

# Clock Control Sequence Reconstruction in the Generalized Shrinking Generator

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## Abstract

The shrinking generator is a popular representation of pseudorandom sequence generators (PRSGs) that employ so-called irregular clocking. This family of PRSGs is characterized by having one or more linear feedback shift registers (LFSRs), whose clocking is controlled by the output sequence of one of the subgenerators. Such a sequence is called clock-control sequence. In this thesis, an algorithm is presented that reconstructs the clock-control sequence in the generalized shrinking generator in the presence of noise. The shrinking generator is first reduced to a step1-stepE generator (another class of PRSGs with irregular clocking), where E depends on the maximum length of runs of zeros in the output sequence of the clocking part of the generator. Then a directed depth-first like search for optimal and suboptimal paths in the edit distance matrix corresponding to the generator is performed. The permitted path weight deviation from the optimum is determined by the noise level in the statistical model of the generator. Influence of inadequate estimation of the length of the output sequence of the clocked LFSR without irregular clocking on the level of equivalent noise is discussed. The experimental results that are presented show that the total number of candidate clock-control sequences increases moderately as the necessary clock-control sequence length increases. The advantage of this attack over the other attacks reported in the litterature is that this attack is effective even if the noise level is relatively high and that the solution is guaranteed to be found.



## Sammendrag

"Shrinking" generatoren er en populær representant for pseudotilfeldige sekvensgeneratører (PTSG) som anvender såkalt uregelmessig klokking. Det karakteristiske ved denne familien av PTSG er at de har en eller flere lineært tilbakekoblede skift registre(LTSR) hvis klokking blir kontrollert av sekvensen til en av subgeneratorene. Denne sekvensen kalles klokke-kontroll sekvensen. I denne rapporten presenterer vi en algoritme som rekonstruerer klokke-kontroll sekvensen til en generalisert "shrinking" generator med tilstedeværelse av støy. "Shrinking" generatoren blir først redusert til en step1-stepE generator (en annen klasse av PTSG med uregelmessig klokking), hvor E avhenger av den maksimale lengden på etterfølgende nuller i ut-sekvensen til den klokkende delen av generatoren. Deretter utføres det et retningsstyrte søk for optimale og suboptimale stier i "edit-distance" matrisen som samsvarer med generatoren. Det tillatte vekt-avviket fra det optimale bestemmes av støy-nivået i den statistiske modellen av generatoren. Innvirkningen som utilstrekkelig estimering av lengden på ut-sekvensen fra den klokkede LTSR uten uregelmessig klokking har på nivået tilsvarende støy, diskuteres. Forsøksresultane som presenteres viser at totalt antall klokke-kontroll sekvens kandidater øker moderat etter hvert som den nødvendige klokke-kontroll sekvens lengden øker. Fordelen med dette angrepet i forhold til andre angrep rapportert i litteraturen, er at dette angrepet er effektivt selv om støy-nivået er relativt høyt og at løsningen garantert vil bli funnet.



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## 1 Introduction

The shrinking generator [1] (Figure 1) is a popular example of a pseudorandom sequence generator with irregular clocking. The generator produces output sequences with good cryptographic characteristics (long period, high linear complexity, good statistical properties, etc.) However, if a sufficiently long prefix of the output sequence of such a generator is known, it is possible to reconstruct the initial state of the LFSR<sub>A</sub> by means of a generalized correlation attack. In [2] it was shown for a general type pseudorandom generator with irregular clocking that, by making use of a special statistical model, it is possible to determine a set of candidate initial states of the clocked LFSR, which could generate the intercepted output sequence. The statistical model employs the edit distance with the constraint on the maximum length of runs of deletions. Once the set of candidate initial states is known, the attack continues by determining the clock control sequence that, together with one of the candidate initial states of the LFSR, could generate the intercepted sequence. In this thesis, we describe a deterministic method of reconstruction

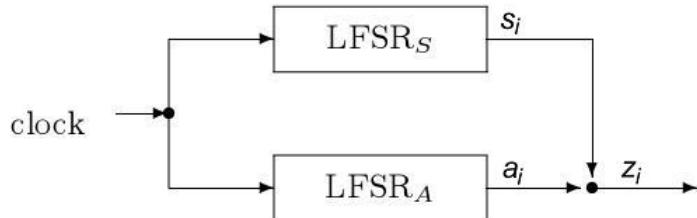


Figure 1: The shrinking generator

of clock control sequence in the generalized shrinking generator in the ciphertext only attack scenario. The shrinking generator from Figure 1 is generalized in the sense that LFSR<sub>S</sub> may be replaced with a general type subgenerator. Such a generalized shrinking generator is first reduced to a step1-stepE generator, where E depends on the maximum length of runs of zeros in the output sequence of the clocking part of the generator. Then a "depth-first"-like search through the constrained edit distance matrix associated with every candidate initial state of the LFSR<sub>A</sub> is used. The paths in this matrix that correspond to candidate clock control sequences are reconstructed. Influence of noise is taken into account by relating the noise level with the permitted weight deviation from the optimum path weight used in the search process. By starting with the reconstruction of paths whose weight deviation from the optimum is 0 (the optimal paths - without noise) and then by increasing this weight deviation according to the noise level (the suboptimal paths), we make our search a directed one.

## 1.1 Preliminaries

To generate a random sequence of 0s and 1s, a coin could be tossed with a head landing up recorded as a 1 and a tail as a 0. It is assumed that the coin is unbiased, which means that the probability of a 1 on a given toss is exactly 1/2. This will depend on how well the coin is made and how the toss is performed. This method would be of little value in a system where random sequences must be generated quickly and often. It has no practical value other than to serve as an example of the idea of random number generation.

Since most true sources of random sequences (the concept of "randomness" is also questionable, but discussing this would be beyond the scope of this thesis) come from physical means, they tend to be either costly or slow in their generation. To overcome these problems, methods have been devised to construct pseudorandom<sup>1</sup> sequences in a deterministic manner from a shorter random sequence called a *seed*. The pseudorandom sequences appear to be generated by a truly random source to anyone not knowing the method of generation. Often the generation algorithm is known to all, but the seed is unknown except by the entity generating the sequence. A plethora of algorithms has been developed to generate pseudorandom bit sequences of various types.

A pseudorandom sequence generator (PRSG) is a deterministic<sup>2</sup> algorithm which, given a truly random binary sequence of length  $k$ , outputs a binary sequence of length  $l \gg k$  which "appears" to be random. The input to the PRSG is called the seed, while the output of the PRSG is called a pseudorandom bit sequence or the output sequence.

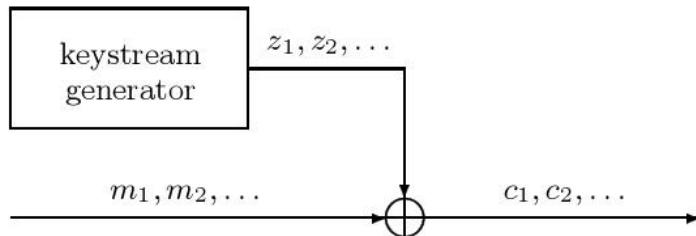


Figure 2: A PRSG used as a keystream generator in a simple stream cipher

Pseudorandom sequence generators are often used in cryptography for enciphering/deciphering (although their use is not limited to this). Such a generator is used as a keystream generator in a cipher system (Figure 2).

When used in cryptography, the output sequence of a PRSG should have at least the following properties

- long period
- good statistical properties and
- large linear complexity.

<sup>1</sup>From Clue: Pseudo- is used to form nouns and adjectives that refer to or describe things that are not really what they seem or claim to be

<sup>2</sup>Deterministic here means that given the same initial seed, the generator will always produce the same output sequence

An LFSR with length  $L$  and a primitive connection polynomial of degree  $L$  is often referred to as a maximum length LFSR [3, 4]. This means that such an LFSR will produce an output sequence, called an  $m$ -sequence, of maximum length, i.e. the period of the LFSR will be of length  $2^L - 1$ . Thus, one could say that a maximum-length LFSR is capable of generating an output sequence with *long period*. In the rest of this thesis, when referring to LFSR, we mean LFSR with period of maximum length.

Golomb's randomness postulates establish some *necessary* conditions for a periodic pseudorandom sequence to look random. The first postulate states that in a periodic binary sequence, the number of 1s differs from the number of 0s by at most 1. The second postulate states that of the total number of runs<sup>3</sup> in a periodic binary sequence, approximately half of the runs have length 1, approximately one-fourth have length 2, approximately one-eighth have length 3, etc. The output sequence of an LFSR also satisfies Golomb's randomness postulates. Thus, one could say that the output sequence also has *good statistical properties*, i.e. it looks random.

The last desireable property of a PRSG is that it should also have large linear complexity (LC). LC is defined as the length of the smallest LFSR capable of generating the given output sequence. LC is a measure of unpredictability. The Berlekamp-Massey algorithm [4] may be used to calculate the LC of a binary sequence (if LC is not too large, since the Berlekamp-Massey algorithm has quadratic time complexity). The algorithm also determines the characteristics of the LFSR, namely the connection polynomial and the initial state. What makes an LFSR unsuitable for the generation of pseudorandom sequences in cipher systems, is that by using the Berlekamp-Massey algorithm we need only a relatively few consecutive bits of the LFSR's output sequence in order to easily predict the rest of the sequence.

Although LFSRs cannot be used directly as a PRSG in cipher systems, they are popular building blocks in PRSGs. The goal when designing LFSR-based PRSG is to increase the LC of the LFSR's output sequence to a level that is too high for the Berlekamp-Massey algorithm, while preserving the good statistical properties. There are various approaches to this. One of them, which we analyse in this thesis, is to use PRSG with irregular clocking. These are constructions where one or more LFSRs are used to control the clocking of one or more other LFSRs.

An example of a typical clock controlled PRSG is depicted in Figure 3.  $\text{LFSR}_S$  is used to control the clocking of  $\text{LFSR}_A$ . If the current bit from  $\text{LFSR}_S$  is 0 then  $\text{LFSR}_A$  is clocked regularly, i.e. stepped once. If the current bit from  $\text{LFSR}_S$  is 1 then  $\text{LFSR}_A$  is clocked twice, i.e. one bit from  $\text{LFSR}_A$  is deleted or omitted.

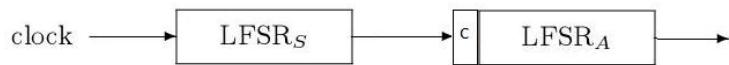


Figure 3: The step1-step2 generator

The generator from Figure 3 is easily generalized by using more than one bit of the output of the  $\text{LFSR}_S$  (for example by combining bits from several positions of the  $\text{LFSR}_S$ ) for clocking. In this thesis, we call such a generalized generator the step1-stepE gen-

<sup>3</sup>Let  $s$  be a binary sequence. A *run* of  $s$  is a subsequence of  $s$  consisting of consecutive 0s or consecutive 1s which is neither preceded nor succeeded by the same symbol.

erator, where  $E$  is the maximum number of skipped bits from the output sequence of  $\text{LFSR}_A$ .

The shrinking generator is another popular example of a PRSG with irregular clocking. It is a simple construction that consists of two linear feedback shift registers (LFSRs),  $\text{LFSR}_A$  and  $\text{LFSR}_S$ . If the output sequence from  $\text{LFSR}_A$  is  $\mathbf{a} = a_1, a_2, \dots$ , and the output sequence from  $\text{LFSR}_S$  is  $\mathbf{s} = s_1, s_2, \dots$ , then the output sequence  $\mathbf{z} = z_1, z_2, \dots$  of the shrinking generator is the sequence obtained from  $\mathbf{a}$  by removing all  $a_i$ 's for which  $s_i = 0$ . Let  $l_A$  and  $l_S$  be the lengths of the  $\text{LFSR}_A$  and  $\text{LFSR}_S$  respectively. If  $\gcd(l_A, l_S) = 1$  then the output sequence  $\mathbf{z}$  of the shrinking generator has period  $(2^{l_A} - 1)(2^{l_S} - 1)$ . The linear complexity  $\text{LC}(\mathbf{z})$  of  $\mathbf{z}$  satisfies  $l_A(2^{l_S} - 2) < \text{LC}(\mathbf{z}) \leq l_A(2^{l_S} - 1)$ , see [4].

There are many PRSGs with high linear complexity that are still vulnerable to correlation attacks. The basic idea behind a correlation attack is to identify correlation between the output of the generator and the output of one of its LFSRs. One way of measuring correlation between two sequences is by using the Hamming distance. However, the Hamming distance is used to compare sequences of the same length. If irregular clocking is employed in a PRSG, one should determine correlation (more precisely, generalized correlation in this case), by comparing two (or more) sequences of different lengths. Since Hamming distance cannot be used for this, using the Levenshtein Distance is an alternative. The Levenshtein or edit distance may be defined as the minimum number of elementary operations (insertions, deletions and substitutions) required to transform one sequence  $X$  of length  $N$  into another sequence  $Y$  of length  $M$ . The dynamic programming approach is a classical solution for computing the edit distance matrix, where the distances between longer and longer prefixes of the sequences are successively evaluated until the final result is achieved. When applying an edit distance attack to a PRSG and depending on the generator design, some edit operations may be restricted. In this case, a so-called constrained edit distance may be necessary [5]. An efficient dynamic programming algorithm for the computation of the constrained edit distance was given in [2].

## 2 Previous work

When attacking clock-controlled generators like the shrinking generator (SG) the goal is to recover the generator's initial settings. In the case of the SG, these settings include both initial states of the LFSRs in the design. The attack can be divided into two phases. In the phase one the goal is to recover the initial state of the clocked LFSR. In the second phase the goal is to recover the clock-control sequence and consequently the initial state of the clocking subgenerator. A common assumption by most authors is that the connection polynomials are known and that they possess a noiseless segment of the output sequence (i.e. the attack is in the known-plaintext scenario).

Several approaches to the problem of reconstructing the initial settings of the SG and the related generators can be found in the literature. One approach, described in [6, 7] is the coding theory approach. In [6] two fast techniques for reconstructing the initial state of the clock-control shift register in the SG, one based on list decoding and the other on information set decoding, are investigated. Both start from computing the a'priori probabilities for the clock-control bits from a known segment of the output sequence, under the assumption that the initial state of the clock-controlled shift register is already recovered. The influence of noise is not considered, e.g. techniques are developed in the known-plaintext attack scenario.

In [7] a MAP<sup>1</sup> decoding technique is considered. Correlation attacks on the SG and the Alternating Step Generator based on an identified relation to the decoding problem for the deletion channel and the insertion channel, respectively, are presented. Several ways of reducing the decoding complexity are proposed and investigated, resulting in "divide-and-conquer" attacks on the two generators having considerably lower complexity than previously known attacks. The influence of noise is not considered.

[8] describes methods for fast correlation attacks, based on an interesting observation, namely that one can identify an embedded low-rate convolutional code in the code generated by the LFSR sequences. This embedded convolutional code can then be decoded with low complexity, using the Viterbi algorithm (see for example [9]). From the result of the decoding phase, the secret key can be obtained. These algorithms provide a remarkable improvement over previous methods, however they are also developed in the known-plaintext attack scenario.

In [10] a relatively new and interesting linear consistency attack, with lower complexity than previous, on a general model for clock controlled generators is presented. The attack is tested in software and confirmed as having a very low running time that follows the expected complexity  $O(2^{l_s})$ , where  $l_s$  is the length of the clocking register. Thus the run time complexity is independent of the degree/length ( $l_A$ ) of the clocked register. This attack is also developed in the known-plaintext attack scenario, however, the authors indicate that if they modify the algorithm it will be able to handle noise. Other approaches based on the Linear Consistency Test (LCT) [11] are presented in [12, 13].

In [14] a correlation attack using a non-linear boolean filter is described. The at-

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<sup>1</sup>Maximum a'priori probability

tack uses a correlation property of Boolean functions, that gives higher correlation than previous methods. The influence of noise is not considered.

In [15] a probabilistic analysis of the shrinking generator, which shows that this generator can be vulnerable to a specific fast correlation attack is conducted. The first stage of the attack is based on a recursive computation of the posterior probabilities of individual bits of the regularly clocked LFSR<sub>A</sub> sequence when conditioned on a given segment of the output sequence. Theoretical analysis shows that these probabilities are significantly different from one half and can hence be used for reconstructing the initial state of LFSR<sub>A</sub> by iterative probabilistic decoding algorithms for fast correlation attacks on regularly clocked LFSR's. In the second stage of the attack, the initial state of LFSR<sub>S</sub> is reconstructed in a similar way, which is based on a recursive computation of the posterior probabilities of individual bits of the LFSR<sub>S</sub> sequence when conditioned on the output sequence and on the reconstructed LFSR<sub>A</sub> sequence. The influence of noise is not considered.

In [16] a probabilistic correlation attack on irregularly clocked shift registers is applied in a divide and conquer attack on the shrinking generator. Systematic computer simulations show that the joint probability is a suitable basis for the correlation attack and that, given an output sequence segment of length linear in the length of the clock-controlled shift register, the shift register initial states can be identified with high probability. This approach is also developed in the known-plaintext attack scenario.

In [17], the possibility of clock control sequence reconstruction by backtracking through the edit distance matrix is mentioned in the context of cryptanalysis of the alternating step generator. The method used in [17] considers the known plaintext attack scenario, i.e. with the noise probability equal to zero.

In [5] a new algorithm based on two different and independent ways to improve a known constrained edit distance attack on clock-controlled LFSR-based generators is proposed. The described algorithm avoids the exhaustive search over all the initial states of the involved LFSRs. The most remarkable aspect of this work is that the general ideas that have been proposed may be applied to attack any clock-controlled LFSR-based stream cipher. Again, the known-plaintext attack scenario is studied.

In the process of reconstructing the clock-control sequence, the influence of noise on the effectiveness of the procedure applied is decisive. Namely, the noise can either prevent a procedure from functioning or significantly reduce its effectiveness. Besides, the ciphertext-only attack on stream ciphers, as the most difficult one, is the most realistic attack scenario.

In [2] a generalized correlation attack determining a set of candidate initial states of the clocked register is described. A special statistical model that employs the Constrained Levenshtein or edit distance with the constraint on the maximum number of consecutive deletions is used.

In [18], a clock-control sequence reconstruction procedure capable of handling noise (ciphertext-only attack scenario) is presented. In the phase one, candidate initial states of the clocked register are reconstructed by means of the procedure given in [2]. In the phase two, the edit distance matrices created in the phase one are used to reconstruct the clock-control sequence. The procedure is applied on the step1-step2 generator and results from these experiments show that the average number of paths that have to be reconstructed increases moderately with the level of noise. The procedure uses a depth

first like search through the edit distance matrix, when reconstructing the paths. In [19] an attempt to improve this search process by the use of a k-best-paths algorithm is described.



### 3 The attack

#### 3.1 Reduction to the step1-stepE generator

The SG is a member of the family of clock-controlled generators. The procedure in [18] uses a statistical model (see Figure 4) of the basic structure of this family. The statistical model employs the constrained edit distance. One could readily observe that this model could be used to describe the step1-stepE generator. However, it is not that trivial to observe how the model could be used to describe the SG. In [1], where the SG was first introduced, it was stressed that the SG is not a special case of a clock-controlled generator. Intuitively, this seems reasonable when considering the SG as depicted in Figure 1. The two LFSRs are both clocked regularly by the same clock.

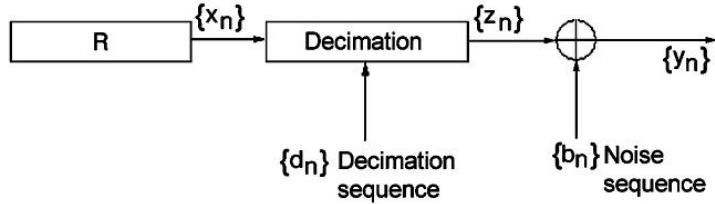


Figure 4: The statistical model of the analyzed generator

In Figure 5 we depict a scheme of a generator equivalent to the SG. It is a special case of the step1-stepE generator. The output from LFSR<sub>S</sub> runs through a filter which generates the clock control sequence used to control the clocking of LFSR<sub>A</sub>.

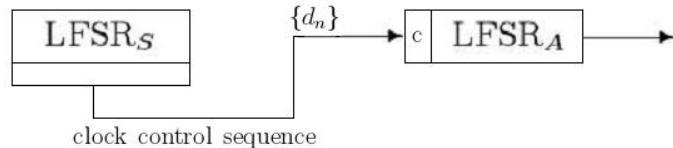


Figure 5: The step1-stepE generator

The clock control sequence  $\{d_n\}$  is a sequence of integers. Let  $s$  denote a subsequence of the output sequence  $S$  of LFSR<sub>S</sub>. In Table 1 one could see how  $\{d_n\}$  is constructed and how LFSR<sub>A</sub> is clocked according to it.

The step1-stepE generator is closely related to the step1-step2 generator. Both generators delete or omit bits from the LFSR<sub>A</sub> output sequence. The step1-step2 will never omit more than  $E = 1$  consecutive bits. Let  $l_s$  denote the length of LFSR<sub>S</sub>. The special case of the step1-stepE generator that is equivalent to the SG will at maximum delete  $l_s - 1$  consecutive bits.

To illustrate the equivalence of the SG and the step1-stepE special case generator from Figure 5, let us consider the following output sequence from LFSR<sub>S</sub> in the SG.

Table 1: The step1-stepE clock-control sequence

$\{d_n\}$	s	step
0	1	1
1	01	2
2	001	3
3	0001	4
4	00001	5

$$\mathbf{s} = (0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 1)$$

This would correspond to following clock-control sequence in the step1-stepE

$$\mathbf{d} = (2, 0, 0, 1, 2, 3)$$

Let  $\mathbf{a}$  be the output sequence of LFSR<sub>A</sub>

$$\mathbf{a} = (0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1)$$

Now, let the SG select which bits to become part of the output sequence  $\mathbf{z}$  according to  $\mathbf{s}$

$$\mathbf{z}_{SG} = (0, 0, 0, 0, 1, 1)$$

Then, let the step1-stepE decimate  $\mathbf{a}$  according to  $\mathbf{d}$

$$\mathbf{z}_{1-E} = (0, 0, 0, 0, 1, 1)$$

The output sequences  $\mathbf{z}_{1-E}$  and  $\mathbf{z}_{SG}$  are equal. From Figure 1 one could readily observe that the statistical model (see Figure 4) can be used to describe the step1-stepE. Assuming the step1-stepE and the SG are equal one could assume that the procedure in [18] could be applied to the cryptanalysis of the SG.

The goal for a cryptanalyst is to reconstruct the initial settings of the generator. These settings include both initial states of the internal LFSRs. In phase two of the procedure, the clock-control sequence is reconstructed. The clock-control sequence can then easily be transformed into a sequence of bits corresponding to the initial state of the clocking register. Let  $\mathbf{d} = (2, 0, 1, 3, 0, 1)$  denote a reconstructed clock-control sequence. According to Table 1, this sequence is transformed into the binary sequence  $\mathbf{b} = (0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 0, 1)$ . If  $l$  is the length of clocking register then the initial state of the clocking register are the first  $l$  bits of  $\mathbf{b}$ .

### 3.2 Determining candidate initial states

The register R in the statistical model corresponds to the LFSR<sub>A</sub> from Figure 5 without irregular clocking, and the plain text sequence is modelled by the noise sequence.

Let  $\{x_n\}$  be the binary sequence produced by the shift register R. Let  $\{d_n\}$  be a sequence of integers, named decimation sequence,  $0 \leq d_n \leq E$ , where E is given in advance. In the decimation process, the sequence  $\{z_n\}$  is obtained in the following way:

$$z_n = x_{f(n)}, \quad f(n) = n + \sum_{i=0}^n d_i, \quad n = 0, 1, 2, \dots \quad (3.1)$$

In the statistical model, it is supposed that  $\{d_n\}$  is the realization of the sequence  $\{D_n\}$  of independent and identically distributed (i.i.d.) random variables, with the probability  $\Pr(D_n = i) = \frac{1}{E+1}$ ,  $0 \leq i \leq E$ ,  $\forall n$ .

The binary noise sequence,  $\{b_n\}$ , is the realization of the sequence of random i.i.d. variables  $\{B_n\}$  with the probability  $\Pr(B_n = 1) = p < 0.5, \forall n$ , where  $p$  is the correlation parameter.

The cryptanalyst possesses  $M$  consecutive bits of the sum modulo 2  $\{y_n\}$  of the decimated sequence  $\{z_n\}$  and the noise sequence  $\{b_n\}$ . His/her task is to determine the initial state of the generator that produced the  $M$  intercepted bits of the sequence  $\{y_n\}$ .

The correlation attack described in [2] is based on the edit distance measure with the constraint on the maximum length of runs of deletions. This distance measure is defined as follows:

Let  $X$  and  $Y$  be two binary sequences of lengths  $N$  and  $M$ , respectively. Let us consider the transformation of  $X$  into  $Y$  using elementary edit operations – substitutions and deletions. The constrained edit distance between  $X$  and  $Y$  is defined as the minimum number of elementary edit operations needed to transform  $X$  into  $Y$ , where the number of consecutive deletions is  $\leq E$ . Besides, the elementary edit operations are ordered in the sense that first the deletions are performed and then the substitutions. This order of elementary edit operations conforms to the real situation in the pseudorandom generator (see Figure 4).

The edit distance defined above can be determined in an iterative way, by filling the matrix of partial constrained edit distances (see, for example, [20]). For the rest of the paper, we shall use the term *edit distance matrix*, for simplicity. In the edit transformation, if  $e$  represents the number of deletions and  $s$  represents the number of substitutions, then the edit distance between the prefix  $X_{e+s}$  of the sequence  $X$  and the prefix  $Y_s$  of the sequence  $Y$  is given by the following expression:

$$\begin{aligned} W[e, s] = \min & \{W[e - e_1, s - 1] + e_1 d_e + d(x_{e+s}, y_s) | \\ & \max\{0, e - \min\{N - M, (s - 1)E\}\} \leq e_1 \leq \min\{e, E\}\} \\ & s = 1, \dots, M \quad e = 1, \dots, \min\{N - M, sE\}, \end{aligned} \tag{3.2}$$

where  $d_e$  represents the elementary edit distance associated with a deletion (we assume that this value is constant),  $d(x, y)$  represents the elementary edit distance associated with the substitution of the symbol  $x$  by the symbol  $y$  and  $E$  is the maximum number of consecutive deletions. From now on, we shall assume that  $d(x, y) = 0$  iff  $x = y$ .

Any permitted sequence of elementary edit operations (i.e. the one that satisfies the given constraints) can be represented by means of a two dimensional *edit sequence*  $S = (\alpha, \beta)$  over the alphabet  $\{0, 1, \emptyset\}$ , where the 'empty' symbol  $\emptyset$  is introduced in order to represent deletions,  $\alpha = X$  and  $Y$  is obtained by removing the empty symbols from  $\beta$ . The length of the sequences  $\alpha$  and  $\beta$  is  $N$ , which is the total number of deletions and substitutions in the edit transformation of  $X$  into  $Y$ . The edit sequence is constructed according to the following rules:

1. If both  $\alpha(i)$  and  $\beta(i)$  are non-empty symbols, then the substitution of the symbol  $\alpha(i)$  by  $\beta(i)$  takes place,  $1 \leq i \leq N$ .
2. If  $\beta(i)$  is the empty symbol, then the deletion of the symbol  $\alpha(i)$  takes place,  $1 \leq i \leq N$ .

Having defined the basic concepts, we can now proceed with the description of the attack. The first phase of the attack consists of the following steps:

1. Since the real clock control sequence is unknown, the length  $N$  of the output sequence of the LFSR  $R$  without decimation has to be estimated.  $N$  depends on the maximum number of consecutive deletions  $E$ . A proper estimation of  $N$  is very important not only for the process of determining the candidate initial states of  $R$ , but also for the posterior phase of the clock-control sequence reconstruction. Each estimation of  $N$  introduces additional noise in the statistical model. In this thesis we use the mathematical expectation of  $N$  calculated on the basis of  $E$ . This is not an optimal estimation, since the *average* value of  $E$  is much lower than  $E$  in sequences satisfying the Golomb's postulates.

Next, the threshold  $T$  necessary for the classification of the initial states of  $R$  should be determined, using the probability of "false alarm"  $P_f$  as well as the probability of "missing the event"  $P_m$ , selected in advance [2]. The probability  $P_f$  determines the mathematical expectation of the cardinality of the set of candidate initial states. The threshold is computed by checking  $1/P_m$  initial states, selected at random. For each of them, the edit distance defined above between the output sequence generated by the actual initial state without decimation and the intercepted output sequence is calculated.

2. For every possible initial state of  $R$ , not used in the step 1, the constrained edit distance between its corresponding output sequence of length  $N$  and the intercepted sequence of length  $M$  is computed. All the initial states that produce the output sequences from  $R$ , whose edit distance from the intercepted output sequence is less than the threshold  $T$ , are included in the set of candidate initial states.

### 3.3 Clock Control Sequence Reconstruction

We call the *optimal paths* the paths through the edit distance matrix that begin at  $W[N - M, M]$ . Let  $p_l \leq M$  be the length of the clock-control sequence needed to reconstruct the initial state of the clocking subgenerator. The optimal paths pass through the cells  $W[e_{p_1}, p_l], \dots, W[e_{p_n}, p_l]$  in the column  $p_l$  of the matrix  $W$ , where  $n$  depends on the particular sequences.

To determine the points in the column  $p_l$ , through which the optimal paths that start at  $W[N - M, M]$  pass, every cell  $W[e, s]$  has, besides the value  $c$  of the edit distance, four associated vectors:

1. The vector of 'primary' pointers  $vp$  to the cells  $W[vp[1], s - 1], \dots, W[vp[k], s - 1]$  from which it is possible to arrive to the cell  $W[e, s]$  with the minimum weight increment,  $k \leq E + 2$ .
2. The vector of 'updated' pointers  $vu$  to the cells  $W[vu[1], p_l], \dots, W[vu[l], p_l]$ , through which it is possible to arrive to the cell  $W[e, s]$  with the minimum weight increment (see for example [21]),  $l \leq \min\{N - M + 1, E(1 + p_l)\}$ .
3. The vector of pointers  $ve$  to the cells  $W[ve[1], s - 1], \dots, W[ve[j], s - 1]$  from which it is possible to arrive to the cell  $W[e, s]$  regardless of the weight increment,  $j \leq E + 2$ .

4. The vector of values  $v_j$  of the edit distances corresponding to the elements of the vector  $v_e$ . The cardinality of this vector is also  $j$ .

The actual values of  $k$ ,  $l$ , and  $j$  depend on the particular sequences. The matrix  $W$  is filled by means of the algorithm, which implements the equation (3.2) together with the updating of the four vectors mentioned above. The complete algorithm is given below.

**Algorithm 1**

**Input:**

- The sequences  $X$  and  $Y$  of lengths  $N$  and  $M$ , respectively.
- The necessary length  $p_l$  of the clock control sequence.
- The maximum length  $E$  of runs of deletions.
- The elementary distance  $d_e$  associated with the deletion of a symbol.
- The elementary edit distance  $d[x, y]$  associated with the substitution of the symbol  $x$  by the symbol  $y$ ,  $\forall(x, y)$ .

**Output:**

- The matrix  $W$  of edit distances with the vectors  $v_p$ ,  $v_u$ ,  $v_j$ , and  $v_e$  associated with every cell.

**comment** Initialization

$W[e, s].c \leftarrow \infty$ ,  $e = 0, \dots, N - M$ ,  $s = 0, \dots, M$ ; the vectors  $v_p$ ,  $v_u$ ,  $v_j$ , and  $v_e$  associated with every cell  $W[e, s]$  are empty.

$W[0, 0].c \leftarrow 0$ ;

**comment** The row 0 of the matrix  $W$ :

**for**  $s \leftarrow 1$  **until**  $M$  **do**

**begin**

$W[0, s].k \leftarrow 1$  ;  $W[0, s].vp[1] \leftarrow 0$  ;

$W[0, s].c \leftarrow W[0, s - 1].c + d[X[s], Y[s]]$  ;

**end** ;

**comment** Main loop

**for**  $s \leftarrow 1$  **until**  $M$  **do**

**begin**

**for**  $e \leftarrow 1$  **until**  $\min\{N - M, s * E\}$  **do**

**begin**

                Let  $q$  be the minimum value of the expression

$$W[e - e_1, s - 1].c + e_1 * d_e + d[X[e + s], Y[s]], \quad (1)$$

$$e_1 = \max\{0, e - \min\{N - M, (s - 1) * E\}, \dots, \min\{e, E\}\}.$$

Let  $n_q$  be the number of values of  $e_1$  for which the expression (1) takes the value  $q$ . Then

---

```

 $W[e, s].c \leftarrow q ; W[e, s].k \leftarrow n_q ;$ 
 $W[e, s].vp$  is filled with  $n_q$  values of the expression  $e - e_1$  corresponding
to the values  $e_1$  for which the expression (1) takes the value  $q$ .
 $W[e, s].vj$  is filled with all the values of the expression (1).
 $W[e, s].ve$  is filled with the values  $e - e_1$  corresponding to the values of
 $W[e, s].vj$ .
end ;
comment Determining updated pointers  $v_u$ .
if  $s = pl + 1$  then
     $W[e, s].vu \leftarrow W[e, s].vp,$ 
     $e = 0, \dots, \min\{N - M, s * E\} ;$ 
else if  $s > pl + 1$ 
    For every element of  $W[e, s].vp$ ,  $e = 0, \dots, \min\{N - M, s * E\}$ , the ele-
    ments of  $W[W[e, s].vp[i], s - 1].vu$ ,  $i = 1, \dots, W[e, s].k$  are placed into
     $W[e, s].vu$ , deleting the repeated ones.
end.

```

Having constructed the edit distance matrix, the next step is reconstructing the candidate clock control sequences. From now on, by *paths* we mean fragments of paths that start in the column  $pl$  of the matrix  $W$ . There are three sets of paths to be reconstructed. The first one consists of optimal paths that start at the points  $e_{p_i} = W[N - M, M].vu[i]$ ,  $i = 1, \dots, W[e, s].l$ . The second one consists of suboptimal paths, whose weight-difference from the optimal ones is  $\leq D$ , a threshold given in advance that depends on the noise level, that start at  $e_{p_i} = W[N - M, M].vu[i]$ ,  $i = 1, \dots, W[e, s].l$ . The third set consists of suboptimal paths, whose weight-difference from the optimal ones is  $\leq D$ , that start at other points in the column  $pl$ .

The elements of the vector  $W[N - M, M].vu$  at the end of the execution of the Algorithm 1 represent the initial points of the search for the elements of the first and second set mentioned above. As for the third set, if  $|W[e_{p_i}, pl].c - W[e, pl].c| \leq D$ ,  $e = 0, \dots, \min\{N - M, sE\}$ ,  $e \neq e_{p_i}$ , for at least one  $i$ , then the point  $W[e, pl]$  is an initial point of the search for the paths of the third set.

In order to determine the optimal and suboptimal paths that start at every initial point  $E$  of any set, a special depth-first like search algorithm has been devised. In this algorithm, every branching point is processed by enumerating systematically all the paths that start in it. In this search, a special kind of stack is used. A reconstructed path is rejected if at some point its weight becomes greater than the optimal weight plus  $D$ . The complete algorithm is given below.

**Algorithm 2:**

**Input:**

- The matrix  $W$  of edit distances, obtained by means of the Algorithm 1.
- The values of  $pl$ ,  $E$  and  $D$ .

**Output:**

- All the paths that start at the point  $W[\mathcal{E}, pl]$  that belong to the corresponding set(s) (see text).

**comment** The stack consists of the elements of the matrix  $W$  together with their coordinates; the cardinality of the stack is  $nsp$ ; the current length of the reconstructed path of the edit sequence is  $t$ .

**comment** Initialization

$t \leftarrow 0 ; e \leftarrow \mathcal{E} ; s \leftarrow pl ; nsp \leftarrow 0 ;$

**comment** Main loop

**repeat**

$badpath \leftarrow \text{false} ;$  **comment** This is the path overweight indicator

**while**  $((e > 0) \text{ or } (s > 0)) \text{ and not } badpath \text{ do}$

**begin**

**if** ( $W[e, s]$  is a branching point **then**

**comment** Put  $e$ ,  $s$  and  $W[e, s]$  on the stack

$nsp \leftarrow nsp + 1 ;$

$stack[nsp] \leftarrow (W[e, s], e, s) ;$

**comment** Process a branching point

$badpath \leftarrow \text{false} ;$

**repeat**

Consider the possibility of branching from the current branching point to one of the possible successors, i.e. the point  $j$ . If this possibility is chosen, and after that only the branchings to the points that lead to the optimal subpaths are followed, then the total weight of the chosen subpath is

$aw \leftarrow stack[nsp].W.vj[stack[nsp].W.j] ;$

and the total weight of the corresponding path is

$tw \leftarrow \text{weight}(\alpha, \beta, t) + aw ;$

where  $\text{weight}$  is the function that returns the weight of the path before the branching and  $(\alpha, \beta)$  is the prefix of the edit sequence of length  $t$ .

**comment**  $eprev$  is the value of  $e$  that corresponds to the previous path element

**if**  $tw \leq W[\mathcal{E}, pl].c + \mathcal{D}$  **then**

$eprev \leftarrow stack[nsp].W ve[stack[nsp].W.j] ;$

$stack[nsp].W.j \leftarrow stack[nsp].W.j - 1 ;$

**if**  $stack[nsp].W.j = 0$  **then**  $nsp \leftarrow nsp - 1 ;$

**until** ( $eprev$  was initialized from the stack) **or**

(all the successors were examined) ;

**if**  $eprev$  was not initialized from the stack **then**

$badpath \leftarrow \text{true} ;$

```
comment Process a non-branching point
if (eprev was not initialized from the stack) and
(not badpath) then
begin
    aw ← W[e, s].vj[W[e, s].j] ; tw ← weight(α, β, t) + aw ;
    if tw <= W[ε, p].c + D then
        eprev ← W[e, s].ve[W[e, s].j] ;
    else
        badpath ← true ;
    end ;

comment Reconstruct the current path element
if not badpath then
begin comment Substitution
    t ← t + 1 ; α[t] ← X[s + e] ; β[t] ← Y[s] ;
    for ii ← 1 until e - eprev do
        begin comment The run of deletions
            t ← t + 1 ; α[t] ← X[e + s - ii] ; β[t] ← ∅ ;
            end ;
        e ← eprev ; s ← s - 1 ;
        end ;
    end ;
if not badpath then
    Store the obtained clock control sequence ;
comment Back to the current branching point
if nsp > 0 then
begin
    t ← t - stack[nsp].e - stack[nsp].s ;
    (e, s) ← stack[nsp].(e, s) ;
end
until nsp = 0 .
```

## 4 Experimental work

From the cryptanalytic point of view, the number of paths necessary to find the clock control sequence should be as small as possible. This number depends on  $\mathcal{D}$ . Given a certain level of noise in the statistical model, the behaviour of the maximum value of  $\mathcal{D}$ , denoted by  $\mathcal{D}_{\max}$ , has been analysed experimentally. The experiment has been carried out in the following way: 50 initial states of SG are chosen at random. For each of them, the output sequence corrupted by the noise sequence generated at random is produced. The noise level  $p$  is the control variable of the experiment. The set of candidates for the initial state of LFSR<sub>A</sub> is determined. Once the candidates have been obtained, for a fixed value of  $\mathcal{D}$ , the optimal and suboptimal paths are determined from the edit distance matrix corresponding to each of them. This process is repeated starting from  $\mathcal{D} = 0$  and incrementing the value of  $\mathcal{D}$  until the clock control sequence generated by LFSR<sub>S</sub> is found. The maximum value  $\mathcal{D}_{\max}$  obtained in this process is stored. At the end of the experiment, the mean value  $\overline{\mathcal{D}_{\max}}$  over the values of  $\mathcal{D}_{\max}$  obtained in every case is calculated. The experiment was performed on an ordinary laptop PC. The implementation of the procedure is written in C++ (see Appendix A for the most important portions of it).

### 4.1 Results

The dependence of  $\overline{\mathcal{D}_{\max}}$  on  $p$  for different values of  $pl$  is depicted in Figure 6.

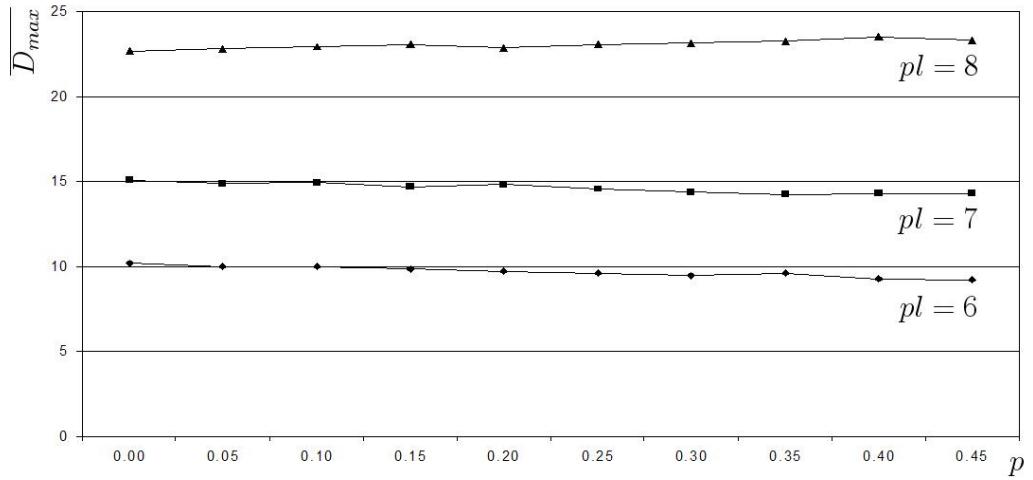


Figure 6: Dependence of  $\overline{\mathcal{D}_{\max}}$  on  $p$

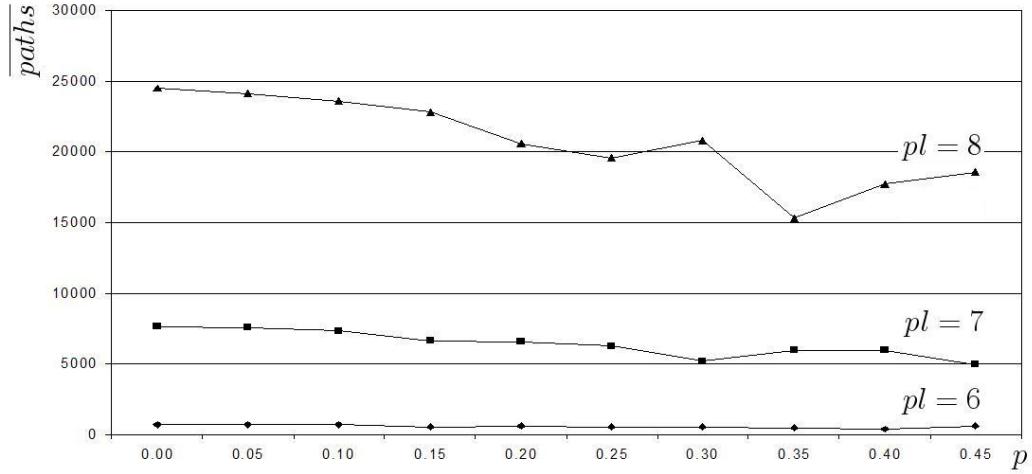
From Figure 6 it can be concluded that:

1.  $\overline{\mathcal{D}_{\max}}$  depends approximately linearly on  $pl$ .
2.  $\overline{\mathcal{D}_{\max}}$  does not depend on  $p$  at all.

Table 2: Average number of reconstructed paths for different levels of noise

$p_l$	$p_{0.00}$	$p_{0.05}$	$p_{0.10}$	$p_{0.15}$	$p_{0.20}$	$p_{0.25}$	$p_{0.30}$	$p_{0.35}$	$p_{0.40}$	$p_{0.45}$
6	677	665	661	532	611	535	562	476	414	600
7	7652	7592	7368	6686	6536	6243	5219	5923	5948	4974
8	24549	24128	23592	22781	20564	19563	20821	15312	17703	18566

From Table 2 one could see how the average number of reconstructed paths depend on the length of necessary clock control sequence,  $p_l$ .

Figure 7: Average number of reconstructed paths for different values of  $p_l$  and  $p$ 

The experimental results presented in Figure 6 and Figure 7 are discussed in the following chapter.

## 5 Discussion

The experiment tells us that the procedure found the solution, i.e. the correct clock-control sequence, in all the tests that were performed. This means that we succeeded in applying the procedure to the SG. It tells us that it is possible to use this procedure in the cryptanalysis of the SG. We reconstructed the generator's initial settings by analyzing a prefix of its output sequence. We even succeeded when we applied noise to the output sequence, meaning the procedure can operate successfully in the ciphertext-only attack scenario.

From Figure 6 we see that  $\mathcal{D}_{\max}$  depends approximately linearly on  $p_l$ . This is what we expected. The dependence of  $\mathcal{D}_{\max}$  on  $p$ , however, is not as expected. We would expect  $\mathcal{D}_{\max}$  to increase as we added more noise, instead it does not seem to be affected by the noise level at all. With zero noise applied we would expect that only the optimal paths would need to be reconstructed, i.e. we would expect  $\mathcal{D}_{\max}$  to be zero when  $p$  was zero. We also expected the average number of reconstructed paths to increase as we applied more noise. In Figure 7 we see that this is not the case.

The lengths of the registers (6,7,8) we chose are small since time and computing resources don't allow trying longer LFSRs. As we can see from Figure 7, with  $p_l = 8$  and zero noise,  $\mathcal{D}_{\max}$  has a value of approximately 22. This means the procedure reconstructs a lot of paths. The experiment with  $p_l = 8$  took several days.

The reason for the "unexpected" results has to do with the estimation of  $N$ . Experiments with a fixed optimal value of  $N$  proves this (see Figure 8). We see that an optimal estimation of  $N$  would make the procedure perform significantly better. This can also be seen in the individual cases of the experiment (see Table 4) where the actual  $N$  was close to the estimated  $N$ , the value of  $\mathcal{D}_{\max}$  was significantly smaller. As one could see in Table 4, with LFSR's length 6, zero noise and an estimated  $N$  of 21, the value of  $\mathcal{D}_{\max}$  (diff) varies from 1 to 16 depending of the actual  $N$  ( $n\_skipped$ ).

We expected  $\mathcal{D}_{\max}$  to depend on the level of noise. The experimental results tell us that  $\mathcal{D}_{\max}$  does not depend on  $p$  at all, however, it still depends on the level of noise. Inexact estimation of  $N$  is equivalent to **introducing a certain level of noise into the statistical model**. The more the estimated  $N$  differs from the "actual"  $N$ , the more "noise" is introduced. Thus, the results are as expected, however, in most of the cases there was no point in increasing  $p$  as the the level of noise was already at a high level due to the inexact estimation of  $N$ .

An adequate estimation of  $N$  is important and becomes more important as  $E$  increases.  $N$  is estimated as a function of  $E$  and  $M$ .  $E$  is the maximum number of consecutive deletions or the maximum length of runs of 0s in the clock control sequence. In the case of the SG,  $E$  depends on the length of the clocking register,  $E = l_S - 1$ . We know from Golomb's postulates[4] that the probability of finding a run of 0s of length  $E$  in the clock control sequence is very small. Perhaps a better solution would be to estimate  $N$  based on the average value of  $E$  which is much lower than  $E$ . However, in order to be sure we shall find the solution, we must estimate  $N$  based on a "worst case scenario". If  $N$  is

too small we shall never find the solution. If  $N$  is too large the procedure will have to reconstruct many more paths than necessary, as was the case in this experiment.

From Table 4 and the rest of the experimental results (see Appendix B) one could see that the value of  $\mathcal{D}_{\max}$  is almost exactly the same as the difference between the estimated  $N$  and the "actual"  $N$  ( $n_{\text{skipped}}$ ).

Table 3: Average number of reconstructed paths for different levels of noise with optimal  $N$

$pl$	$p_{0.00}$	$p_{0.05}$	$p_{0.10}$	$p_{0.15}$	$p_{0.20}$	$p_{0.25}$	$p_{0.30}$	$p_{0.35}$	$p_{0.40}$	$p_{0.45}$
6	35	87	169	180	452	411	505	429	578	700
7	224	527	924	3130	3687	3811	8678	5858	6265	9912
8	281	629	3555	7436	8334	5015	12536	13164	22623	30015

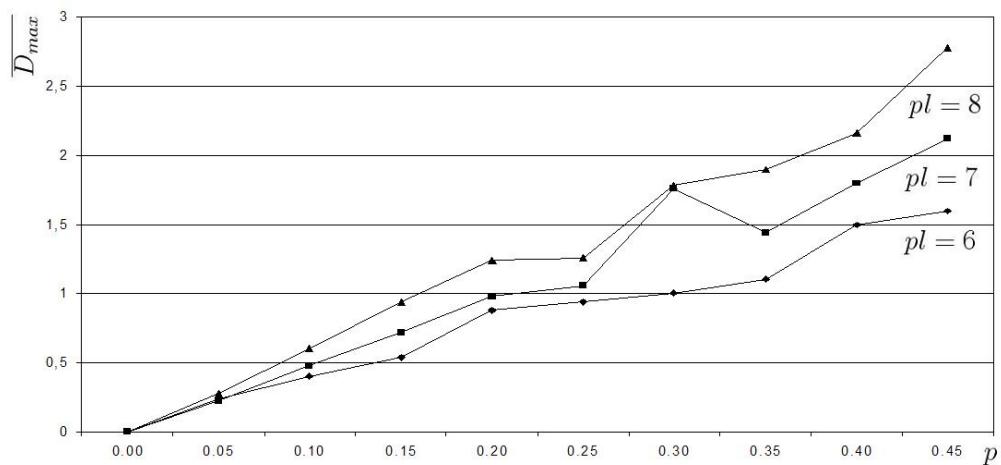


Figure 8: Dependence of  $\overline{\mathcal{D}_{\max}}$  on  $p$  with optimal  $N$

Table 4: Raw data from the experiment

pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.000000	12	144	5	3	10	21
6	0.000000	12	142	5	2	10	21
6	0.000000	11	423	5	2	11	21
6	0.000000	15	7	5	1	7	21
6	0.000000	12	58	5	1	9	21
6	0.000000	6	1327	5	5	15	21
6	0.000000	16	1	5	0	6	21
6	0.000000	3	1985	5	4	19	21
6	0.000000	10	462	5	2	12	21
6	0.000000	3	3685	5	5	18	21
6	0.000000	12	167	5	2	10	21
6	0.000000	9	429	5	2	12	21
6	0.000000	12	80	5	1	9	21
6	0.000000	6	1483	5	3	15	21
6	0.000000	12	183	5	2	10	21
6	0.000000	10	512	5	3	12	21
6	0.000000	6	2682	5	3	15	21
6	0.000000	11	116	5	3	11	21
6	0.000000	1	250	5	4	20	21
6	0.000000	11	373	5	2	11	21
6	0.000000	13	56	5	2	9	21
6	0.000000	15	7	5	1	7	21
6	0.000000	6	1633	5	3	15	21
6	0.000000	13	18	5	2	9	21
6	0.000000	5	4149	5	4	17	21
6	0.000000	13	122	5	3	10	21
6	0.000000	14	7	5	1	7	21
6	0.000000	11	183	5	2	10	21
6	0.000000	12	126	5	3	10	21
6	0.000000	14	7	5	1	8	21
6	0.000000	12	165	5	1	10	21
6	0.000000	10	310	5	3	12	21
6	0.000000	5	2549	5	4	16	21
6	0.000000	13	113	5	3	10	21
6	0.000000	8	836	5	2	13	21
6	0.000000	9	685	5	3	12	21
6	0.000000	7	1482	5	3	14	21
6	0.000000	12	111	5	2	10	21
6	0.000000	15	6	5	1	7	21
6	0.000000	12	209	5	2	10	21
6	0.000000	3	2232	5	5	18	21
6	0.000000	6	2530	5	3	15	21
6	0.000000	14	14	5	1	8	21
6	0.000000	11	194	5	2	10	21
6	0.000000	11	147	5	2	10	21
6	0.000000	12	43	5	1	9	21
6	0.000000	13	75	5	1	9	21
6	0.000000	11	372	5	2	11	21
6	0.000000	10	579	5	3	12	21
6	0.000000	10	399	5	2	11	21



## 6 Conclusion

In this thesis, we describe a deterministic method of reconstruction of clock-control sequence in the generalized shrinking generator in the ciphertext only attack scenario. Such a generalized shrinking generator is first reduced to a step<sub>1</sub>-step<sub>E</sub> generator, where E depends on the maximum length of runs of zeros in the output sequence of the clocking part of the generator. Then a "depth-first"-like search through the constrained edit distance matrix associated with every candidate initial state of the LFSR<sub>A</sub> is used. The paths in this matrix that correspond to candidate clock control sequences are reconstructed. Influence of noise is taken into account by relating the noise level with the permitted weight deviation from the optimum path weight used in the search process. By starting with the reconstruction of paths whose weight deviation from the optimum is 0 (the optimal paths - without noise) and then by increasing this weight deviation according to the noise level (the suboptimal paths), we make our search a directed one. The inexact estimation of N based on the the maximum value of E introduces additional noise in the statistical model. The procedure always finds the correct clock-control sequence. The maximum value of weight deviation necessary for the reconstruction of the actual clock-control sequence depends on the noise level. Experimental results show that the average number of paths that have to be reconstructed in order to find the true clock-control sequence depends on the noise level.



## 7 Future work

In this thesis we use the mathematical expectation of  $N$  calculated on the basis of the maximum length of runs of 0s,  $E$ , in the output sequence from the clocking part of the SG. This is not an optimal estimation, since the average value of  $E$  is much lower than  $E$  in sequences satisfying the Golombs postulates. A more exact estimation of  $N$  will significantly improve the procedure.

In order to further confirm the significance of the experimental results obtained in the thesis, experiments should be performed on much longer LFSRs, for example 20, 30 and even more. This would consume a significant amount of time, but it would give more weight to the conclusions of the thesis.

It would also be interesting to apply the procedure described in this thesis to the cryptanalysis of the Alternating Step Generator (ASG), a very similar generator from the same family, but still reported difficult to cryptanalyse.



## Bibliography

- [1] Coppersmith, D., Krawczyk, H., & Mansour, Y. January 1994. The shrinking generator. *Lecture Notes in Computer Science*, 773, 22–.
- [2] Golić, J. D. & Mihaljević, M. J. January 1991. A generalized correlation attack on a class of stream ciphers based on the levenshtein distance. *Journal of Cryptology*, 3(3), 201–212.
- [3] Gollmann, D. & Chambers, W. G. May 1989. Clock-controlled shift registers: A review. *IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS*, 7(4), 525–533.
- [4] Menezes, A. J., Oorschot, P. C. v., & Vanstone, S. A. 1997. *Handbook of Applied Cryptography*. CRC Press.
- [5] Caballero-Gil, P. & Fúster-Sabater, A. October 2005. Improvement of the edit distance attack to clock-controlled lfsr-based stream ciphers. *Lecture Notes in Computer Science*, 3643, 355–364.
- [6] Chambers, W. G. & Golić, J. September 2002. Fast reconstruction of clock-control sequence. *Electronic Letters*, 38(20), 1174–1175.
- [7] Johansson, T. January 1998. Reduced complexity correlation attacks on two clock-controlled generators. *Lecture Notes in Computer Science*, 1514, 342–.
- [8] Johansson, T. & Jönsson, F. January 1999. Improved fast correlation attacks on stream ciphers via convolutional codes. *Lecture Notes in Computer Science*, 1592, 347–.
- [9] Johannesson, R. & Zigangirov, K. S. 1999. *Fundamentals Of Convolutional Coding*. IEEE Press.
- [10] Molland, H. July 2004. Improved linear consistency attack on irregular clocked keystream generators. *Lecture Notes in Computer Science*, 3017, 109–126.
- [11] Zeng, K., Yang, C., & Rao, T. January 1990. On the linear consistency test (lct) in cryptanalysis with applications. *Lecture Notes in Computer Science*, 435, 164–.
- [12] Zenner, E., Krause, M., & Lucks, S. January 2001. Improved cryptanalysis of the self-shrinking generator. *Lecture Notes in Computer Science*, 2119, 21–.
- [13] Zenner, E. January 2003. On the efficiency of the clock control guessing attack. *Lecture Notes in Computer Science*, 2587, 200–.
- [14] Molland, H. & Helleseth, T. November 2004. An improved correlation attack against irregular clocked and filtered keystream generators. *Lecture Notes in Computer Science*, 3152, 373–389.

- [15] Golić, J. D. January 2001. Correlation analysis of the shrinking generator. *Lecture Notes in Computer Science*, 2139, 440–.
- [16] Simpson, L., Golic, J. D., & Dawson, E. January 1998. A probabilistic correlation attack on the shrinking generator. *Lecture Notes in Computer Science*, 1438, 147–.
- [17] Golić, J. D. & Menicocci, R. January 1997. Edit distance correlation attack on the alternating step generator. *Lecture Notes in Computer Science*, 1294, 499–.
- [18] Petrović, S. & Fúster-Sabater, A. October 2004. Clock control sequence reconstruction in the ciphertext only attack scenario. *Lecture Notes in Computer Science*, 3269, 427–439.
- [19] Herland, T. The use of k-best-paths algorithms in clock-control sequence reconstruction. Master's thesis, Høgskolen i Gjøvik, 2006.
- [20] Oommen, B. 1986. Constrained string editing. *Inform. Sci.*, 40(9), 267–284.
- [21] Hirschberg, D. S. 1997. Serial computations of Levenshtein distances. In *Pattern matching algorithms*, Apostolico, A. & Galil, Z., eds, 123–141. Oxford University Press.

## A Implementation in C++

```

#include "stdafx.h"

bool backtrack( int e, int s, int *n_path, int *x, int *y, int
    diff_min, int diff, mat_cell **W,
    int n_vp_max, int n_vu_max, int *solution, int E ){
    // This function reconstructs all the optimal and suboptimal
    // paths that correspond to the edit sequence of the
    // prefix X[e+s] to the prefix Y[s].
    // It only prints the paths, whose weight is between
    // optimal_weight+diff_min and optimal_weight+diff
    int i, len_prev, e_prev, b_sp, e1, s1, weight_increment,
        total_weight, counter, num_equal_bits ;
    int j, num_2 ;
    int *edit_x, *edit_y, *clock_seq ;
    branch_rec *branch_stack ;
    bool from_branch, bad_path, remove ;
    // Initialization
    branch_stack = (branch_rec *) calloc(s+1,sizeof(branch_rec)) ;
    for(i=0;i<s+1;i++){
        branch_stack[i].cell.vp = (int *)calloc(n_vp_max,sizeof(int))
            ) ;
        branch_stack[i].cell.vu = (int *)calloc(n_vu_max,sizeof(int))
            ) ;
        branch_stack[i].cell.ve = (int *)calloc(n_vp_max,sizeof(int))
            ) ;
        branch_stack[i].cell.vj = (int *)calloc(n_vp_max,sizeof(int))
            ) ;
    }
    edit_x = (int *) calloc(e+s+1,sizeof(int)) ;
    edit_y = (int *) calloc(e+s+1,sizeof(int)) ;
    clock_seq = (int *) calloc(e+s+1,sizeof(int)) ;
    len_prev = 0 ; e1 = e ; s1 = s ; b_sp = 0 ;
    branch_stack[0].e = INT_MAX ;
    branch_stack[0].s = INT_MAX ;
    // Path reconstruction
    do{
        bad_path = false ;
        while( ( (e1>0) || (s1>0) ) && (!bad_path)){
            // Check for a branching at the actual point (e,s)
            from_branch = false ;
            if( (W[e1][s1].n_vej>1) && ( (e1!=branch_stack[b_sp].e) ||
                (s1!=branch_stack[b_sp].s) ) ){
                // There is a branching, so update the branching stack
                b_sp++ ;
                branch_stack[b_sp].cell.n_vp = W[e1][s1].n_vp ;
                branch_stack[b_sp].cell.n_vu = W[e1][s1].n_vu ;
                branch_stack[b_sp].cell.n_vej = W[e1][s1].n_vej ;
            }
        }
    }
}

```

```

branch_stack[b_sp].cell.c = W[e1][s1].c ;
for(i=1;i<=W[e1][s1].n_vp ; i++)
    branch_stack[b_sp].cell.vp[i] = W[e1][s1].vp[i] ;
for(i=1;i<=W[e1][s1].n_vu ; i++)
    branch_stack[b_sp].cell.vu[i] = W[e1][s1].vu[i] ;
for(i=1;i<=W[e1][s1].n_vej ; i++)
    branch_stack[b_sp].cell.ve[i] = W[e1][s1].ve[i] ;
for(i=1;i<=W[e1][s1].n_vej ; i++)
    branch_stack[b_sp].cell.vj[i] = W[e1][s1].vj[i] ;
branch_stack[b_sp].e = e1 ;
branch_stack[b_sp].s = s1 ;
}
// If there is no branching, continue with the path
// reconstruction
if( (branch_stack[b_sp].e==e1) && ( branch_stack[b_sp].s==
s1 ) ){
bad_path = false ;
remove = false ;
do{
    weight_increment = branch_stack[b_sp].cell.vj[
        branch_stack[b_sp].cell.n_vej] ;
    total_weight = weight(edit_x,edit_y,len_prev) + abs(
        weight_increment) ;
    if( total_weight<=W[e][s].c+diff ){
        // We can continue this way: select next point in
        // the path
        from_branch = true ;
        e_prev = branch_stack[b_sp].cell.ve[branch_stack[
            b_sp].cell.n_vej--] ;
        // Check if the branching point is empty
        if( branch_stack[b_sp].cell.n_vej == 0 ){
            b_sp-- ;
            remove = true ;
        }
    }
    else{
        // If it is not possible to select the next point in
        // the path,
        // cancel the corresponding direction
        branch_stack[b_sp].cell.n_vej-- ;
        if( branch_stack[b_sp].cell.n_vej == 0 ){
            b_sp-- ;
            remove = true ;
        }
    }
} while(!from_branch && !remove) ;
// If we cannot continue with the reconstruction of any
// path, raise the flag bad_path
if (!from_branch)
    bad_path = true ;
}
// If there is no branching at the point (e,s),
// continue with the path reconstruction

```

```

if ( (!from_branch) && (!bad_path) ){
    weight_increment = W[e1][s1].vj[W[e1][s1].n_vej] ;
    total_weight = weight(edit_x,edit_y,len_prev) + abs(
        weight_increment) ;
    if( total_weight<=W[e][s].c+diff ){
        // We can continue this way: select next point in the
        // path
        e_prev = W[e1][s1].ve[W[e1][s1].n_vej] ;
    }
    else{
        // If it is not possible to select the next point in
        // the path,
        // cancel the corresponding direction
        bad_path = true ;
    }
}
// If the path is acceptable, in spite of everything, then
// add the corresponding part of the edit sequence
if (!bad_path){
    // Current substitution
    if(s1>0){
        edit_x[++len_prev] = x[s1+e1] ;
        edit_y[len_prev] = y[s1] ;
    }
    // Current run of deletions (if it exists)
    for(i=1;i<=e1-e_prev;i++){
        edit_x[++len_prev] = x[e1+s1-i] ;
        edit_y[len_prev] = 2 ;
    }
    // The coordinates of the point to be processed next
    e1 = e_prev ;
    s1-- ;
}
if (!bad_path){
    // If diff_min < 0 print all the reconstructed paths ;
    // else print only the ones that have not been already
    // reconstructed
    // in the previous calls to backtrack
    if ((diff_min < 0)|| (weight(edit_x,edit_y,len_prev)>W[e][s
        ].c+diff_min)){
        // Reconstruct the clock control sequence from edit_x
        // and edit_y
        i = len_prev + 1 ;
        counter = 0 ;
        do {
            /*
                i-- ;
                counter++ ;
                if (counter<=s){
                    if (edit_y[i]==2){
                        i-- ;
                        clock_seq[counter] = 1 ;
                    }
            */
        }
    }
}

```

```

        else
            clock_seq[counter] = 0 ;
        } */
num_2 = 0 ;
i— ;
while(edit_y[i]==2)
{
    num_2++ ;
    i— ;
}
counter++ ;
if(counter<=s)
{
    for(j=0;j<=E;j++)
    {
        if(num_2==j)
        {
            clock_seq[counter] = j ;
            break ;
        }
    }
} while(i>0) ;
if(counter>s)
    counter = s ;
// Check if the solution has been found
++(*n_path) ;
num_equal_bits = 0 ;
for(i=1;i<=counter;i++) {
    if(clock_seq[i] == solution[i])
        num_equal_bits++ ;
}
if(num_equal_bits == counter){
    free(edit_x) ;
    free(edit_y) ;
    free(clock_seq) ;
    for(i=0;i<s+1;i++){
        free(branch_stack[i].cell.vp) ;
        free(branch_stack[i].cell.vu) ;
        free(branch_stack[i].cell.ve) ;
        free(branch_stack[i].cell.vj) ;
    }
    free(branch_stack) ;
    return true ;
}
}
}
// If the branching stack is not empty, then erase
// everything up to
// the branching point at the top of the branching stack
if(b_sp>0{
    len_prev -= (branch_stack[b_sp].e+branch_stack[b_sp].s) ;
    e1 = branch_stack[b_sp].e ;
}

```

```
s1 = branch_stack[b_sp].s ;
}
}while(b_sp>0) ;
free(edit_x) ;
free(edit_y) ;
free(clock_seq) ;
for(i=0;i<s+1;i++){
    free(branch_stack[i].cell.vp) ;
    free(branch_stack[i].cell.vu) ;
    free(branch_stack[i].cell.ve) ;
    free(branch_stack[i].cell.vj) ;
}
free(branch_stack) ;
return false ;
}
```



## B Raw data from the experiment

pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.000000	12	144	5	3	10	21
6	0.000000	12	142	5	2	10	21
6	0.000000	11	423	5	2	11	21
6	0.000000	15	7	5	1	7	21
6	0.000000	12	58	5	1	9	21
6	0.000000	6	1327	5	5	15	21
6	0.000000	16	1	5	0	6	21
6	0.000000	3	1985	5	4	19	21
6	0.000000	10	462	5	2	12	21
6	0.000000	3	3685	5	5	18	21
6	0.000000	12	167	5	2	10	21
6	0.000000	9	429	5	2	12	21
6	0.000000	12	80	5	1	9	21
6	0.000000	6	1483	5	3	15	21
6	0.000000	12	183	5	2	10	21
6	0.000000	10	512	5	3	12	21
6	0.000000	6	2682	5	3	15	21
6	0.000000	11	116	5	3	11	21
6	0.000000	1	250	5	4	20	21
6	0.000000	11	373	5	2	11	21
6	0.000000	13	56	5	2	9	21
6	0.000000	15	7	5	1	7	21
6	0.000000	6	1633	5	3	15	21
6	0.000000	13	18	5	2	9	21
6	0.000000	5	4149	5	4	17	21
6	0.000000	13	122	5	3	10	21
6	0.000000	14	7	5	1	7	21
6	0.000000	11	183	5	2	10	21
6	0.000000	12	126	5	3	10	21
6	0.000000	14	7	5	1	8	21
6	0.000000	12	165	5	1	10	21
6	0.000000	10	310	5	3	12	21
6	0.000000	5	2549	5	4	16	21
6	0.000000	13	113	5	3	10	21
6	0.000000	8	836	5	2	13	21
6	0.000000	9	685	5	3	12	21
6	0.000000	7	1482	5	3	14	21
6	0.000000	12	111	5	2	10	21
6	0.000000	15	6	5	1	7	21
6	0.000000	12	209	5	2	10	21
6	0.000000	3	2232	5	5	18	21
6	0.000000	6	2530	5	3	15	21
6	0.000000	14	14	5	1	8	21

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.000000	11	194	5	2	10	21
6	0.000000	11	147	5	2	10	21
6	0.000000	12	43	5	1	9	21
6	0.000000	13	75	5	1	9	21
6	0.000000	11	372	5	2	11	21
6	0.000000	10	579	5	3	12	21
6	0.000000	10	399	5	2	11	21
6	0.050000	12	144	5	3	10	21
6	0.050000	12	142	5	2	10	21
6	0.050000	11	423	5	2	11	21
6	0.050000	15	7	5	1	7	21
6	0.050000	12	56	5	1	9	21
6	0.050000	6	1327	5	5	15	21
6	0.050000	14	1	5	0	6	21
6	0.050000	3	1985	5	4	19	21
6	0.050000	10	462	5	2	12	21
6	0.050000	3	3685	5	5	18	21
6	0.050000	12	167	5	2	10	21
6	0.050000	9	429	5	2	12	21
6	0.050000	12	80	5	1	9	21
6	0.050000	6	1483	5	3	15	21
6	0.050000	10	126	5	2	10	21
6	0.050000	10	512	5	3	12	21
6	0.050000	6	2682	5	3	15	21
6	0.050000	9	116	5	3	11	21
6	0.050000	1	85	5	4	20	21
6	0.050000	10	367	5	2	11	21
6	0.050000	13	56	5	2	9	21
6	0.050000	15	7	5	1	7	21
6	0.050000	6	1633	5	3	15	21
6	0.050000	12	9	5	2	9	21
6	0.050000	5	3498	5	4	17	21
6	0.050000	13	122	5	3	10	21
6	0.050000	14	7	5	1	7	21
6	0.050000	11	183	5	2	10	21
6	0.050000	12	126	5	3	10	21
6	0.050000	14	7	5	1	8	21
6	0.050000	12	163	5	1	10	21
6	0.050000	10	310	5	3	12	21
6	0.050000	5	2549	5	4	16	21
6	0.050000	13	113	5	3	10	21
6	0.050000	8	836	5	2	13	21
6	0.050000	9	685	5	3	12	21
6	0.050000	7	1482	5	3	14	21
6	0.050000	12	136	5	2	10	21
6	0.050000	13	5	5	1	7	21
6	0.050000	12	110	5	2	10	21
6	0.050000	3	2596	5	5	18	21
6	0.050000	6	2505	5	3	15	21
6	0.050000	14	21	5	1	8	21

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.050000	11	194	5	2	10	21
6	0.050000	11	146	5	2	10	21
6	0.050000	12	43	5	1	9	21
6	0.050000	13	75	5	1	9	21
6	0.050000	11	372	5	2	11	21
6	0.050000	10	579	5	3	12	21
6	0.050000	10	399	5	2	11	21
6	0.100000	12	144	5	3	10	21
6	0.100000	10	85	5	2	10	21
6	0.100000	11	423	5	2	11	21
6	0.100000	16	6	5	1	7	21
6	0.100000	12	58	5	1	9	21
6	0.100000	6	1236	5	5	15	21
6	0.100000	15	1	5	0	6	21
6	0.100000	3	1985	5	4	19	21
6	0.100000	10	462	5	2	12	21
6	0.100000	3	3685	5	5	18	21
6	0.100000	12	167	5	2	10	21
6	0.100000	9	423	5	2	12	21
6	0.100000	12	70	5	1	9	21
6	0.100000	6	1483	5	3	15	21
6	0.100000	10	122	5	2	10	21
6	0.100000	10	512	5	3	12	21
6	0.100000	6	2634	5	3	15	21
6	0.100000	10	116	5	3	11	21
6	0.100000	1	250	5	4	20	21
6	0.100000	11	373	5	2	11	21
6	0.100000	13	56	5	2	9	21
6	0.100000	15	7	5	1	7	21
6	0.100000	6	1633	5	3	15	21
6	0.100000	13	18	5	2	9	21
6	0.100000	5	4634	5	4	17	21
6	0.100000	13	122	5	3	10	21
6	0.100000	13	7	5	1	7	21
6	0.100000	11	183	5	2	10	21
6	0.100000	12	126	5	3	10	21
6	0.100000	14	7	5	1	8	21
6	0.100000	12	162	5	1	10	21
6	0.100000	10	310	5	3	12	21
6	0.100000	5	2549	5	4	16	21
6	0.100000	11	107	5	3	10	21
6	0.100000	8	836	5	2	13	21
6	0.100000	9	810	5	3	12	21
6	0.100000	7	1458	5	3	14	21
6	0.100000	10	55	5	2	10	21
6	0.100000	15	5	5	1	7	21
6	0.100000	12	209	5	2	10	21
6	0.100000	3	2232	5	5	18	21
6	0.100000	6	1600	5	3	15	21
6	0.100000	14	14	5	1	8	21

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.100000	11	194	5	2	10	21
6	0.100000	11	148	5	2	10	21
6	0.100000	12	43	5	1	9	21
6	0.100000	13	70	5	1	9	21
6	0.100000	9	225	5	2	11	21
6	0.100000	10	579	5	3	12	21
6	0.100000	10	399	5	2	11	21
6	0.150000	12	144	5	3	10	21
6	0.150000	12	142	5	2	10	21
6	0.150000	9	367	5	2	11	21
6	0.150000	15	7	5	1	7	21
6	0.150000	12	58	5	1	9	21
6	0.150000	6	106	5	5	15	21
6	0.150000	15	1	5	0	6	21
6	0.150000	3	1985	5	4	19	21
6	0.150000	8	210	5	2	12	21
6	0.150000	3	3393	5	5	18	21
6	0.150000	10	167	5	2	10	21
6	0.150000	9	174	5	2	12	21
6	0.150000	12	80	5	1	9	21
6	0.150000	6	858	5	3	15	21
6	0.150000	12	183	5	2	10	21
6	0.150000	10	492	5	3	12	21
6	0.150000	6	2682	5	3	15	21
6	0.150000	11	9	5	3	11	21
6	0.150000	1	24	5	4	20	21
6	0.150000	9	244	5	2	11	21
6	0.150000	12	44	5	2	9	21
6	0.150000	15	7	5	1	7	21
6	0.150000	6	1333	5	3	15	21
6	0.150000	12	9	5	2	9	21
6	0.150000	5	2568	5	4	17	21
6	0.150000	13	122	5	3	10	21
6	0.150000	13	7	5	1	7	21
6	0.150000	11	179	5	2	10	21
6	0.150000	11	2	5	3	10	21
6	0.150000	14	7	5	1	8	21
6	0.150000	12	165	5	1	10	21
6	0.150000	10	308	5	3	12	21
6	0.150000	5	2516	5	4	16	21
6	0.150000	13	113	5	3	10	21
6	0.150000	8	826	5	2	13	21
6	0.150000	9	675	5	3	12	21
6	0.150000	7	1482	5	3	14	21
6	0.150000	12	111	5	2	10	21
6	0.150000	14	6	5	1	7	21
6	0.150000	12	209	5	2	10	21
6	0.150000	3	523	5	5	18	21
6	0.150000	6	2530	5	3	15	21
6	0.150000	14	14	5	1	8	21

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.150000	11	166	5	2	10	21
6	0.150000	10	141	5	2	10	21
6	0.150000	11	43	5	1	9	21
6	0.150000	12	49	5	1	9	21
6	0.150000	11	331	5	2	11	21
6	0.150000	10	364	5	3	12	21
6	0.150000	10	399	5	2	11	21
6	0.200000	11	115	5	3	10	21
6	0.200000	12	72	5	2	10	21
6	0.200000	11	423	5	2	11	21
6	0.200000	14	7	5	1	7	21
6	0.200000	12	51	5	1	9	21
6	0.200000	6	106	5	5	15	21
6	0.200000	16	1	5	0	6	21
6	0.200000	3	1402	5	4	19	21
6	0.200000	10	412	5	2	12	21
6	0.200000	3	3996	5	5	18	21
6	0.200000	11	164	5	2	10	21
6	0.200000	9	410	5	2	12	21
6	0.200000	11	70	5	1	9	21
6	0.200000	6	1483	5	3	15	21
6	0.200000	12	164	5	2	10	21
6	0.200000	8	47	5	3	12	21
6	0.200000	6	2682	5	3	15	21
6	0.200000	11	116	5	3	11	21
6	0.200000	1	462	5	4	20	21
6	0.200000	11	373	5	2	11	21
6	0.200000	12	46	5	2	9	21
6	0.200000	14	5	5	1	7	21
6	0.200000	6	1599	5	3	15	21
6	0.200000	13	18	5	2	9	21
6	0.200000	5	4149	5	4	17	21
6	0.200000	11	122	5	3	10	21
6	0.200000	13	5	5	1	7	21
6	0.200000	11	183	5	2	10	21
6	0.200000	11	126	5	3	10	21
6	0.200000	13	6	5	1	8	21
6	0.200000	12	160	5	1	10	21
6	0.200000	10	156	5	3	12	21
6	0.200000	5	2509	5	4	16	21
6	0.200000	9	113	5	3	10	21
6	0.200000	8	826	5	2	13	21
6	0.200000	9	685	5	3	12	21
6	0.200000	7	726	5	3	14	21
6	0.200000	11	114	5	2	10	21
6	0.200000	13	4	5	1	7	21
6	0.200000	12	108	5	2	10	21
6	0.200000	3	2157	5	5	18	21
6	0.200000	6	2505	5	3	15	21
6	0.200000	10	14	5	1	8	21

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.200000	11	151	5	2	10	21
6	0.200000	11	143	5	2	10	21
6	0.200000	11	42	5	1	9	21
6	0.200000	13	61	5	1	9	21
6	0.200000	11	352	5	2	11	21
6	0.200000	10	579	5	3	12	21
6	0.200000	10	319	5	2	11	21
6	0.250000	10	58	5	3	10	21
6	0.250000	11	66	5	2	10	21
6	0.250000	11	419	5	2	11	21
6	0.250000	11	6	5	1	7	21
6	0.250000	11	9	5	1	9	21
6	0.250000	6	1236	5	5	15	21
6	0.250000	16	1	5	0	6	21
6	0.250000	3	1536	5	4	19	21
6	0.250000	10	389	5	2	12	21
6	0.250000	3	1332	5	5	18	21
6	0.250000	9	166	5	2	10	21
6	0.250000	9	429	5	2	12	21
6	0.250000	12	69	5	1	9	21
6	0.250000	6	1222	5	3	15	21
6	0.250000	12	108	5	2	10	21
6	0.250000	10	512	5	3	12	21
6	0.250000	6	2682	5	3	15	21
6	0.250000	11	115	5	3	11	21
6	0.250000	1	462	5	4	20	21
6	0.250000	10	358	5	2	11	21
6	0.250000	10	34	5	2	9	21
6	0.250000	14	7	5	1	7	21
6	0.250000	6	1633	5	3	15	21
6	0.250000	12	9	5	2	9	21
6	0.250000	5	2568	5	4	17	21
6	0.250000	13	2	5	3	10	21
6	0.250000	13	6	5	1	7	21
6	0.250000	11	132	5	2	10	21
6	0.250000	10	5	5	3	10	21
6	0.250000	12	5	5	1	8	21
6	0.250000	12	123	5	1	10	21
6	0.250000	10	311	5	3	12	21
6	0.250000	5	2509	5	4	16	21
6	0.250000	13	113	5	3	10	21
6	0.250000	8	836	5	2	13	21
6	0.250000	9	675	5	3	12	21
6	0.250000	7	1482	5	3	14	21
6	0.250000	12	136	5	2	10	21
6	0.250000	12	5	5	1	7	21
6	0.250000	10	171	5	2	10	21
6	0.250000	3	2130	5	5	18	21
6	0.250000	6	1369	5	3	15	21
6	0.250000	14	14	5	1	8	21

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.250000	11	138	5	2	10	21
6	0.250000	11	73	5	2	10	21
6	0.250000	12	43	5	1	9	21
6	0.250000	12	75	5	1	9	21
6	0.250000	9	100	5	2	11	21
6	0.250000	10	579	5	3	12	21
6	0.250000	10	283	5	2	11	21
6	0.300000	12	144	5	3	10	21
6	0.300000	9	85	5	2	10	21
6	0.300000	9	363	5	2	11	21
6	0.300000	14	7	5	1	7	21
6	0.300000	12	10	5	1	9	21
6	0.300000	6	106	5	5	15	21
6	0.300000	10	1	5	0	6	21
6	0.300000	3	1609	5	4	19	21
6	0.300000	10	401	5	2	12	21
6	0.300000	3	3393	5	5	18	21
6	0.300000	10	111	5	2	10	21
6	0.300000	9	429	5	2	12	21
6	0.300000	12	42	5	1	9	21
6	0.300000	6	1479	5	3	15	21
6	0.300000	12	178	5	2	10	21
6	0.300000	8	237	5	3	12	21
6	0.300000	6	2291	5	3	15	21
6	0.300000	10	88	5	3	11	21
6	0.300000	1	250	5	4	20	21
6	0.300000	11	342	5	2	11	21
6	0.300000	13	46	5	2	9	21
6	0.300000	13	7	5	1	7	21
6	0.300000	6	1602	5	3	15	21
6	0.300000	11	18	5	2	9	21
6	0.300000	5	4659	5	4	17	21
6	0.300000	9	58	5	3	10	21
6	0.300000	12	6	5	1	7	21
6	0.300000	12	132	5	2	10	21
6	0.300000	10	5	5	3	10	21
6	0.300000	14	4	5	1	8	21
6	0.300000	12	162	5	1	10	21
6	0.300000	7	30	5	3	12	21
6	0.300000	5	2549	5	4	16	21
6	0.300000	11	113	5	3	10	21
6	0.300000	8	736	5	2	13	21
6	0.300000	9	685	5	3	12	21
6	0.300000	7	724	5	3	14	21
6	0.300000	12	136	5	2	10	21
6	0.300000	15	6	5	1	7	21
6	0.300000	12	209	5	2	10	21
6	0.300000	3	2130	5	5	18	21
6	0.300000	6	1327	5	3	15	21
6	0.300000	14	13	5	1	8	21

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.300000	11	135	5	2	10	21
6	0.300000	11	148	5	2	10	21
6	0.300000	11	37	5	1	9	21
6	0.300000	12	70	5	1	9	21
6	0.300000	10	242	5	2	11	21
6	0.300000	10	365	5	3	12	21
6	0.300000	9	196	5	2	11	21
6	0.350000	10	58	5	3	10	21
6	0.350000	11	140	5	2	10	21
6	0.350000	11	75	5	2	11	21
6	0.350000	14	7	5	1	7	21
6	0.350000	12	56	5	1	9	21
6	0.350000	6	106	5	5	15	21
6	0.350000	12	1	5	0	6	21
6	0.350000	3	5539	5	4	19	21
6	0.350000	10	521	5	2	12	21
6	0.350000	3	283	5	5	18	21
6	0.350000	12	167	5	2	10	21
6	0.350000	9	429	5	2	12	21
6	0.350000	12	53	5	1	9	21
6	0.350000	6	858	5	3	15	21
6	0.350000	10	118	5	2	10	21
6	0.350000	10	511	5	3	12	21
6	0.350000	6	1521	5	3	15	21
6	0.350000	11	193	5	3	11	21
6	0.350000	1	213	5	4	20	21
6	0.350000	11	367	5	2	11	21
6	0.350000	11	45	5	2	9	21
6	0.350000	15	3	5	1	7	21
6	0.350000	6	1633	5	3	15	21
6	0.350000	13	18	5	2	9	21
6	0.350000	6	2543	5	4	17	21
6	0.350000	11	122	5	3	10	21
6	0.350000	12	4	5	1	7	21
6	0.350000	10	69	5	2	10	21
6	0.350000	11	17	5	3	10	21
6	0.350000	13	5	5	1	8	21
6	0.350000	10	67	5	1	10	21
6	0.350000	10	32	5	3	12	21
6	0.350000	5	797	5	4	16	21
6	0.350000	11	58	5	3	10	21
6	0.350000	8	370	5	2	13	21
6	0.350000	9	320	5	3	12	21
6	0.350000	7	1268	5	3	14	21
6	0.350000	12	86	5	2	10	21
6	0.350000	14	5	5	1	7	21
6	0.350000	10	171	5	2	10	21
6	0.350000	3	2445	5	5	18	21
6	0.350000	6	1173	5	3	15	21
6	0.350000	13	13	5	1	8	21

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pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.350000	11	155	5	2	10	21
6	0.350000	11	141	5	2	10	21
6	0.350000	11	43	5	1	9	21
6	0.350000	13	75	5	1	9	21
6	0.350000	9	176	5	2	11	21
6	0.350000	10	564	5	3	12	21
6	0.350000	10	183	5	2	11	21
6	0.400000	12	109	5	3	10	21
6	0.400000	9	7	5	2	10	21
6	0.400000	11	102	5	2	11	21
6	0.400000	13	7	5	1	7	21
6	0.400000	11	56	5	1	9	21
6	0.400000	6	1236	5	5	15	21
6	0.400000	16	1	5	0	6	21
6	0.400000	3	932	5	4	19	21
6	0.400000	10	461	5	2	12	21
6	0.400000	3	1165	5	5	18	21
6	0.400000	10	110	5	2	10	21
6	0.400000	9	154	5	2	12	21
6	0.400000	12	52	5	1	9	21
6	0.400000	6	1274	5	3	15	21
6	0.400000	10	122	5	2	10	21
6	0.400000	10	284	5	3	12	21
6	0.400000	6	2453	5	3	15	21
6	0.400000	11	191	5	3	11	21
6	0.400000	1	250	5	4	20	21
6	0.400000	11	367	5	2	11	21
6	0.400000	10	23	5	2	9	21
6	0.400000	12	6	5	1	7	21
6	0.400000	6	862	5	3	15	21
6	0.400000	11	3	5	2	9	21
6	0.400000	5	2568	5	4	17	21
6	0.400000	11	86	5	3	10	21
6	0.400000	12	6	5	1	7	21
6	0.400000	11	200	5	2	10	21
6	0.400000	9	2	5	3	10	21
6	0.400000	14	5	5	1	8	21
6	0.400000	10	67	5	1	10	21
6	0.400000	8	32	5	3	12	21
6	0.400000	5	2542	5	4	16	21
6	0.400000	10	78	5	3	10	21
6	0.400000	7	746	5	2	13	21
6	0.400000	9	287	5	3	12	21
6	0.400000	7	632	5	3	14	21
6	0.400000	10	55	5	2	10	21
6	0.400000	13	6	5	1	7	21
6	0.400000	10	102	5	2	10	21
6	0.400000	3	276	5	5	18	21
6	0.400000	6	1558	5	3	15	21
6	0.400000	13	14	5	1	8	21

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pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.400000	10	96	5	2	10	21
6	0.400000	10	71	5	2	10	21
6	0.400000	11	21	5	1	9	21
6	0.400000	13	39	5	1	9	21
6	0.400000	9	219	5	2	11	21
6	0.400000	10	578	5	3	12	21
6	0.400000	10	186	5	2	11	21
6	0.450000	12	114	5	3	10	21
6	0.450000	11	141	5	2	10	21
6	0.450000	9	140	5	2	11	21
6	0.450000	11	6	5	1	7	21
6	0.450000	12	10	5	1	9	21
6	0.450000	6	1236	5	5	15	21
6	0.450000	14	1	5	0	6	21
6	0.450000	3	1923	5	4	19	21
6	0.450000	9	395	5	2	12	21
6	0.450000	4	3208	5	5	18	21
6	0.450000	9	164	5	2	10	21
6	0.450000	9	168	5	2	12	21
6	0.450000	10	64	5	1	9	21
6	0.450000	6	594	5	3	15	21
6	0.450000	12	175	5	2	10	21
6	0.450000	9	509	5	3	12	21
6	0.450000	6	1146	5	3	15	21
6	0.450000	9	115	5	3	11	21
6	0.450000	3	3598	5	4	20	21
6	0.450000	11	342	5	2	11	21
6	0.450000	10	24	5	2	9	21
6	0.450000	11	7	5	1	7	21
6	0.450000	6	874	5	3	15	21
6	0.450000	11	9	5	2	9	21
6	0.450000	5	4634	5	4	17	21
6	0.450000	11	122	5	3	10	21
6	0.450000	13	7	5	1	7	21
6	0.450000	10	172	5	2	10	21
6	0.450000	12	2	5	3	10	21
6	0.450000	13	6	5	1	8	21
6	0.450000	10	81	5	1	10	21
6	0.450000	8	35	5	3	12	21
6	0.450000	5	797	5	4	16	21
6	0.450000	10	72	5	3	10	21
6	0.450000	8	826	5	2	13	21
6	0.450000	9	319	5	3	12	21
6	0.450000	7	1482	5	3	14	21
6	0.450000	10	53	5	2	10	21
6	0.450000	12	5	5	1	7	21
6	0.450000	12	201	5	2	10	21
6	0.450000	3	2471	5	5	18	21
6	0.450000	6	2514	5	3	15	21
6	0.450000	13	13	5	1	8	21

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pl	p	diff	n_path	E	largest_step	n_skipped	N
6	0.450000	11	128	5	2	10	21
6	0.450000	11	76	5	2	10	21
6	0.450000	11	21	5	1	9	21
6	0.450000	9	33	5	1	9	21
6	0.450000	9	226	5	2	11	21
6	0.450000	10	570	5	3	12	21
6	0.450000	10	173	5	2	11	21
7	0.000000	17	84	6	4	12	28
7	0.000000	15	881	6	2	13	28
7	0.000000	17	172	6	3	12	28
7	0.000000	8	19568	6	6	21	28
7	0.000000	15	280	6	2	13	28
7	0.000000	18	69	6	1	10	28
7	0.000000	16	819	6	3	13	28
7	0.000000	3	64214	6	6	25	28
7	0.000000	15	838	6	4	13	28
7	0.000000	6	60137	6	6	22	28
7	0.000000	13	4242	6	4	15	28
7	0.000000	17	448	6	2	12	28
7	0.000000	5	25867	6	6	23	28
7	0.000000	17	291	6	2	11	28
7	0.000000	15	709	6	3	13	28
7	0.000000	18	288	6	4	11	28
7	0.000000	13	4041	6	3	15	28
7	0.000000	16	620	6	3	12	28
7	0.000000	14	1442	6	2	14	28
7	0.000000	15	888	6	3	14	28
7	0.000000	19	27	6	1	9	28
7	0.000000	18	102	6	1	10	28
7	0.000000	19	23	6	1	10	28
7	0.000000	20	23	6	1	9	28
7	0.000000	15	1590	6	4	14	28
7	0.000000	5	73125	6	6	23	28
7	0.000000	18	112	6	2	10	28
7	0.000000	17	112	6	2	11	28
7	0.000000	4	45596	6	6	25	28
7	0.000000	15	1315	6	2	13	28
7	0.000000	18	91	6	1	10	28
7	0.000000	17	452	6	2	12	28
7	0.000000	18	112	6	3	10	28
7	0.000000	17	721	6	2	12	28
7	0.000000	18	108	6	1	10	28
7	0.000000	18	201	6	1	11	28
7	0.000000	14	4511	6	4	15	28
7	0.000000	18	198	6	1	11	28
7	0.000000	18	284	6	2	11	28
7	0.000000	16	259	6	4	12	28
7	0.000000	19	28	6	1	9	28
7	0.000000	17	294	6	1	11	28
7	0.000000	7	59670	6	6	22	28

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pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.000000	19	112	6	1	10	28
7	0.000000	16	774	6	2	12	28
7	0.000000	18	84	6	2	10	28
7	0.000000	18	307	6	2	11	28
7	0.000000	13	4652	6	4	15	28
7	0.000000	16	1384	6	4	13	28
7	0.000000	16	458	6	2	12	28
7	0.050000	17	84	6	4	12	28
7	0.050000	15	881	6	2	13	28
7	0.050000	17	172	6	3	12	28
7	0.050000	8	19552	6	6	21	28
7	0.050000	15	280	6	2	13	28
7	0.050000	18	69	6	1	10	28
7	0.050000	14	819	6	3	13	28
7	0.050000	3	64214	6	6	25	28
7	0.050000	15	838	6	4	13	28
7	0.050000	6	60137	6	6	22	28
7	0.050000	13	3891	6	4	15	28
7	0.050000	17	448	6	2	12	28
7	0.050000	5	24362	6	6	23	28
7	0.050000	17	291	6	2	11	28
7	0.050000	14	709	6	3	13	28
7	0.050000	18	288	6	4	11	28
7	0.050000	13	4041	6	3	15	28
7	0.050000	16	458	6	3	12	28
7	0.050000	14	1442	6	2	14	28
7	0.050000	15	806	6	3	14	28
7	0.050000	19	27	6	1	9	28
7	0.050000	18	102	6	1	10	28
7	0.050000	19	23	6	1	10	28
7	0.050000	20	23	6	1	9	28
7	0.050000	15	1590	6	4	14	28
7	0.050000	5	72646	6	6	23	28
7	0.050000	18	97	6	2	10	28
7	0.050000	16	106	6	2	11	28
7	0.050000	4	45596	6	6	25	28
7	0.050000	15	1315	6	2	13	28
7	0.050000	18	91	6	1	10	28
7	0.050000	17	452	6	2	12	28
7	0.050000	18	112	6	3	10	28
7	0.050000	16	538	6	2	12	28
7	0.050000	17	108	6	1	10	28
7	0.050000	18	197	6	1	11	28
7	0.050000	14	4511	6	4	15	28
7	0.050000	18	198	6	1	11	28
7	0.050000	18	284	6	2	11	28
7	0.050000	15	259	6	4	12	28
7	0.050000	19	28	6	1	9	28
7	0.050000	17	294	6	1	11	28
7	0.050000	7	59670	6	6	22	28

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.050000	19	97	6	1	10	28
7	0.050000	16	774	6	2	12	28
7	0.050000	18	84	6	2	10	28
7	0.050000	16	161	6	2	11	28
7	0.050000	13	4641	6	4	15	28
7	0.050000	16	1384	6	4	13	28
7	0.050000	15	423	6	2	12	28
7	0.100000	17	84	6	4	12	28
7	0.100000	15	881	6	2	13	28
7	0.100000	16	87	6	3	12	28
7	0.100000	8	12829	6	6	21	28
7	0.100000	15	71	6	2	13	28
7	0.100000	18	41	6	1	10	28
7	0.100000	16	819	6	3	13	28
7	0.100000	3	60796	6	6	25	28
7	0.100000	15	838	6	4	13	28
7	0.100000	6	60137	6	6	22	28
7	0.100000	13	2473	6	4	15	28
7	0.100000	17	310	6	2	12	28
7	0.100000	5	25867	6	6	23	28
7	0.100000	17	291	6	2	11	28
7	0.100000	15	709	6	3	13	28
7	0.100000	18	288	6	4	11	28
7	0.100000	13	4041	6	3	15	28
7	0.100000	16	458	6	3	12	28
7	0.100000	14	981	6	2	14	28
7	0.100000	15	888	6	3	14	28
7	0.100000	19	26	6	1	9	28
7	0.100000	18	100	6	1	10	28
7	0.100000	19	23	6	1	10	28
7	0.100000	20	23	6	1	9	28
7	0.100000	15	1571	6	4	14	28
7	0.100000	5	73125	6	6	23	28
7	0.100000	18	55	6	2	10	28
7	0.100000	17	112	6	2	11	28
7	0.100000	4	45596	6	6	25	28
7	0.100000	15	1315	6	2	13	28
7	0.100000	18	91	6	1	10	28
7	0.100000	17	452	6	2	12	28
7	0.100000	17	112	6	3	10	28
7	0.100000	17	704	6	2	12	28
7	0.100000	18	108	6	1	10	28
7	0.100000	18	201	6	1	11	28
7	0.100000	14	3786	6	4	15	28
7	0.100000	18	198	6	1	11	28
7	0.100000	18	200	6	2	11	28
7	0.100000	15	259	6	4	12	28
7	0.100000	18	27	6	1	9	28
7	0.100000	17	294	6	1	11	28
7	0.100000	7	59670	6	6	22	28

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.100000	18	97	6	1	10	28
7	0.100000	16	711	6	2	12	28
7	0.100000	18	84	6	2	10	28
7	0.100000	17	307	6	2	11	28
7	0.100000	13	4652	6	4	15	28
7	0.100000	16	1132	6	4	13	28
7	0.100000	16	458	6	2	12	28
7	0.150000	17	84	6	4	12	28
7	0.150000	15	1441	6	2	13	28
7	0.150000	17	172	6	3	12	28
7	0.150000	8	19568	6	6	21	28
7	0.150000	14	279	6	2	13	28
7	0.150000	18	68	6	1	10	28
7	0.150000	15	819	6	3	13	28
7	0.150000	3	64214	6	6	25	28
7	0.150000	15	836	6	4	13	28
7	0.150000	6	49439	6	6	22	28
7	0.150000	13	764	6	4	15	28
7	0.150000	15	446	6	2	12	28
7	0.150000	5	25867	6	6	23	28
7	0.150000	17	291	6	2	11	28
7	0.150000	14	708	6	3	13	28
7	0.150000	18	120	6	4	11	28
7	0.150000	13	4040	6	3	15	28
7	0.150000	16	620	6	3	12	28
7	0.150000	14	1442	6	2	14	28
7	0.150000	15	888	6	3	14	28
7	0.150000	18	27	6	1	9	28
7	0.150000	17	102	6	1	10	28
7	0.150000	19	23	6	1	10	28
7	0.150000	19	21	6	1	9	28
7	0.150000	15	1072	6	4	14	28
7	0.150000	5	39803	6	6	23	28
7	0.150000	18	112	6	2	10	28
7	0.150000	16	106	6	2	11	28
7	0.150000	4	47599	6	6	25	28
7	0.150000	15	1063	6	2	13	28
7	0.150000	17	91	6	1	10	28
7	0.150000	15	452	6	2	12	28
7	0.150000	17	112	6	3	10	28
7	0.150000	17	721	6	2	12	28
7	0.150000	18	18	6	1	10	28
7	0.150000	18	201	6	1	11	28
7	0.150000	14	4367	6	4	15	28
7	0.150000	17	187	6	1	11	28
7	0.150000	18	284	6	2	11	28
7	0.150000	15	259	6	4	12	28
7	0.150000	18	28	6	1	9	28
7	0.150000	17	294	6	1	11	28
7	0.150000	7	59135	6	6	22	28

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pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.150000	18	112	6	1	10	28
7	0.150000	16	416	6	2	12	28
7	0.150000	18	84	6	2	10	28
7	0.150000	18	307	6	2	11	28
7	0.150000	13	3961	6	4	15	28
7	0.150000	14	793	6	4	13	28
7	0.150000	16	458	6	2	12	28
7	0.200000	17	82	6	4	12	28
7	0.200000	15	1399	6	2	13	28
7	0.200000	17	172	6	3	12	28
7	0.200000	8	19321	6	6	21	28
7	0.200000	15	1035	6	2	13	28
7	0.200000	18	68	6	1	10	28
7	0.200000	16	819	6	3	13	28
7	0.200000	3	46391	6	6	25	28
7	0.200000	15	815	6	4	13	28
7	0.200000	6	49439	6	6	22	28
7	0.200000	13	4242	6	4	15	28
7	0.200000	15	411	6	2	12	28
7	0.200000	5	24594	6	6	23	28
7	0.200000	17	291	6	2	11	28
7	0.200000	15	709	6	3	13	28
7	0.200000	18	288	6	4	11	28
7	0.200000	13	4040	6	3	15	28
7	0.200000	16	620	6	3	12	28
7	0.200000	14	1441	6	2	14	28
7	0.200000	15	742	6	3	14	28
7	0.200000	18	6	6	1	9	28
7	0.200000	18	100	6	1	10	28
7	0.200000	19	21	6	1	10	28
7	0.200000	19	14	6	1	9	28
7	0.200000	15	1489	6	4	14	28
7	0.200000	5	72802	6	6	23	28
7	0.200000	18	112	6	2	10	28
7	0.200000	16	106	6	2	11	28
7	0.200000	4	20419	6	6	25	28
7	0.200000	15	1113	6	2	13	28
7	0.200000	18	91	6	1	10	28
7	0.200000	17	452	6	2	12	28
7	0.200000	17	112	6	3	10	28
7	0.200000	17	554	6	2	12	28
7	0.200000	17	52	6	1	10	28
7	0.200000	16	201	6	1	11	28
7	0.200000	14	4501	6	4	15	28
7	0.200000	18	198	6	1	11	28
7	0.200000	17	266	6	2	11	28
7	0.200000	16	217	6	4	12	28
7	0.200000	18	28	6	1	9	28
7	0.200000	17	294	6	1	11	28
7	0.200000	7	59664	6	6	22	28

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.200000	18	41	6	1	10	28
7	0.200000	16	416	6	2	12	28
7	0.200000	18	82	6	2	10	28
7	0.200000	18	305	6	2	11	28
7	0.200000	13	4594	6	4	15	28
7	0.200000	16	1132	6	4	13	28
7	0.200000	16	494	6	2	12	28
7	0.250000	15	81	6	4	12	28
7	0.250000	15	593	6	2	13	28
7	0.250000	17	66	6	3	12	28
7	0.250000	8	19507	6	6	21	28
7	0.250000	15	1035	6	2	13	28
7	0.250000	18	69	6	1	10	28
7	0.250000	16	687	6	3	13	28
7	0.250000	3	30306	6	6	25	28
7	0.250000	15	793	6	4	13	28
7	0.250000	6	49598	6	6	22	28
7	0.250000	13	3891	6	4	15	28
7	0.250000	17	737	6	2	12	28
7	0.250000	5	25867	6	6	23	28
7	0.250000	15	25	6	2	11	28
7	0.250000	13	709	6	3	13	28
7	0.250000	17	204	6	4	11	28
7	0.250000	13	2678	6	3	15	28
7	0.250000	16	325	6	3	12	28
7	0.250000	14	978	6	2	14	28
7	0.250000	15	803	6	3	14	28
7	0.250000	18	6	6	1	9	28
7	0.250000	18	44	6	1	10	28
7	0.250000	19	21	6	1	10	28
7	0.250000	17	21	6	1	9	28
7	0.250000	15	1466	6	4	14	28
7	0.250000	5	54094	6	6	23	28
7	0.250000	18	92	6	2	10	28
7	0.250000	15	28	6	2	11	28
7	0.250000	4	45224	6	6	25	28
7	0.250000	15	1208	6	2	13	28
7	0.250000	18	22	6	1	10	28
7	0.250000	17	413	6	2	12	28
7	0.250000	18	112	6	3	10	28
7	0.250000	16	721	6	2	12	28
7	0.250000	17	52	6	1	10	28
7	0.250000	16	201	6	1	11	28
7	0.250000	14	3779	6	4	15	28
7	0.250000	16	190	6	1	11	28
7	0.250000	18	294	6	2	11	28
7	0.250000	16	259	6	4	12	28
7	0.250000	17	28	6	1	9	28
7	0.250000	17	231	6	1	11	28
7	0.250000	7	59670	6	6	22	28

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pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.250000	16	85	6	1	10	28
7	0.250000	16	711	6	2	12	28
7	0.250000	18	84	6	2	10	28
7	0.250000	17	245	6	2	11	28
7	0.250000	13	2079	6	4	15	28
7	0.250000	16	1384	6	4	13	28
7	0.250000	15	423	6	2	12	28
7	0.300000	16	7	6	4	12	28
7	0.300000	15	448	6	2	13	28
7	0.300000	16	102	6	3	12	28
7	0.300000	8	19549	6	6	21	28
7	0.300000	15	951	6	2	13	28
7	0.300000	17	68	6	1	10	28
7	0.300000	16	817	6	3	13	28
7	0.300000	3	55750	6	6	25	28
7	0.300000	15	755	6	4	13	28
7	0.300000	6	31723	6	6	22	28
7	0.300000	13	944	6	4	15	28
7	0.300000	17	315	6	2	12	28
7	0.300000	5	3693	6	6	23	28
7	0.300000	15	25	6	2	11	28
7	0.300000	14	709	6	3	13	28
7	0.300000	16	260	6	4	11	28
7	0.300000	13	2332	6	3	15	28
7	0.300000	16	515	6	3	12	28
7	0.300000	13	2702	6	2	14	28
7	0.300000	15	718	6	3	14	28
7	0.300000	18	6	6	1	9	28
7	0.300000	16	109	6	1	10	28
7	0.300000	16	15	6	1	10	28
7	0.300000	19	14	6	1	9	28
7	0.300000	15	2751	6	4	14	28
7	0.300000	5	72122	6	6	23	28
7	0.300000	17	89	6	2	10	28
7	0.300000	17	99	6	2	11	28
7	0.300000	2	20577	6	6	25	28
7	0.300000	15	1315	6	2	13	28
7	0.300000	18	76	6	1	10	28
7	0.300000	15	252	6	2	12	28
7	0.300000	17	112	6	3	10	28
7	0.300000	16	553	6	2	12	28
7	0.300000	17	108	6	1	10	28
7	0.300000	17	225	6	1	11	28
7	0.300000	14	4367	6	4	15	28
7	0.300000	18	198	6	1	11	28
7	0.300000	16	284	6	2	11	28
7	0.300000	15	259	6	4	12	28
7	0.300000	17	28	6	1	9	28
7	0.300000	17	228	6	1	11	28
7	0.300000	5	31256	6	6	22	28

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pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.300000	17	69	6	1	10	28
7	0.300000	16	745	6	2	12	28
7	0.300000	18	76	6	2	10	28
7	0.300000	18	273	6	2	11	28
7	0.300000	13	958	6	4	15	28
7	0.300000	14	893	6	4	13	28
7	0.300000	16	494	6	2	12	28
7	0.350000	17	84	6	4	12	28
7	0.350000	15	593	6	2	13	28
7	0.350000	16	87	6	3	12	28
7	0.350000	6	19568	6	6	21	28
7	0.350000	12	69	6	2	13	28
7	0.350000	17	41	6	1	10	28
7	0.350000	14	812	6	3	13	28
7	0.350000	4	29826	6	6	25	28
7	0.350000	15	770	6	4	13	28
7	0.350000	6	49439	6	6	22	28
7	0.350000	13	477	6	4	15	28
7	0.350000	15	291	6	2	12	28
7	0.350000	5	26267	6	6	23	28
7	0.350000	16	137	6	2	11	28
7	0.350000	14	709	6	3	13	28
7	0.350000	17	204	6	4	11	28
7	0.350000	13	2719	6	3	15	28
7	0.350000	15	430	6	3	12	28
7	0.350000	13	1441	6	2	14	28
7	0.350000	15	364	6	3	14	28
7	0.350000	19	27	6	1	9	28
7	0.350000	18	66	6	1	10	28
7	0.350000	18	48	6	1	10	28
7	0.350000	19	21	6	1	9	28
7	0.350000	14	2975	6	4	14	28
7	0.350000	5	72802	6	6	23	28
7	0.350000	18	95	6	2	10	28
7	0.350000	17	107	6	2	11	28
7	0.350000	4	20520	6	6	25	28
7	0.350000	15	922	6	2	13	28
7	0.350000	17	35	6	1	10	28
7	0.350000	17	251	6	2	12	28
7	0.350000	17	104	6	3	10	28
7	0.350000	14	285	6	2	12	28
7	0.350000	17	37	6	1	10	28
7	0.350000	15	201	6	1	11	28
7	0.350000	14	4511	6	4	15	28
7	0.350000	15	187	6	1	11	28
7	0.350000	16	118	6	2	11	28
7	0.350000	14	256	6	4	12	28
7	0.350000	18	27	6	1	9	28
7	0.350000	16	228	6	1	11	28
7	0.350000	7	51798	6	6	22	28

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pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.350000	18	112	6	1	10	28
7	0.350000	15	753	6	2	12	28
7	0.350000	18	71	6	2	10	28
7	0.350000	16	201	6	2	11	28
7	0.350000	13	3339	6	4	15	28
7	0.350000	16	1384	6	4	13	28
7	0.350000	14	329	6	2	12	28
7	0.400000	15	3	6	4	12	28
7	0.400000	15	593	6	2	13	28
7	0.400000	16	172	6	3	12	28
7	0.400000	8	19276	6	6	21	28
7	0.400000	14	277	6	2	13	28
7	0.400000	17	69	6	1	10	28
7	0.400000	15	230	6	3	13	28
7	0.400000	3	42096	6	6	25	28
7	0.400000	15	816	6	4	13	28
7	0.400000	6	20324	6	6	22	28
7	0.400000	13	390	6	4	15	28
7	0.400000	15	447	6	2	12	28
7	0.400000	5	25885	6	6	23	28
7	0.400000	17	21	6	2	11	28
7	0.400000	15	653	6	3	13	28
7	0.400000	15	260	6	4	11	28
7	0.400000	13	2679	6	3	15	28
7	0.400000	16	514	6	3	12	28
7	0.400000	13	978	6	2	14	28
7	0.400000	15	284	6	3	14	28
7	0.400000	19	27	6	1	9	28
7	0.400000	17	100	6	1	10	28
7	0.400000	16	20	6	1	10	28
7	0.400000	18	15	6	1	9	28
7	0.400000	15	558	6	4	14	28
7	0.400000	5	72566	6	6	23	28
7	0.400000	18	56	6	2	10	28
7	0.400000	17	112	6	2	11	28
7	0.400000	4	46016	6	6	25	28
7	0.400000	15	849	6	2	13	28
7	0.400000	17	75	6	1	10	28
7	0.400000	15	252	6	2	12	28
7	0.400000	17	77	6	3	10	28
7	0.400000	15	705	6	2	12	28
7	0.400000	18	30	6	1	10	28
7	0.400000	15	201	6	1	11	28
7	0.400000	14	3781	6	4	15	28
7	0.400000	17	225	6	1	11	28
7	0.400000	16	132	6	2	11	28
7	0.400000	14	217	6	4	12	28
7	0.400000	18	18	6	1	9	28
7	0.400000	17	217	6	1	11	28
7	0.400000	7	50150	6	6	22	28

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pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.400000	18	54	6	1	10	28
7	0.400000	16	288	6	2	12	28
7	0.400000	18	76	6	2	10	28
7	0.400000	17	306	6	2	11	28
7	0.400000	13	2941	6	4	15	28
7	0.400000	14	903	6	4	13	28
7	0.400000	15	455	6	2	12	28
7	0.450000	17	123	6	4	12	28
7	0.450000	15	249	6	2	13	28
7	0.450000	15	87	6	3	12	28
7	0.450000	8	12521	6	6	21	28
7	0.450000	13	277	6	2	13	28
7	0.450000	18	41	6	1	10	28
7	0.450000	14	812	6	3	13	28
7	0.450000	4	6868	6	6	25	28
7	0.450000	14	801	6	4	13	28
7	0.450000	6	20175	6	6	22	28
7	0.450000	13	2471	6	4	15	28
7	0.450000	17	285	6	2	12	28
7	0.450000	5	24445	6	6	23	28
7	0.450000	17	16	6	2	11	28
7	0.450000	15	653	6	3	13	28
7	0.450000	17	120	6	4	11	28
7	0.450000	12	2594	6	3	15	28
7	0.450000	15	220	6	3	12	28
7	0.450000	13	1428	6	2	14	28
7	0.450000	15	882	6	3	14	28
7	0.450000	18	6	6	1	9	28
7	0.450000	18	66	6	1	10	28
7	0.450000	18	17	6	1	10	28
7	0.450000	16	22	6	1	9	28
7	0.450000	15	2975	6	4	14	28
7	0.450000	5	72078	6	6	23	28
7	0.450000	18	112	6	2	10	28
7	0.450000	16	19	6	2	11	28
7	0.450000	4	37298	6	6	25	28
7	0.450000	15	460	6	2	13	28
7	0.450000	18	74	6	1	10	28
7	0.450000	15	284	6	2	12	28
7	0.450000	17	109	6	3	10	28
7	0.450000	14	301	6	2	12	28
7	0.450000	17	9	6	1	10	28
7	0.450000	15	201	6	1	11	28
7	0.450000	14	3633	6	4	15	28
7	0.450000	17	185	6	1	11	28
7	0.450000	17	119	6	2	11	28
7	0.450000	14	256	6	4	12	28
7	0.450000	18	28	6	1	9	28
7	0.450000	17	293	6	1	11	28
7	0.450000	7	51226	6	6	22	28

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
7	0.450000	16	20	6	1	10	28
7	0.450000	16	398	6	2	12	28
7	0.450000	18	91	6	2	10	28
7	0.450000	16	194	6	2	11	28
7	0.450000	13	1630	6	4	15	28
7	0.450000	16	1083	6	4	13	28
7	0.450000	15	423	6	2	12	28
8	0.000000	15	109175	7	7	21	36
8	0.000000	19	5747	7	4	17	36
8	0.000000	28	3995	7	2	16	36
8	0.000000	15	102311	7	4	22	36
8	0.000000	28	2484	7	4	16	36
8	0.000000	26	451	7	3	14	36
8	0.000000	24	98	7	2	12	36
8	0.000000	26	700	7	2	14	36
8	0.000000	27	1071	7	3	15	36
8	0.000000	14	243826	7	4	23	36
8	0.000000	20	6131	7	3	17	36
8	0.000000	25	176	7	4	13	36
8	0.000000	26	576	7	3	14	36
8	0.000000	28	2509	7	3	16	36
8	0.000000	24	66	7	2	12	36
8	0.000000	25	65	7	1	11	36
8	0.000000	27	927	7	2	15	36
8	0.000000	14	88229	7	4	23	36
8	0.000000	25	209	7	2	12	36
8	0.000000	26	46	7	1	11	36
8	0.000000	27	1453	7	2	15	36
8	0.000000	19	9079	7	2	17	36
8	0.000000	28	3536	7	2	16	36
8	0.000000	29	1	7	0	8	36
8	0.000000	17	45074	7	7	19	36
8	0.000000	26	358	7	3	14	36
8	0.000000	15	136731	7	7	22	36
8	0.000000	15	76986	7	4	21	36
8	0.000000	26	34	7	1	10	36
8	0.000000	24	55	7	3	12	36
8	0.000000	15	52040	7	4	21	36
8	0.000000	25	176	7	3	13	36
8	0.000000	28	3654	7	2	16	36
8	0.000000	19	11228	7	3	17	36
8	0.000000	19	27267	7	5	18	36
8	0.000000	24	21	7	2	12	36
8	0.000000	27	1820	7	3	15	36
8	0.000000	26	27	7	1	10	36
8	0.000000	26	275	7	2	14	36
8	0.000000	28	2869	7	2	16	36
8	0.000000	16	95024	7	6	21	36
8	0.000000	26	670	7	2	14	36
8	0.000000	17	32901	7	3	19	36

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
8	0.000000	28	1975	7	3	16	36
8	0.000000	16	71146	7	6	21	36
8	0.000000	19	30205	7	2	18	36
8	0.000000	17	26244	7	7	19	36
8	0.000000	27	1268	7	3	15	36
8	0.000000	25	505	7	2	13	36
8	0.000000	17	26032	7	3	19	36
8	0.050000	15	109175	7	7	21	36
8	0.050000	19	2742	7	4	17	36
8	0.050000	28	3995	7	2	16	36
8	0.050000	15	102311	7	4	22	36
8	0.050000	29	5513	7	4	16	36
8	0.050000	26	451	7	3	14	36
8	0.050000	26	267	7	2	12	36
8	0.050000	26	700	7	2	14	36
8	0.050000	27	1071	7	3	15	36
8	0.050000	14	242819	7	4	23	36
8	0.050000	20	6131	7	3	17	36
8	0.050000	26	264	7	4	13	36
8	0.050000	26	576	7	3	14	36
8	0.050000	30	7233	7	3	16	36
8	0.050000	24	66	7	2	12	36
8	0.050000	25	65	7	1	11	36
8	0.050000	27	927	7	2	15	36
8	0.050000	14	60677	7	4	23	36
8	0.050000	25	209	7	2	12	36
8	0.050000	26	46	7	1	11	36
8	0.050000	28	2474	7	2	15	36
8	0.050000	19	9079	7	2	17	36
8	0.050000	29	5826	7	2	16	36
8	0.050000	29	1	7	0	8	36
8	0.050000	17	45074	7	7	19	36
8	0.050000	26	358	7	3	14	36
8	0.050000	15	136675	7	7	22	36
8	0.050000	15	75106	7	4	21	36
8	0.050000	25	32	7	1	10	36
8	0.050000	24	55	7	3	12	36
8	0.050000	15	52040	7	4	21	36
8	0.050000	25	176	7	3	13	36
8	0.050000	29	4788	7	2	16	36
8	0.050000	19	11228	7	3	17	36
8	0.050000	19	27267	7	5	18	36
8	0.050000	24	21	7	2	12	36
8	0.050000	27	1820	7	3	15	36
8	0.050000	26	27	7	1	10	36
8	0.050000	26	275	7	2	14	36
8	0.050000	28	2869	7	2	16	36
8	0.050000	16	95024	7	6	21	36
8	0.050000	26	670	7	2	14	36
8	0.050000	17	32901	7	3	19	36

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
8	0.050000	28	1975	7	3	16	36
8	0.050000	16	71146	7	6	21	36
8	0.050000	19	30205	7	2	18	36
8	0.050000	17	26244	7	7	19	36
8	0.050000	27	1268	7	3	15	36
8	0.050000	25	505	7	2	13	36
8	0.050000	17	26032	7	3	19	36
8	0.100000	15	61362	7	7	21	36
8	0.100000	19	5747	7	4	17	36
8	0.100000	29	5011	7	2	16	36
8	0.100000	15	102311	7	4	22	36
8	0.100000	28	2484	7	4	16	36
8	0.100000	26	451	7	3	14	36
8	0.100000	25	144	7	2	12	36
8	0.100000	28	1357	7	2	14	36
8	0.100000	27	1071	7	3	15	36
8	0.100000	14	243826	7	4	23	36
8	0.100000	20	5751	7	3	17	36
8	0.100000	26	300	7	4	13	36
8	0.100000	27	1434	7	3	14	36
8	0.100000	29	4370	7	3	16	36
8	0.100000	24	66	7	2	12	36
8	0.100000	25	65	7	1	11	36
8	0.100000	27	927	7	2	15	36
8	0.100000	14	88229	7	4	23	36
8	0.100000	25	209	7	2	12	36
8	0.100000	26	59	7	1	11	36
8	0.100000	27	1542	7	2	15	36
8	0.100000	19	9079	7	2	17	36
8	0.100000	28	3536	7	2	16	36
8	0.100000	29	1	7	0	8	36
8	0.100000	17	45074	7	7	19	36
8	0.100000	27	722	7	3	14	36
8	0.100000	15	136731	7	7	22	36
8	0.100000	15	75106	7	4	21	36
8	0.100000	26	34	7	1	10	36
8	0.100000	24	55	7	3	12	36
8	0.100000	15	52040	7	4	21	36
8	0.100000	27	614	7	3	13	36
8	0.100000	28	3654	7	2	16	36
8	0.100000	19	4094	7	3	17	36
8	0.100000	19	27267	7	5	18	36
8	0.100000	24	21	7	2	12	36
8	0.100000	27	1820	7	3	15	36
8	0.100000	25	30	7	1	10	36
8	0.100000	26	275	7	2	14	36
8	0.100000	28	2869	7	2	16	36
8	0.100000	16	95024	7	6	21	36
8	0.100000	26	714	7	2	14	36
8	0.100000	17	32901	7	3	19	36

*Continued on next page*

pl	p	diff	n_path	E	largest_step	n_skipped	N
8	0.100000	29	3487	7	3	16	36
8	0.100000	16	71146	7	6	21	36
8	0.100000	19	30205	7	2	18	36
8	0.100000	17	26244	7	7	19	36
8	0.100000	29	3231	7	3	15	36
8	0.100000	27	866	7	2	13	36
8	0.100000	17	26032	7	3	19	36
8	0.150000	15	92487	7	7	21	36
8	0.150000	19	5740	7	4	17	36
8	0.150000	28	3995	7	2	16	36
8	0.150000	15	98522	7	4	22	36
8	0.150000	29	5536	7	4	16	36
8	0.150000	26	508	7	3	14	36
8	0.150000	25	144	7	2	12	36
8	0.150000	27	773	7	2	14	36
8	0.150000	30	3710	7	3	15	36
8	0.150000	14	243826	7	4	23	36
8	0.150000	20	5732	7	3	17	36
8	0.150000	25	143	7	4	13	36
8	0.150000	26	576	7	3	14	36
8	0.150000	32	6599	7	3	16	36
8	0.150000	25	128	7	2	12	36
8	0.150000	24	78	7	1	11	36
8	0.150000	28	2606	7	2	15	36
8	0.150000	14	88087	7	4	23	36
8	0.150000	25	157	7	2	12	36
8	0.150000	26	46	7	1	11	36
8	0.150000	29	3162	7	2	15	36
8	0.150000	19	9069	7	2	17	36
8	0.150000	28	3536	7	2	16	36
8	0.150000	29	1	7	0	8	36
8	0.150000	17	45039	7	7	19	36
8	0.150000	26	358	7	3	14	36
8	0.150000	15	72561	7	7	22	36
8	0.150000	15	76671	7	4	21	36
8	0.150000	25	32	7	1	10	36
8	0.150000	24	65	7	3	12	36
8	0.150000	15	52040	7	4	21	36
8	0.150000	25	238	7	3	13	36
8	0.150000	28	3654	7	2	16	36
8	0.150000	19	4094	7	3	17	36
8	0.150000	19	23003	7	5	18	36
8	0.150000	25	125	7	2	12	36
8	0.150000	29	3338	7	3	15	36
8	0.150000	24	20	7	1	10	36
8	0.150000	26	275	7	2	14	36
8	0.150000	30	5533	7	2	16	36
8	0.150000	16	95024	7	6	21	36
8	0.150000	27	853	7	2	14	36
8	0.150000	17	32853	7	3	19	36

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pl	p	diff	n_path	E	largest_step	n_skipped	N
8	0.150000	30	6362	7	3	16	36
8	0.150000	16	69936	7	6	21	36
8	0.150000	19	29435	7	2	18	36
8	0.150000	17	14291	7	7	19	36
8	0.150000	27	1268	7	3	15	36
8	0.150000	26	801	7	2	13	36
8	0.150000	17	26032	7	3	19	36
8	0.200000	15	108979	7	7	21	36
8	0.200000	19	1232	7	4	17	36
8	0.200000	29	5011	7	2	16	36
8	0.200000	15	39538	7	4	22	36
8	0.200000	29	3840	7	4	16	36
8	0.200000	29	1985	7	3	14	36
8	0.200000	25	152	7	2	12	36
8	0.200000	26	720	7	2	14	36
8	0.200000	29	2780	7	3	15	36
8	0.200000	14	243814	7	4	23	36
8	0.200000	20	5179	7	3	17	36
8	0.200000	25	176	7	4	13	36
8	0.200000	26	576	7	3	14	36
8	0.200000	28	2509	7	3	16	36
8	0.200000	25	143	7	2	12	36
8	0.200000	24	38	7	1	11	36
8	0.200000	29	3023	7	2	15	36
8	0.200000	12	60677	7	4	23	36
8	0.200000	25	147	7	2	12	36
8	0.200000	26	51	7	1	11	36
8	0.200000	29	3658	7	2	15	36
8	0.200000	19	8998	7	2	17	36
8	0.200000	28	3585	7	2	16	36
8	0.200000	26	1	7	0	8	36
8	0.200000	17	45038	7	7	19	36
8	0.200000	26	416	7	3	14	36
8	0.200000	15	72551	7	7	22	36
8	0.200000	15	76983	7	4	21	36
8	0.200000	25	28	7	1	10	36
8	0.200000	24	65	7	3	12	36
8	0.200000	15	44542	7	4	21	36
8	0.200000	25	238	7	3	13	36
8	0.200000	29	5707	7	2	16	36
8	0.200000	19	11172	7	3	17	36
8	0.200000	17	15891	7	5	18	36
8	0.200000	25	12	7	2	12	36
8	0.200000	27	1820	7	3	15	36
8	0.200000	26	12	7	1	10	36
8	0.200000	27	732	7	2	14	36
8	0.200000	29	5031	7	2	16	36
8	0.200000	16	84985	7	6	21	36
8	0.200000	27	1159	7	2	14	36
8	0.200000	17	13740	7	3	19	36

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pl	p	diff	n_path	E	largest_step	n_skipped	N
8	0.200000	30	4546	7	3	16	36
8	0.200000	16	71146	7	6	21	36
8	0.200000	19	29558	7	2	18	36
8	0.200000	17	26243	7	7	19	36
8	0.200000	27	1268	7	3	15	36
8	0.200000	26	867	7	2	13	36
8	0.200000	17	17620	7	3	19	36
8	0.250000	15	92291	7	7	21	36
8	0.250000	19	1827	7	4	17	36
8	0.250000	28	4765	7	2	16	36
8	0.250000	15	101841	7	4	22	36
8	0.250000	30	5113	7	4	16	36
8	0.250000	27	1117	7	3	14	36
8	0.250000	25	160	7	2	12	36
8	0.250000	28	1167	7	2	14	36
8	0.250000	29	2096	7	3	15	36
8	0.250000	14	158076	7	4	23	36
8	0.250000	20	5835	7	3	17	36
8	0.250000	27	377	7	4	13	36
8	0.250000	26	576	7	3	14	36
8	0.250000	29	5086	7	3	16	36
8	0.250000	24	51	7	2	12	36
8	0.250000	24	37	7	1	11	36
8	0.250000	28	2502	7	2	15	36
8	0.250000	12	87689	7	4	23	36
8	0.250000	25	209	7	2	12	36
8	0.250000	26	46	7	1	11	36
8	0.250000	30	3137	7	2	15	36
8	0.250000	19	4673	7	2	17	36
8	0.250000	29	5189	7	2	16	36
8	0.250000	27	1	7	0	8	36
8	0.250000	17	45074	7	7	19	36
8	0.250000	28	1241	7	3	14	36
8	0.250000	13	65519	7	7	22	36
8	0.250000	15	12621	7	4	21	36
8	0.250000	25	34	7	1	10	36
8	0.250000	24	110	7	3	12	36
8	0.250000	15	52040	7	4	21	36
8	0.250000	27	569	7	3	13	36
8	0.250000	29	5228	7	2	16	36
8	0.250000	19	11228	7	3	17	36
8	0.250000	17	15888	7	5	18	36
8	0.250000	25	57	7	2	12	36
8	0.250000	29	3674	7	3	15	36
8	0.250000	26	27	7	1	10	36
8	0.250000	26	275	7	2	14	36
8	0.250000	29	5431	7	2	16	36
8	0.250000	16	86491	7	6	21	36
8	0.250000	30	1387	7	2	14	36
8	0.250000	17	31445	7	3	19	36

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pl	p	diff	n_path	E	largest_step	n_skipped	N
8	0.250000	29	3487	7	3	16	36
8	0.250000	16	67976	7	6	21	36
8	0.250000	19	29558	7	2	18	36
8	0.250000	17	26205	7	7	19	36
8	0.250000	28	2923	7	3	15	36
8	0.250000	25	565	7	2	13	36
8	0.250000	17	25259	7	3	19	36
8	0.300000	15	30902	7	7	21	36
8	0.300000	19	5738	7	4	17	36
8	0.300000	30	7787	7	2	16	36
8	0.300000	15	102311	7	4	22	36
8	0.300000	30	7375	7	4	16	36
8	0.300000	27	1096	7	3	14	36
8	0.300000	26	191	7	2	12	36
8	0.300000	28	1189	7	2	14	36
8	0.300000	29	3346	7	3	15	36
8	0.300000	14	158056	7	4	23	36
8	0.300000	20	5835	7	3	17	36
8	0.300000	27	391	7	4	13	36
8	0.300000	26	576	7	3	14	36
8	0.300000	32	9907	7	3	16	36
8	0.300000	24	45	7	2	12	36
8	0.300000	25	108	7	1	11	36
8	0.300000	28	2502	7	2	15	36
8	0.300000	12	87864	7	4	23	36
8	0.300000	26	238	7	2	12	36
8	0.300000	24	20	7	1	11	36
8	0.300000	28	2760	7	2	15	36
8	0.300000	19	4285	7	2	17	36
8	0.300000	30	7633	7	2	16	36
8	0.300000	24	1	7	0	8	36
8	0.300000	17	45074	7	7	19	36
8	0.300000	27	890	7	3	14	36
8	0.300000	15	136631	7	7	22	36
8	0.300000	15	76658	7	4	21	36
8	0.300000	26	17	7	1	10	36
8	0.300000	25	180	7	3	12	36
8	0.300000	15	52039	7	4	21	36
8	0.300000	26	373	7	3	13	36
8	0.300000	30	6911	7	2	16	36
8	0.300000	19	10002	7	3	17	36
8	0.300000	19	22933	7	5	18	36
8	0.300000	25	137	7	2	12	36
8	0.300000	28	2425	7	3	15	36
8	0.300000	25	32	7	1	10	36
8	0.300000	30	1758	7	2	14	36
8	0.300000	29	5431	7	2	16	36
8	0.300000	16	86775	7	6	21	36
8	0.300000	27	948	7	2	14	36
8	0.300000	17	23227	7	3	19	36

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<b>pl</b>	<b>p</b>	<b>diff</b>	<b>n_path</b>	<b>E</b>	<b>largest_step</b>	<b>n_skipped</b>	<b>N</b>
8	0.300000	29	3830	7	3	16	36
8	0.300000	14	58526	7	6	21	36
8	0.300000	19	30174	7	2	18	36
8	0.300000	17	19162	7	7	19	36
8	0.300000	28	2747	7	3	15	36
8	0.300000	25	550	7	2	13	36
8	0.300000	17	13484	7	3	19	36
8	0.350000	15	109043	7	7	21	36
8	0.350000	19	4835	7	4	17	36
8	0.350000	30	7848	7	2	16	36
8	0.350000	15	98624	7	4	22	36
8	0.350000	30	7375	7	4	16	36
8	0.350000	29	1880	7	3	14	36
8	0.350000	25	219	7	2	12	36
8	0.350000	27	1399	7	2	14	36
8	0.350000	29	3327	7	3	15	36
8	0.350000	12	45016	7	4	23	36
8	0.350000	18	5541	7	3	17	36
8	0.350000	27	468	7	4	13	36
8	0.350000	29	1709	7	3	14	36
8	0.350000	32	6254	7	3	16	36
8	0.350000	25	157	7	2	12	36
8	0.350000	25	108	7	1	11	36
8	0.350000	29	3088	7	2	15	36
8	0.350000	12	44148	7	4	23	36
8	0.350000	25	218	7	2	12	36
8	0.350000	24	28	7	1	11	36
8	0.350000	29	3078	7	2	15	36
8	0.350000	19	8964	7	2	17	36
8	0.350000	31	6456	7	2	16	36
8	0.350000	28	1	7	0	8	36
8	0.350000	17	26811	7	7	19	36
8	0.350000	26	358	7	3	14	36
8	0.350000	13	460	7	7	22	36
8	0.350000	14	75106	7	4	21	36
8	0.350000	24	20	7	1	10	36
8	0.350000	25	66	7	3	12	36
8	0.350000	15	43561	7	4	21	36
8	0.350000	26	580	7	3	13	36
8	0.350000	31	6989	7	2	16	36
8	0.350000	19	9946	7	3	17	36
8	0.350000	17	9543	7	5	18	36
8	0.350000	25	114	7	2	12	36
8	0.350000	28	1918	7	3	15	36
8	0.350000	25	2	7	1	10	36
8	0.350000	31	1500	7	2	14	36
8	0.350000	29	6615	7	2	16	36
8	0.350000	16	94992	7	6	21	36
8	0.350000	26	774	7	2	14	36
8	0.350000	17	5913	7	3	19	36

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pl	p	diff	n_path	E	largest_step	n_skipped	N
8	0.350000	32	9714	7	3	16	36
8	0.350000	16	52742	7	6	21	36
8	0.350000	17	2567	7	2	18	36
8	0.350000	17	26244	7	7	19	36
8	0.350000	30	3354	7	3	15	36
8	0.350000	27	637	7	2	13	36
8	0.350000	17	25296	7	3	19	36
8	0.400000	15	30968	7	7	21	36
8	0.400000	19	1060	7	4	17	36
8	0.400000	30	9565	7	2	16	36
8	0.400000	15	36296	7	4	22	36
8	0.400000	29	3840	7	4	16	36
8	0.400000	27	917	7	3	14	36
8	0.400000	24	106	7	2	12	36
8	0.400000	29	1377	7	2	14	36
8	0.400000	32	3741	7	3	15	36
8	0.400000	12	141195	7	4	23	36
8	0.400000	18	5224	7	3	17	36
8	0.400000	27	450	7	4	13	36
8	0.400000	30	1966	7	3	14	36
8	0.400000	31	8874	7	3	16	36
8	0.400000	28	207	7	2	12	36
8	0.400000	25	100	7	1	11	36
8	0.400000	28	1331	7	2	15	36
8	0.400000	14	72638	7	4	23	36
8	0.400000	26	293	7	2	12	36
8	0.400000	25	34	7	1	11	36
8	0.400000	30	3116	7	2	15	36
8	0.400000	19	4250	7	2	17	36
8	0.400000	32	6812	7	2	16	36
8	0.400000	28	1	7	0	8	36
8	0.400000	17	14425	7	7	19	36
8	0.400000	30	1583	7	3	14	36
8	0.400000	15	136693	7	7	22	36
8	0.400000	15	62167	7	4	21	36
8	0.400000	24	17	7	1	10	36
8	0.400000	24	16	7	3	12	36
8	0.400000	15	52040	7	4	21	36
8	0.400000	29	781	7	3	13	36
8	0.400000	18	2015	7	2	16	36
8	0.400000	19	9898	7	3	17	36
8	0.400000	19	20761	7	5	18	36
8	0.400000	29	355	7	2	12	36
8	0.400000	29	3338	7	3	15	36
8	0.400000	24	4	7	1	10	36
8	0.400000	28	1058	7	2	14	36
8	0.400000	30	6700	7	2	16	36
8	0.400000	16	82555	7	6	21	36
8	0.400000	27	777	7	2	14	36
8	0.400000	17	13264	7	3	19	36

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pl	p	diff	n_path	E	largest_step	n_skipped	N
8	0.400000	30	6345	7	3	16	36
8	0.400000	16	69856	7	6	21	36
8	0.400000	19	15067	7	2	18	36
8	0.400000	17	26200	7	7	19	36
8	0.400000	30	3195	7	3	15	36
8	0.400000	28	616	7	2	13	36
8	0.400000	17	21065	7	3	19	36
8	0.450000	15	95400	7	7	21	36
8	0.450000	19	2693	7	4	17	36
8	0.450000	32	10223	7	2	16	36
8	0.450000	13	101834	7	4	22	36
8	0.450000	29	4013	7	4	16	36
8	0.450000	31	2089	7	3	14	36
8	0.450000	25	311	7	2	12	36
8	0.450000	29	1397	7	2	14	36
8	0.450000	29	3055	7	3	15	36
8	0.450000	14	239850	7	4	23	36
8	0.450000	20	6087	7	3	17	36
8	0.450000	27	423	7	4	13	36
8	0.450000	27	1242	7	3	14	36
8	0.450000	31	6294	7	3	16	36
8	0.450000	24	83	7	2	12	36
8	0.450000	24	65	7	1	11	36
8	0.450000	30	3322	7	2	15	36
8	0.450000	12	87308	7	4	23	36
8	0.450000	28	314	7	2	12	36
8	0.450000	24	19	7	1	11	36
8	0.450000	29	3526	7	2	15	36
8	0.450000	19	5053	7	2	17	36
8	0.450000	31	7753	7	2	16	36
8	0.450000	24	1	7	0	8	36
8	0.450000	17	27514	7	7	19	36
8	0.450000	30	1646	7	3	14	36
8	0.450000	15	71748	7	7	22	36
8	0.450000	15	14254	7	4	21	36
8	0.450000	24	28	7	1	10	36
8	0.450000	26	311	7	3	12	36
8	0.450000	15	14204	7	4	21	36
8	0.450000	26	549	7	3	13	36
8	0.450000	33	7353	7	2	16	36
8	0.450000	19	1983	7	3	17	36
8	0.450000	19	22882	7	5	18	36
8	0.450000	27	339	7	2	12	36
8	0.450000	30	3710	7	3	15	36
8	0.450000	25	11	7	1	10	36
8	0.450000	27	607	7	2	14	36
8	0.450000	18	5124	7	2	16	36
8	0.450000	14	38355	7	6	21	36
8	0.450000	28	1562	7	2	14	36
8	0.450000	17	13717	7	3	19	36

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pl	p	diff	n_path	E	largest_step	n_skipped	N
8	0.450000	30	6345	7	3	16	36
8	0.450000	16	41332	7	6	21	36
8	0.450000	17	18796	7	2	18	36
8	0.450000	17	26239	7	7	19	36
8	0.450000	30	3567	7	3	15	36
8	0.450000	27	637	7	2	13	36
8	0.450000	17	23129	7	3	19	36