry 1. Then 2 plus 3 plus 1 equals to 6. So, 63 is the answer"

501. Me: Good, which other method did you use to find the answer?

502. S5: “place value”

503. Me Ok, how did you find the answer using the place value?

504. S5: “20 plus 8 plus 30 plus 5 equals to 50 plus 13. Then 50 plus 13 equals to 63.”

505. Me: Good.

Q 10

506. S5: Reads question 10

507. Me: How did you find the answer?

508. S5: “nina count ma pages mu buku 2 ya ma maths” [I counted pages in maths book 2].

509. Me: Explain how you counted the pages using the mathematics textbook?

510. S5: “48, 49, 50, 51, 52, 53, 54, 55, the answer is 55.” [Flapped the pages in a mathematics textbook, pupils book 2]

511. Me: Ok, which other method did use to find the answer?

512. S5: “adding”

513. Me: Ok, how did you add?

514. S5: “47 plus 8 equals to 55.”

515. Me: Good, how did you get 55?

516. S5: “7 plus 8 equals 15, leave 5 and carry 1, then 4 plus 1 carried is 5, then the answer is 55”.

517. Me: What do the numbers 4 and 7 represent in place value?

518. S5: “4 stand for tens and 7 for units”

519. Me: How many tens and units are in 55?

520. S5: “5 tens and 5 units, oh I have remembered another method I used”

521. Me: Which method have you remembered and how did you use it?

522. S5: “First I added 7 plus 8 equals 15 put 5 and carry 1ten. 40 plus 0 equals 40. Then 40 plus 10 equals 50. So 50 plus 5 equals 55.”

523. Me: Where did you get the 40 from?

524. S5: “4 tens is the same as 40”

525. Me: Alright